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GRANTISM: An Excel^{TM} model for Greenland and Antarctic ice-sheet response to climate changes

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Abstract

Over the last decades, the response of large ice sheets on Earth, such as the Greenland and Antarctic ice sheets, to changes in climate has been successfully simulated with large-scale numerical ice-sheet models. Since these models are highly sophisticated, they are only applicable on the scientific level as they demand a large amount of CPU time. Based on similar physics, a computationally fast flowline model of the Greenland and Antarctic ice sheet is presented here, primarily designed for educational purposes. Using an over-implicit numerical scheme, the model runs fast and behaves in a similar way to changes in background temperature forcing as major ice-sheet models do. A user-friendly interface and the implementation within a common spreadsheet program (Excel^{TM}) make the model suitable for the classroom.

Keywords: Antarctica; Greenland; Ice-sheet model; Spreadsheet; Climate change

1. Introduction

The Antarctic and Greenland ice sheets are the two largest ice masses on Earth. The Antarctic ice sheet covers an area of more than $13 \times 10^6\,\mathrm{km}^2$ and contains more than 80% of the world's fresh-water supply. The Greenland ice sheet—the Northern Hemisphere counterpart of Antarctica—is ten times smaller and has a shorter mass turnover time, so that it has a faster response time to environmental change. Due to the generally long time scales involved, i.e. longer than direct observations can account for, numerical modelling is a vital tool for understanding and predicting the past, current and future behaviour of these ice masses. Nowadays, sophisticated three-dimensional thermomechanical ice-sheet models are capable of explaining and predicting the waxing and waning of the Greenland and Antarctic ice sheet under changing environmental conditions with a relatively high degree of confidence. Such large-scale models are based on continuum modelling and encompass conservation laws of mass, momentum and energy. Examples for such thermomechanical models are Budd and Smith (1982), Huybrechts (1990), Huybrechts (1992), Fastook and Prentice (1994), Hulbe and MacAyeal (1999), Savvin et al. (2000), and Ritz et al. (2001) for the Antarctic and Letreguilly et al. (1991), Huybrechts (1994b), Greve (1997), and Ritz et al. (1997) for the Greenland ice sheet.

Due to the problem complexity and the large amount of input data required, such models need much CPU time to calculate. Here, I present an ice-sheet model at low resolution along a flowline through the Antarctic and Greenland ice sheet, respectively, that

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runs fast even on a modest PC. The model has the same physical basis as the models cited above. Complex interactions are parameterized in such a way that the behaviour of the classroom model is comparable to the findings with the more complex and high-resolution counterparts. However, ice stream behaviour and proper grounding line migration are not accounted for (as is the case in some 3D models), which limits the application of the Antarctic model essentially to an East-Antarctic type of ice sheet instead of a marine ice sheet such as the West-Antarctic ice sheet. Similar simplified models exist in the literature (MacAyeal, 1994; Oerlemans, 2002, 2003). A major difference lies in the use of a common spreadsheet program to perform the calculations, which makes it also easy to use. Furthermore, the numerical over-implicit scheme allows for a fast execution time of the model over time spans that cover several hundred thousands of years. Finally, the model is designed in such a way that it can be employed on different levels. At High-school level it is aimed at investigating the impact of background temperature change on ice sheets (in such case only one parameter has to be changed). At undergraduate level the impact of physical and environmental processes—of which temperature is one of them—on the behaviour of ice sheets can be studied. At graduate level, the model can be applied to other ice masses or glaciers, and adapted to different boundary conditions.

The response of the Antarctic ice sheet to changes in background temperature is likely to be different for the Greenland ice sheet. Because of the low mass turnover—due to the extremely cold surface temperatures, low accumulation rates and large size—the Antarctic ice sheet is considered to remain relatively stable for 100-year time scales under warming scenarios of up to 20 °C (Huybrechts, 1994a). This follows because the increased moisture content in warmer air masses leads to increased precipitation rates. The situation in Greenland is quite different. Surface melting is prevalent during summer around the ice sheet perimeter, but not higher up as temperatures remain year-round below freezing. Surface melting in the ablation zone presently accounts for roughly half of the mass loss from the Greenland ice sheet. The general consensus suggests that modest-to-moderate warming over the next millennium will lead to a gradual, dynamic interplay between marginal mass loss in the ablation zone and increased mass gain in the accumulation zone resulting in a slow, monotonic ice-sheet retreat (Church et al., 2001). This different ice-sheet behaviour for both Greenland and Antarctic ice sheets is clearly demonstrated with the classroom model, even though ice physics are the same for both modeled ice sheets.

