Bayesian Estimation of Sparse Smooth Speckle Shape Models for Motion Tracking in Medical Ultrasound

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***Abstract*— Emerging ultrasound phased-array technologies will soon enable the acquisition of high-resolution 3D+T images for medical applications. Processing the huge amount of spatiotemporal measurements remains a practical challenge. In this work, dynamic ultrasound images are sparsely represented by a mixture of moving speckles. We model the shape of a speckle and its locally linear motion with a weighted multivariate Gaussian kernel. Parameters of the model are estimated with online Bayesian learning from a stream of random measurements. In our preliminary experiments with a simulated phantom of a moving cylindrical structure, the optical flow of speckles is estimated for a vertical line profile and compared to the ground truth. The mean accuracy of the linear motion estimate is of 93.53%, using only a statistically sufficient random subset of the data.**

1 Introduction

Measuring strain in structural soft tissue, such as tendons, could be an invaluable diagnostic tool for physicians [1]. For example, in orthopedics, measuring strain in ligaments will assist the placement procedure of joint implants and ultimately decrease the number of follow-up procedures [2]. Recent developments in ultrasound (US) imaging allows for the acquisition of high resolution volumetric images. This technology enables researchers to leverage existing imaging techniques such as elastography [3] or model-based biomechanical simulations of deformation [4] to the problem of assessing strain in tendons. Unfortunatelly, these techniques are not yet capable of capturing local motion occurring in small scale soft tissues.

In this work, we aim at adapting the speckle tracking echocardiography (STE) technique [5] from cardiology to the prescribed situation. Therefore, we propose a novel method for directly estimating motion of a sparse selection of speckle shape models instead of using detection and explicit tracking [6]. Speckles are forming the characteristic granular appearance in US images, which is due to deterministic interference originating from sub-wavelength scattering sites [7]. We approximate speckles and their local linear motion with a mixture of weighted multivariate Gaussian kernels. A Bayesian method is used for parametric estimation of the mixture components, which are representing the optical flow that is traced by the trajectories of speckles. Preliminary results are shown for simulated one-dimensional vertical line profiles.

2 Data and Image Models

We collect echography measurements in a dense discrete spatiotemporal amplitude field $A(x,t)$, with $x \in \{1,\dots,N_x\}$ and $t \in \{1,\dots,N_t\}$. This matrix of real positive amplitude values

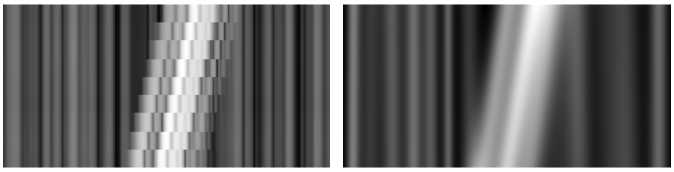


Figure 1: Optical flow images of the input data consisting of 2597 echo measurements over 9 discrete time frames (left) and the smooth continuous spatiotemporal reconstruction using 16 multivariate mixture components (right).

is first normalized for every time frame t such that

$$\sum_{i=1}^{N_x} A(x_i,t) = 1,\ \forall t \in \{1,\dots,N_t\}.$$

We approximate $A(x,t)$ with a continuous and smooth conditional Gaussian mixture model $G(x,t)$ having the form

$$A(x,t) \approx G(x,t) = \sum_{k=1}^{K} w_k\ \frac{g(x,t;\Theta_k)}{g(t;\Theta_k)},$$

with the set of model parameters

$$\Theta = \{w_k,\Theta_k\}_{k=1}^{K} \quad \text{with} \quad \Theta_k = \left(\mu,\tau,\sigma_x^2,\sigma_t^2,\sigma_{xt}\right)_k$$

where $g(x,t;\Theta_k)$ can be decomposed as the product of a marginal (temporal) and the conditional (spatial) distributions:

$$g(x,t;\Theta_k) = g(t;\tau,\sigma_t^2) \times g(x;\bar{\mu},\bar{\sigma}_x^2),$$

where the parameters of the conditional distribution are

$$\bar{\mu} = \mu + \frac{\sigma_{xt}}{\sigma_t^2}(t-\tau) \ \text{ and } \ \bar{\sigma}_x^2 = \sigma_x^2 - \frac{\sigma_{xt}}{\sigma_t^2}\sigma_x^2,$$

and the Gaussian kernel $g(x;\mu,\sigma^2)$ is defined by

$$g(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right).$$

Before estimating the mixture parameters, we first sample $A(x,t)$ by taking a subset of $M \ll N_x \times N_t$ amplitude values

