Total Variation Reconstruction From Quasi-Random Samples

ABSTRACT

Pseudo-random numbers are often used for generating incoherent uniformly distributed sample distributions.

Observation: *Randomness is a sufficient but not necessary condition to ensure incoherence.*

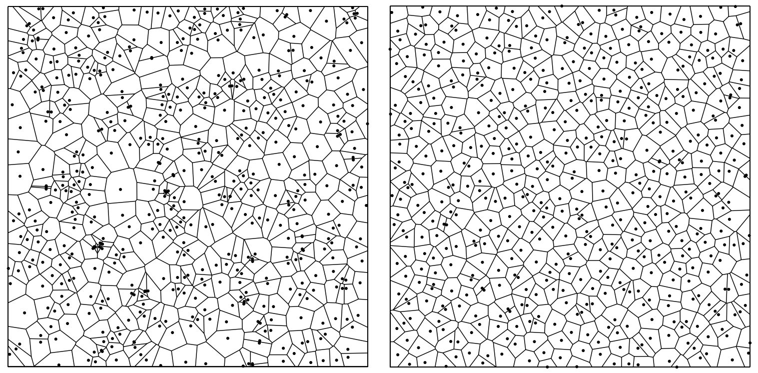
Thesis: *A globally optimized set of evenly distributed points could capture the visual content more efficiently to reconstruct (interpolate) images from few samples.*

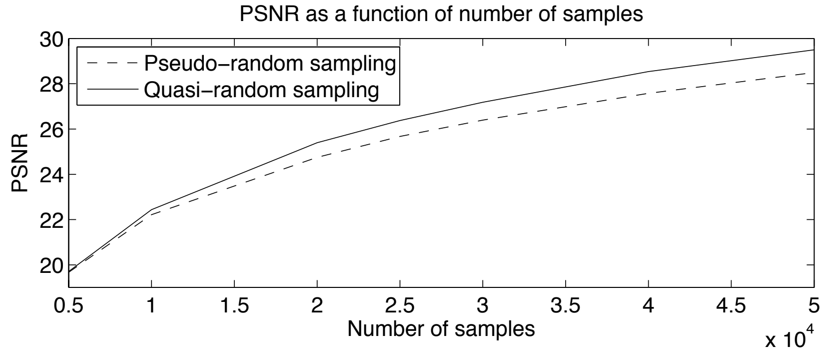
Method: *We compare classical random sampling with a simple construction based on properties of the fractional Golden ratio sequence and the Hilbert space filling curve. Images are reconstructed using a TV prior.*

Results: *We observed improvements in terms of peak signal to noise ratio over pseudo-random sampling.*

Pseudo-random | Quasi-random | Original

Pseudo-random samples | Quasi-random samples





QUASI-RANDOM SAMPLING

Elements of the fractional golden ratio sequence $G_s(i)$ with given seed constant $s \in [0,1)$ are

$$G_s(i) = \{s + i \cdot \phi\},\ \forall\, i \geq 1\,, \quad \text{with} \quad \phi = \frac{1+\sqrt{5}}{2} \tag{1}$$

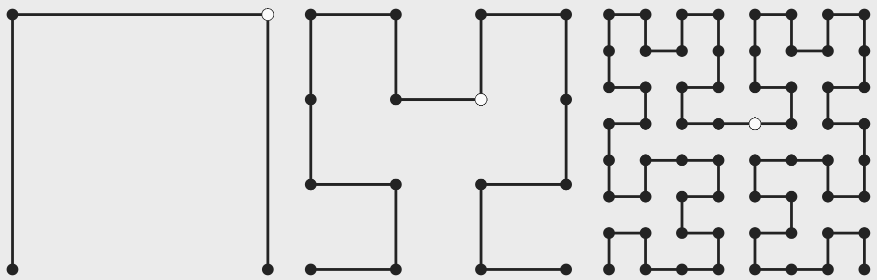
where $\{t\}$ is the fractional part of the real number t.

The Hilbert space filling curve $H(t) = (x, y)$, $t \in [0,1)$ defines a nested recursive grid and a locally-preserving traversal order of grid elements. Plugin-in $G_s(\cdot)$ in $H(\cdot)$, we obtain the following sets of N uniformly distributed points:

$$H(G_s(i)) = H(\{s + i \cdot \tau\}),\ i \in [1 \dots N]. \tag{2}$$

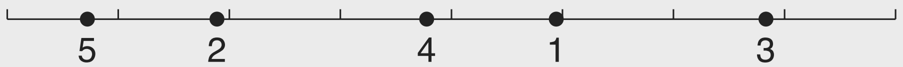
A key corollary of the strong irrationality of ϕ is that the fractional parts of integer multiples will not align on any regular grid.

HILBERT SPACE FILLING CURVE



1st order SFC | 2nd order SFC | 3rd order SFC

Coordinates of the golden ratio sequence (below) are transformed from the unit interval to the unit square with the Hilbert space filling curve.



TOTAL VARIATION MINIMIZATION

Based on samples y of some given image ($y = Su^{\text{orig}}$), the reconstructed image u^{rec} minimizes the total variation and coincides with u^{orig} in the chosen samples:

$$u^{\text{rec}} = \underset{Su=y}{\arg\min}\ \text{TV}(u). \tag{1}$$

Here the total variation $\text{TV}(u)$ is defined in terms of the local differences (image gradients) $D(u) \equiv (D_{\text{h}}u, D_{\text{v}}u) \in \mathbb{R}^2$:

$$\text{TV}(u) = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \sqrt{(D_{\text{h}}u_{i,j})^2 + (D_{\text{v}}u_{i,j})^2}. \tag{2}$$

We use an iterative scheme to compute the minimum TV solution.

Colas Schretter, Ignace Loris, Ann Dooms and Peter Schelkens

iTWIST' 14, Namur, Belgium - August 27-29 2014

iMinds CONNECT.INNOVATE.CREATE
Vrije Universiteit Brussel
ULB UNIVERSITÉ LIBRE DE BRUXELLES