2. Model description

2.1. Field equations

GRANTISM (GReenland and ANTarctic Ice Sheet Model) is a completely dynamic ice-sheet model, based on conservation laws of mass and momentum. For large ice masses such as the Greenland and Antarctic ice sheet, the *shallow-ice approximation* holds, which states that the driving stress τ_d in an ice mass is balanced by the vertical shearing at the bed $\sigma(b)$, or

$$\sigma(b) \sim \tau_d = -\rho_i g H \nabla h, \tag{1}$$

where H is the ice thickness, h the surface elevation of the ice sheet, ρ_i is the ice density and g the acceleration due to gravity. Stresses are related to strain rates by means of Glen's flow law, from which the vertical mean horizontal velocity in an ice sheet due to internal deformation $\bar{u}_d$ can be derived (Paterson, 1994):

$$\bar{u}_d = \frac{2}{n+2} A(T) H \tau_d^n, \tag{2}$$

where $A(T)$ is a temperature dependent flow parameter, defined by a modified Arrhenius relationship (Hooke, 1981):

$$A(T) = m\left(\frac{1}{B_0}\right)^n \exp\left[\frac{3C}{(T_r - T)^K} - \frac{Q}{RT}\right]. \tag{3}$$

Here, T (K) is the ice temperature, and m is a tuning parameter. Values for this and all other parameters and constants are listed in Tables 1 and 2. The velocity at the base of the ice sheet is given by

$$u(b) = A_b \frac{\tau_d^p}{Z^\star}, \tag{4}$$

where $p = 3$ is a sliding law exponent, and $Z^\star$ is the height of the ice surface above buoyancy level, defined

Table 1
List of constants and global parameters

Constant	Value	Units	Definition
ρ_i	910	$\mathrm{kg\,m^{-3}}$	Ice density
ρ_s	1028	$\mathrm{kg\,m^{-3}}$	Sea-water density
ρ_m	3300	$\mathrm{kg\,m^{-3}}$	Mantle density
θ	3000	a	Relaxation time asthenosphere
B_0	2.207	$\mathrm{Pa\,a^{1/n}}$	
C	0.16612	K^K	
K	1.17		
n	3		Flow law exponent
p	3		Sliding law exponent
Q	7.88×10^4	$\mathrm{J\,mol^{-1}}$	Activation energy for creep
R	8.31	$\mathrm{J\,mol^{-1}\,K^{-1}}$	Universal gas constant
T_r	273.39	K	

Table 2
List of experiment-dependent parameters

Constant	Antarctica	Greenland	Units
m	59	7.5	—
Δx	120	36	km
Δt	200	40	a
ω	2.5	1.0	—

by $Z^{\star} = H + \min[(b - H_{\mathrm{sl}}), 0]\rho_s/\rho_i$, where b is the bedrock topography and H_{sl} the eustatic sea-level height (Huybrechts, 1992). The continuity equation for ice thickness then reads

$$\frac{\partial H}{\partial t} = \dot{a} - \nabla(\overline{u}H), \tag{5}$$

where $\dot{a}$ is the surface mass balance and $\overline{u} = \overline{u}_d + u(b)$. Temperature calculation within the ice sheet is not considered, but thermomechanical coupling is taken into account in an implicit way. T (K) is therefore related to the background forcing temperature T_f (°C), which is an anomalous temperature relative to the present conditions, by the following empirical relationship

$$\begin{aligned} T &= T_f + 263.15 \quad (T_f < 0) \\ &= 0.5T_f + 263.15 \quad (T_f \geqslant 0). \end{aligned} \tag{6}$$

A limit is set to the ice temperature, i.e. $T < T_r$ so that (3) remains valid. In reality, most of the ice deformation takes place near the bedrock where surface temperatures are strongly dampened. Due to the simplicity of the model such complexity is not taken into account, so that transient thermodynamic effects are not accounted for. Modelling large ice sheets over the course of a glacial—interglacial transition requires isostatic adjustment of the underlying lithosphere. Bedrock deflection is determined by local isostatic equilibrium and the time-dependent response is given by a simple relaxation scheme with a characteristic time scale θ (Huybrechts and Oerlemans, 1988; Huybrechts, 1993)

$$\frac{\partial b}{\partial t} = \frac{1}{\theta}\left(b_0 - b - \frac{\rho_i}{\rho_m} H\right), \tag{7}$$

where b_0 is the isostatically adjusted bedrock elevation if the present ice load were removed:

$$b_0 = b_{\mathrm{obs}} + \frac{\rho_i}{\rho_m} H_{\mathrm{obs}}. \tag{8}$$

2.2. Mass-balance treatment and eustatic sea level

2.2.1. Antarctica

The surface mass balance of ice sheets is the result of ice accumulation minus ablation. Surface accumulation, $\dot{a}_{\mathrm{acc}}$ (m a^{-1} ice equivalent), over the Antarctic ice sheet is parameterized as an exponential function of mean annual surface temperature T_{ma} (°C), as is common in a number of ice-sheet model studies (for e.g. Huybrechts, 1993):

$$\dot{a}_{\mathrm{acc}} = 2.5 \times 2^{T_{\mathrm{ma}}/10}. \tag{9}$$

Surface ablation, $\dot{a}_{\mathrm{abl}}$ (m a^{-1} ice equivalent), primarily due to surface melting is parameterized in an empirical fashion as a function of the mean summer temperature T_{ms} (°C), comparable to the linear-temperature-index (LTI) method (Ohmura et al., 1996), i.e.