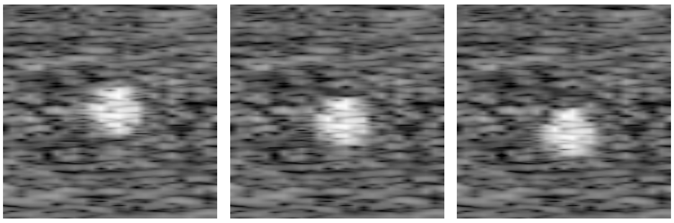
$$D = \{(x_m,t_m,a_m)\}_{m=1}^{M} \ \text{ with } \ a_m = A(\lfloor x_m \rfloor,\lfloor t_m \rfloor)$$

where (x_m,t_m) are drawn from the two-dimensional uniform random distribution $\mathcal{U}(1,N_x)\times\mathcal{U}(1,N_t)$.

3 Reconstruction

The set Θ of all weights and parameters of the mixture model $G(x,t)$ is now estimated by maximizing the log-likelihood

$$L(\Theta;D) = P(D;\Theta) = \sum_{m=1}^{M}\log\sum_{k=1}^{K} w_k\ \frac{g(x_m,t_m;\Theta_k)}{g(t_m;\Theta_k)}.$$



No displacement 1mm displacement 2mm displacement

Figure 2: Three selected frames of the moving cylinder phantom. The total course of the cylinder was 2mm long, such that the measured global displacement of the center of mass was 1.308mm.

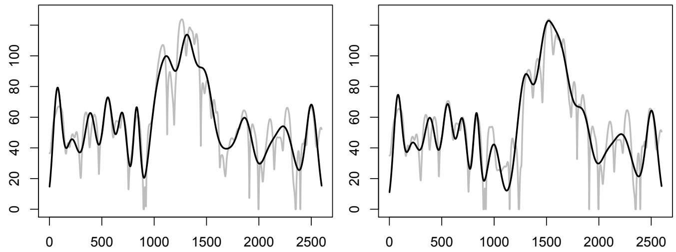


Figure 3: Plot of original (gray) and reconstructed (black) vertical line profiles for two selected time frames corresponding to the start (left) and end (right) of dynamic data acquisition. The sparse reconstruction uses 16 kernels.

We use the non-iterative online expectation-maximization (EM) method [8, 9] for estimating the parameters. Online EM alternates between performing an expectation (E-step) which evaluates membership probabilities for the log-likelihood using the current estimate of the parameters, and a maximization (M-step), which updates parameters for maximizing the expected log-likelihood found in the E-step. The model is first initialized with a regular grid, then the two E- and M- steps are performed successively for every sample, converging to the final model.

E-step When updating the model from the information carried by the m-th sample (x_m, t_m, a_m), we first compute the membership probability $p_k(x_m|t_m)$, expressing the probability that the sample is drawn from the conditional distribution of the component k at given time t_m using

$$p_k(x_m|t_m) = \frac{w_k\ g(x_m; \bar{\mu}, \bar{\sigma}_x^2)}{\sum_{k=1}^{K} w_k\ g(x_m; \bar{\mu}, \bar{\sigma}_x^2)},$$

where the parameters $\bar{\mu}$ and $\bar{\sigma}_x^2$ of the conditional distribution are calculated as previously described.