$$\begin{aligned} \dot{a}_{\mathrm{abl}} &= \min(1.4\,T_{\mathrm{ms}}; 10) \quad (T_{\mathrm{ms}} \geqslant 0) \\ &= 0 \quad (T_{\mathrm{ms}} < 0). \end{aligned} \tag{10}$$

Percolation of surface meltwater within the ice sheet and refreezing processes are not taken into account explicitly, but are implicitly included in the choice of the parameter settings. The ablation limit value of 10 m a^{-1} prevents that the ice sheet forms in zones where this value is reached. Mean annual and summer surface ice temperatures (T_{ma} and T_{ms} (°C), respectively) are parameterized as linear functions of surface elevation and latitude (Huybrechts and Oerlemans, 1988; Huybrechts, 1993):

$$T_{\mathrm{ma}} = -15.15 - 0.012\,h + T_f, \tag{11}$$

$$T_{\mathrm{ms}} = 16.81 - 0.00692\,h - 0.27937\,\phi + T_f, \tag{12}$$

where ϕ is the latitude (°S).

2.2.2. Greenland

Surface accumulation across the Greenland ice sheet was parameterized as a second-order polynomial along the 72° parallel based on the data presented in (Ohmura and Reeh, 1991):

$$\dot{a}_{\mathrm{acc}} = (-2.46257 + 0.1367\lambda - 0.0016\lambda^2) \times 1.0533^{T_f^{\star}}, \tag{13}$$

where λ is the longitude (°W), $T_f^{\star} = T_f$, for $T_f \leqslant 0$, and $T_f^{\star} = 0$ otherwise. As precipitation in warmer climates may also lead to changes in the pattern of accumulation distribution, the increase in accumulation with temperature is limited to its present value (Letreguilly et al., 1991). The use of a second-order polynomial leads to a negative accumulation over the ocean. Since ablation is dominant in this area, this has no influence whatsoever on the overall mass balance. Surface ablation follows from (10). Mean annual and summer surface ice temperatures are parameterized as linear functions of surface elevation.

$$T_{\mathrm{ma}} = -5.31 - 0.007992h + T_f, \tag{14}$$

$$T_{\mathrm{ms}} = 7.29 - 0.006277h + T_f. \tag{15}$$

2.2.3. Eustatic sea-level change

Eustatic sea-level change coupled to colder climates is parameterized as a linear function of background temperature and accounts for the presence of Northern Hemisphere ice sheets during glaciations. This corresponds to a gradual lowering of sea level to -150 m, for background temperatures $0 < T_f \leqslant -10\,^\circ\mathrm{C}$.

2.3. Numerical solution

The model domain is restricted to a flowline crossing each ice sheet (Fig. 1). Present observed ice surface and bedrock is resampled on a regular grid in space with a horizontal resolution specific for each ice sheet (Table 2). The ice-thickness (5) is written as a diffusive equation and solved with a semi-implicit numerical scheme using a weighed average of explicit and implicit terms (Hindmarsh, 2001). If one considers the ice thickness H_t, H_{t+1} at time levels t, $t+1$, the ice-thickness equation is rewritten as

$$H_{t+1} = H_t + \nabla[D_t\{\omega\nabla(H+b)_{t+1} + (1-\omega)\nabla h_t\}]\Delta t + \dot{a}\Delta t, \tag{16}$$

where Δt is the time step and D_t is the diffusivity at time step t, derived by combining (1), (2), (4) and (5):

$$D_t = \left[\frac{2}{5}A(T)H_t + \frac{A_b}{Z^\star}\right](\rho_i g)^n H_t^{n+1}|\nabla h_t|^{n-1}. \tag{17}$$

Eq. (16) can be considered as a tridiagonal system of equations and solved by means of standard methods (e.g. Press et al., 1992):

$$\alpha_{i,t}H_{i-1,t+1} + \beta_{i,t}H_{i,t+1} + \gamma_{i,t}H_{i+1,t+1} = \delta_{i,t}, \tag{18}$$

where $i = 1, N$ is the number of grid points along the central flowline. Let

$$\alpha'_{i,t} = -\frac{\Delta t}{2(\Delta x)^2}(D_{i,t} + D_{i-1,t}) \tag{19}$$

$$\gamma'_{i,t} = -\frac{\Delta t}{2(\Delta x)^2}(D_{i,t} + D_{i+1,t}), \tag{20}$$

then, it follows after some algebraic manipulation that

$$\alpha_{i,t} = \omega\alpha'_{i,t} \tag{21}$$

$$\beta_{i,t} = 1 + \omega\frac{\Delta t}{2(\Delta x)^2}(D_{i-1,t} + 2D_{i,t} + D_{i+1,t}) \tag{22}$$

$$\gamma_{i,t} = \omega\gamma'_{i,t} \tag{23}$$

$$\begin{aligned}\delta_{i,t} = {} & \alpha'_{i,t}[\omega(b_{i,t+1} - b_{i-1,t+1}) + (1-\omega)(h_{i,t} - h_{i-1,t})] \\ & - \gamma'_{i,t}[\omega(b_{i+1,t+1} - b_{i,t+1}) + (1-\omega)(h_{i+1,t} - h_{i,t})] \\ & + H_{i,t} + \dot{a}\Delta t.\end{aligned} \tag{24}$$

An explicit scheme has $\omega = 0$, a semi-implicit scheme has $\omega = 1$, while a Crank-Nicholson scheme has $\omega = \frac{1}{2}$.

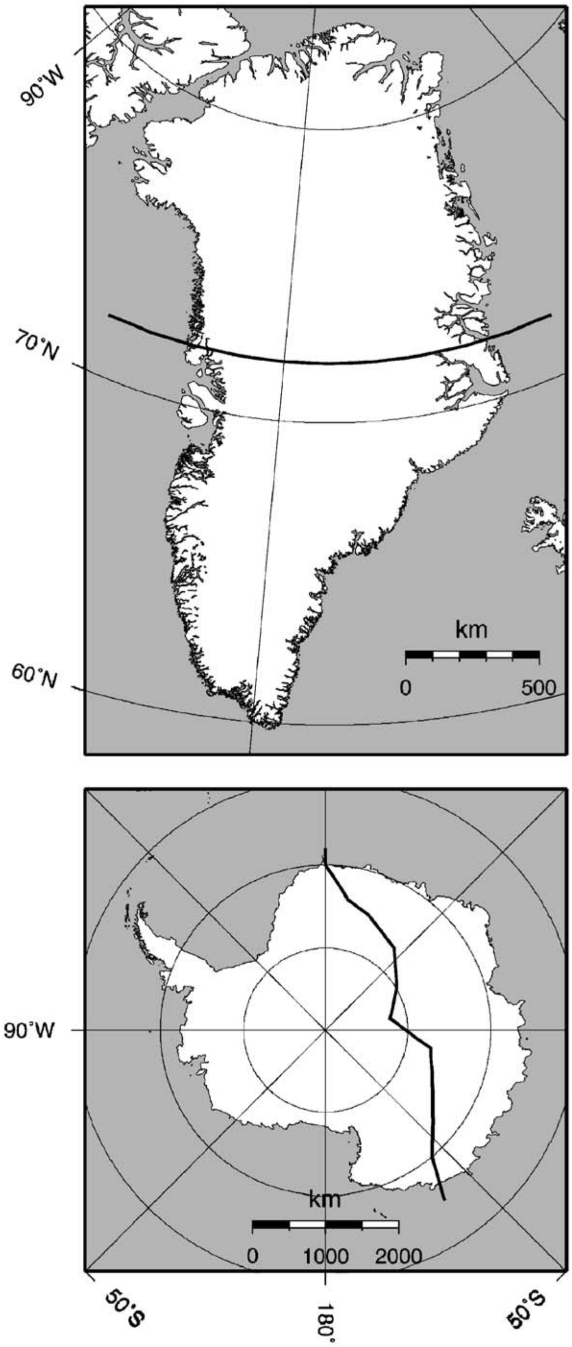

Fig. 1. Situation of modelled flowlines (thick lines) through the Greenland (top) and Antarctic (bottom) ice sheet.

None of these numerical schemes are unconditionally stable for the ice-thickness equation. This scheme becomes over-implicit for a weighing factor $\omega > 1$, which is unconditionally stable in case of an isothermal ice sheet for $\omega \geqslant n/2$ (Hindmarsh, 2001). Greve and Calov

(2002) have demonstrated that for the thermomechanically coupled problem unconditional stability does not hold anymore, but the over-implicit scheme still allows to use larger time-steps than conventional schemes with $\omega \leqslant 1$. The GRANTISM model can be regarded as an isothermal model and should therefore guarantee a better stability. For the Antarctic ice-sheet model, ω was set to 2.5, which allows for time steps of at least 200 years (Table 2), while for the Greenland ice-sheet model, due to its smaller size, a semi-implicit scheme was preferred with $\omega = 1$. In that case a time step of 40 years was employed. These parameters and time steps were chosen in such a way that for any background temperature change T_f between -15 and $+25$ K, a stable result is obtained.