M-step In the second step, the parameters are updated such that the likelihood of observing the new sample is improved by applying the following update rules:

$$\begin{cases} w_k \leftarrow w_k\,(1+\alpha), \\ \mu \leftarrow \mu + \delta_x\,\alpha, \\ \tau \leftarrow \tau + \delta_t\,\alpha, \end{cases} \qquad \begin{cases} \sigma_x^2 \leftarrow (\sigma_x^2 + \delta_x^2\,\alpha)\,(1-\alpha), \\ \sigma_t^2 \leftarrow (\sigma_t^2 + \delta_t^2\,\alpha)\,(1-\alpha), \\ \sigma_{xt} \leftarrow (\sigma_{xt} + \delta_x\,\delta_t\,\alpha)\,(1-\alpha), \end{cases}$$

for each $k \in \{1, \ldots, K\}$ with

$$\alpha = \frac{p_k(x_m|t_m)\,a_m}{w_k} \quad \text{and} \quad \begin{cases} \delta_x = x_m - \mu_k, \\ \delta_t = t_m - \tau_k. \end{cases}$$

Initialization We initialize each component of a given index k with the following weight $w_k = N_t/K$ and parameters

$$\mu = (k - \tfrac{1}{2})\,\tfrac{N_x}{K},\ \tau = \tfrac{N_t}{2}, \sigma_x^2 = \tfrac{N_x^2}{12K^2},\ \sigma_t^2 = \tfrac{N_t^2}{12},\ \sigma_{xt} = 0.$$

As presented, the weights are systematically incremented for every new sample in the M-step. In order to adapt smoothly to new data, a "forgetting" mechanism should be used to adjust parameters to the recent sufficient statistics. We simply apply a multiplicative decay factor $\gamma = 1 - 1/N_t$ to every component such that the total mixture weight remains normalized to N_t.

4 Experiment

For this paper we use the Field II program [7] in order to simulate RF-signals from a linear US probe consisting of 192 (64 active) elements. To this end a 3D phantom is generated containing a 10mm long high-intensity cylinder of 4mm radius. The whole numerical phantom consists of 5000 uniformly distributed and amplitude-weighted random scatterers for a region of $30 \times 30 \times 10\text{mm}^3$. This cylinder is then consecutively moved downwards by 8 consecutive steps of 0.25mm along the vertical axis, while the background is left unaltered.

For the simulation, the probe is positioned 15mm from the phantom emitting a 3.5Mhz pulse focused at a depth of 30mm. Afterwards the return signals is sampled at a frequency of 100Mhz. Using this method a total of 25 RF lines are created on which we perform log envelope detection. The images shown in Figure 2 show three temporal frames of the input data after downsampling along the axial dimension by a factor 10.

For the experiment we created a 2D optical flow image $A(x, t)$ using the central vertical line profile line at all $N_t = 9$ temporal steps which is shown in Figure 1. The number of amplitude measurements for the profile lines is $N_x = 2597$, the number of mixture components was set to $K = 16$, and the average number of samples used for statistical estimations of each component was set to was set to $N = 512$ such that the total number of samples was $M = NK = 8192$.

Reconstructed line profiles in Figure 3 show that the linear motion of speckles covering the cylinder is successfully captured with 16 mixture components. The center of mass in the data shifted by 1.308mm between the start and end of the simulation. The reconstructed displacement was of 1.224mm, reaching a relative error of 6.47% in this experiment. Similar experiments have been conducted with 8 and 32 mixture components. The motion extraction was not accurate with only 8 components, while we obtained comparable results with 32 components. We observed that the simulated speckle pattern slightly deforms over time while following the displacement path of the object. This effect has not yet been accounted for.

5 Conclusion

We represent dynamic ultrasound images as a mixture of moving speckle shape models that are approximated by smooth weighted multivariate Gaussian kernels. Maximum likelihood values of these kernel parameters can therefore be estimated by a conditional expectation-maximization method, using simple closed-form update rules. Experiments have been conducted on a line profile of a simple cylinder phantom. We observed that the sparse reconstruction approximates smoothly the acquired data. The subtle motion of the moving structure on a static background was successfully captured in the model. Future work will consider modeling the space-variant point spread function of speckles and extend the method to 3D+T imaging for assessing results on clinical data.

Acknowledgment

This work is supported by the iMinds ICON 3DUS project and the VUB Strategic Research Programme M^3D^2. S. Bundervoet is supported by the grant nr.131813 from the Agency for Innovation by Science and Technology in Flanders (IWT).

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