3. Model layout

3.1. Control parameters

The model interface is very straightforward. In its simplest form, only two parameters need to be adjusted, i.e. 'RUN' and 'TFOR'. 'RUN' controls the model performance. When set to '1', the model runs forward in time; subsequent pushing 'F9' then brings the model ice sheet towards a steady state condition. Each model run depends on the initial conditions. When the initial conditions were set to '0', the simulation starts without ice and a bedrock profile isostatically adjusted for the removed ice load.[1] On the contrary, when 'RUN' is set to '2' prior to a model run, the simulation starts from the present observed ice sheet configuration. The second parameter 'TFOR' determines the background temperature forcing, that can range for -15 to $+25$ K compared to the present environmental conditions (default is 'TFOR' = '0').

3.2. Options

Optional parameters are listed in the spreadsheet below the control parameters. They are 'BASALSL', 'TKOPP', 'DATASET', 'BEDADJ', and 'SEALEV'. 'BASALSL' controls basal sliding in the model. Set to '0', the ice sheet remains frozen to the bed (default = '1'). 'TKOPP' introduces the ice-temperature coupling according to (6), otherwise an isothermal ice sheet is considered (default = '1'). 'DATASET' selects the model domain: '1' stands for the Antarctic ice sheet, '2' for Greenland and '0' for a initial flat bedrock (ideal ice sheet). 'BEDADJ' controls isostatic bed adjustment (default = '1', activated). Finally, 'SEALEV' introduces the sea-level lowering for colder background temperatures associated with the formation of Northern Hemisphere ice sheets (default = '1').

[1]This is only valid as long as isostatic bedrock adjustment is taken into account, which is a default option. When 'BEDADJ' is set to '0', the present observed bedrock profile is retained.

3.3. Model output and graphics

Numeric model output are ice-sheet volume (km^3), flowline length (km), and elapsed model time (year). Present-day modeled volumes of the Antarctic and Greenland ice sheet are set to be 30×10^6 and 2.6×10^6 km^3, respectively. Besides the global numerical output, four panels display major parameters along the modeled flowline. Panel 1 displays the ice-sheet surface and basal topography, both observed and simulated; Panel 2 displays the vertical mean deformational and basal velocity, $\overline{u}_d$ and $u(b)$; Panel 3 shows the surface mass balance and its two components, i.e. surface accumulation and ablation; and Panel 4 displays the mean summer and year temperatures at the surface of the ice sheet (see Fig. 2 for the Antarctic and Fig. 3 for the Greenland ice sheet). The information given in these panels is continuously updated during the model run.

4. Model results

4.1. Steady-state response to a warmer climate

Two experiments were carried out to investigate the sensitivity of the modeled ice sheets to changes in background temperature. Both start from a steady-state ice-sheet configuration. In the first experiment the background temperature is increased by 1 K to attain a new steady state. Subsequently the background temperature is raised again by 1 K, and the process repeated until the ice sheet has disappeared. A buildup of the ice sheet is again invoked by decreasing the temperature in steps of 1 K until the present conditions are achieved again. Results show a clear hysteresis loop in the volume changes of both ice sheets (Fig. 4). For the Greenland ice sheet, there are two possible steady states of the ice sheet at all temperature increases between 0 and 6 K. At a 1 K temperature increase, for example, one of the steady states consists of a small ice cap centered near the eastern margin of the continent (Fig. 5), while the other consists of a full bodied ice sheet closely resembling the present steady state as displayed in Fig. 3. Such hysteresis for the Greenland ice sheet is also confirmed by experiments with a three-dimensional thermomechanical ice-sheet model (Letreguilly et al., 1991). The GRANTISM model, however, seems to be more sensitive to temperature rise when starting from no ice conditions and more robust when letting the climate gradually warm up. Nevertheless, in both cases the ice sheet has disappeared when temperatures have risen by 6 K. The difference in sensitivity is

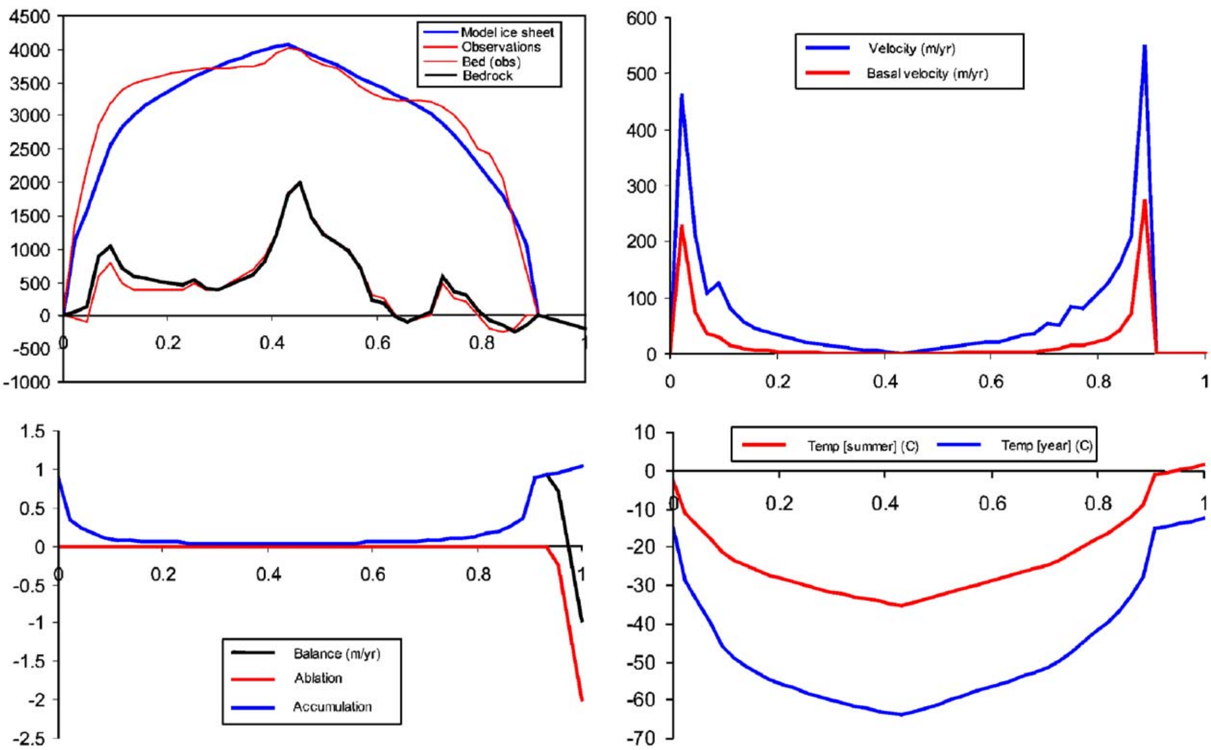

Fig. 2. Steady-state Antarctic ice-sheet configuration under present climate conditions ($T_f = 0\,\mathrm{K}$). Upper left panel: ice-sheet geometry (observed and modelled, m a.s.l.); upper right panel: surface and basal velocity ($\mathrm{m\,a^{-1}}$); lower left panel: surface mass balance, accumulation and ablation ($\mathrm{m\,a^{-1}}$); lower right panel: annual mean and summer mean surface temperature (°C).

obviously due to the simplified geometry (flowline) and the difference in mass-balance (climatic) forcing, compared to the full three-dimensional treatment.

Antarctic steady-state volumes for different temperature increases also show a clear hysteresis loop (Fig. 4), the main difference with the Greenland ice sheet being that melting of the ice sheet occurs only at much higher temperatures. One would need a rise in temperatures of at least 25 K to remove the whole ice mass of Antarctica, while for Greenland a temperature rise of 5–6 K is sufficient. Modeled volume changes are in accord with model simulations by Huybrechts (1993). However, according to the latter study, the hysteresis is less pronounced and regions with multiple solutions only occur for a temperature rise of 9, 18 and 19 K (see Fig. 9 in Huybrechts (1993)).

Although less pronounced in the GRANTISM model, an increase of background temperatures up to 5 K leads to an expansion of the Antarctic ice sheet due to an increase in precipitation (Huybrechts, 1993). The ice volume increase is only marginal due to a softening of the ice with higher temperatures, which leads to higher discharge rates at the coast, hence balancing the increase in surface accumulation. Ablation remains very marginal, as the mean annual temperature at the coast is very low (−15 to −10 °C).

The reason for the hysteresis in both Greenland and Antarctic simulations stems from the mass balance-elevation feedback. The surface elevation of a full-bodied ice sheet is much higher than the surface elevation of a isostatically adjusted bedrock without the presence of an ice sheet. When the equilibrium line altitude (elevation above which accumulation dominates, or zone of positive mass balance, and below which melting prevails, or zone of negative mass balance) is situated between both surface elevations, the ice sheet will remain in the first case, while only a small ice cap is developed in the latter case, due to the negative mass balance. Therefore, the size of the ice sheet is not only determined by climatic conditions, but also by the preceding mass balance history. The hysteresis for the Antarctic ice sheet might thus be a result of the simplified ablation treatment in the GRANTISM model, while in Huybrechts (1993) a positive-degree-day model is applied to account for ice loss.

4.2. Steady-state response to a colder climate

Background temperature decreases lead for both ice sheets to an increase in ice volume (Fig. 6). Here, hysteresis does not occur as for colder temperatures the whole modeled area generally falls within the

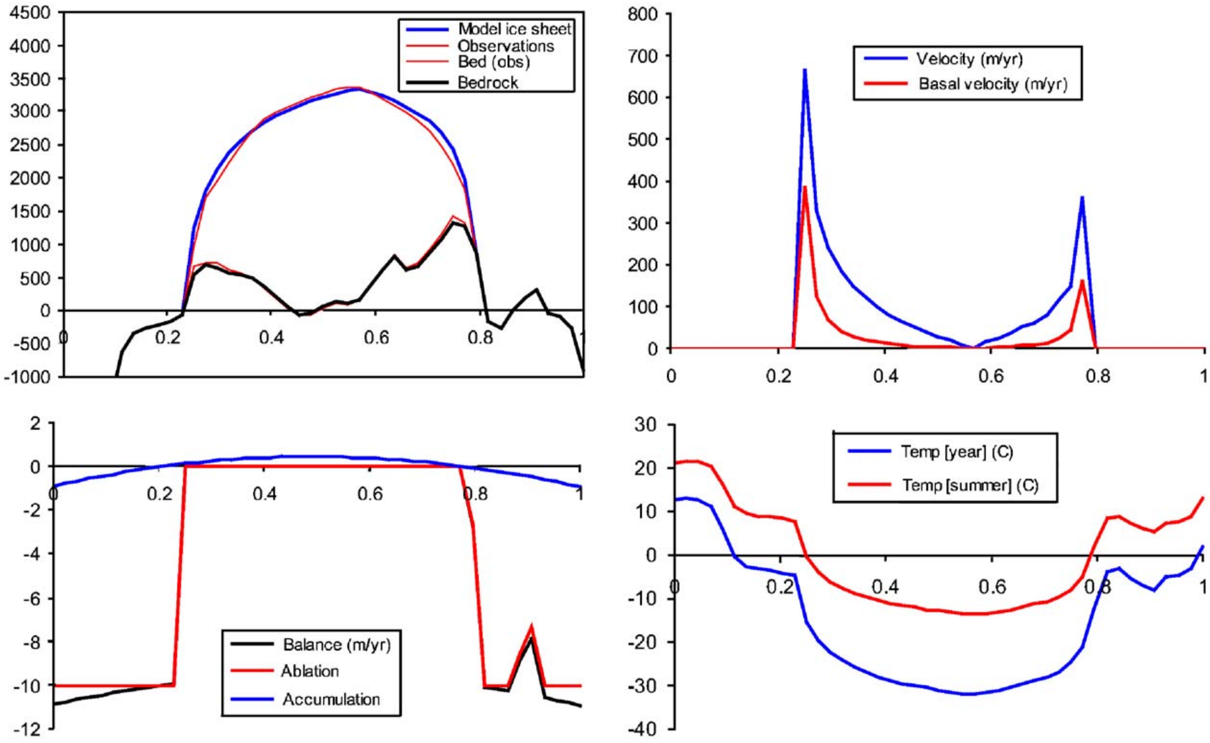

Fig. 3. Steady-state Greenland ice-sheet configuration under present climate conditions ($T_f = 0\,\mathrm{K}$). Upper left panel: ice-sheet geometry (observed and modelled, m a.s.l.); upper right panel: surface and basal velocity ($\mathrm{m\,a^{-1}}$); lower left panel: surface mass balance, accumulation and ablation ($\mathrm{m\,a^{-1}}$); lower right panel: annual mean and summer mean surface temperature (°C).

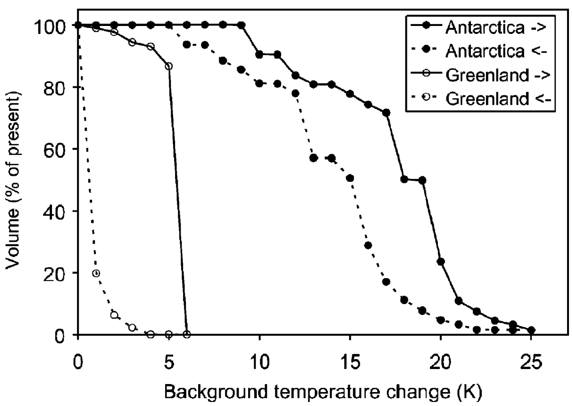

Fig. 4. Steady-state ice-sheet volumes under a warmer climate for Antarctic (solid circles) and Greenland (open circles) ice sheets. Solid lines show evolution starting at present conditions ($T_f = 0$), while dotted lines show evolution starting from a warm climate in which an ice sheet could not exist.

accumulation zone, and hence topographic control on mass balance is less effective. Colder temperatures reduce accumulation rates for the Antarctic ice sheet, but this effect is balanced by stiffer and less deformable ice due to the ice-temperature coupling. The increase in ice volume is mostly due to a lowering of eustatic sea level, so that the Antarctic ice sheet can expand laterally over the continental shelf. Grounding line motion is introduced by allowing the ice sheet to expand over grid points that are lying above sea level. Despite this crude parameterization, the GRANTISM model behaves in a realistic way, allowing Antarctic ice volumes to increase with 20% for a temperature drop of 10 K. On the contrary, the Greenland ice sheet simulation leads to a volume increase of 50% for a similar temperature change. Huybrechts (2002) gives an overview of ice sheet volume changes for Greenland and Antarctica at the Last Glacial Maximum. According to his calculations, maximum ice sheet volumes were 30% and 33% larger than the present Antarctic and Greenland ice sheet volumes, respectively. This leads to a sea-level lowering by 16 m for the Antarctic and 2.5 m for the Greenland ice sheet. The GRANTISM model underestimates Antarctic and overestimates Greenland glacial ice volumes. This is mainly due to the fact that a flowline cannot represent the whole ice sheet in terms of absolute volumes. Especially for the Antarctic simulation one has to keep in mind that the model basically represents the behaviour of the East Antarctic ice sheet.

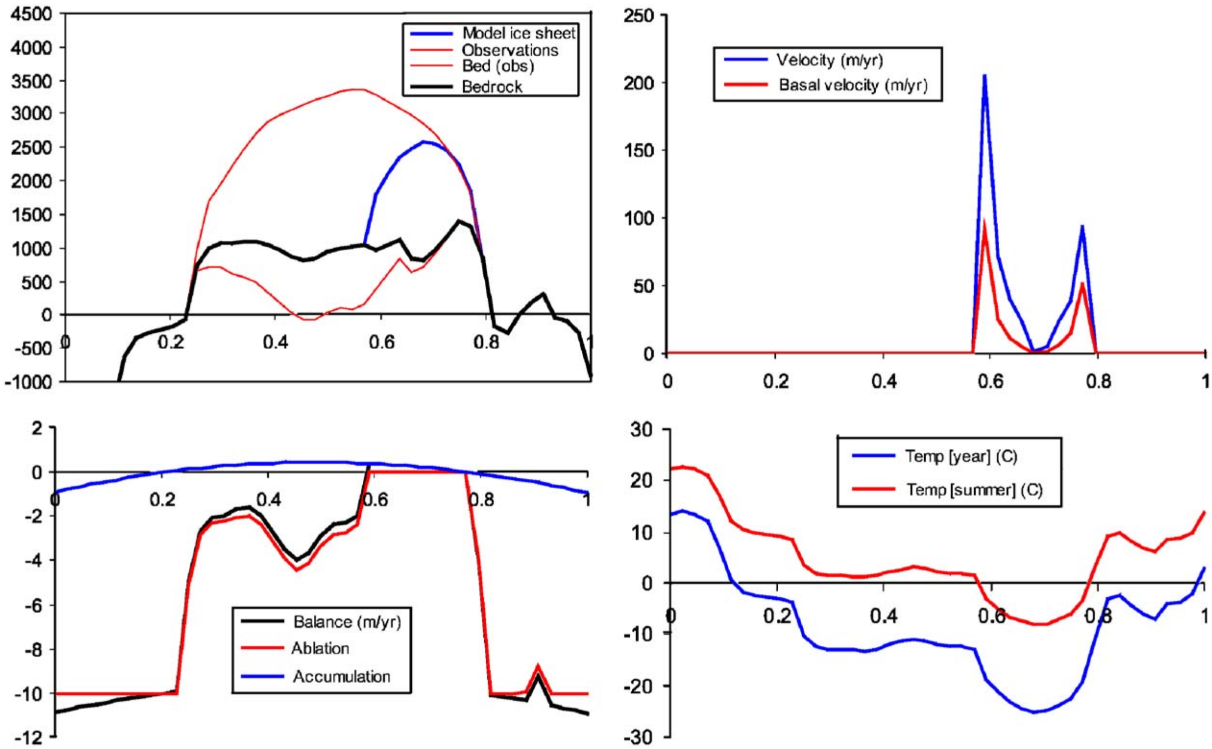

Fig. 5. Steady-state Greenland ice-sheet configuration under climate conditions $T_f = 1\,\mathrm{K}$, starting from zero ice thickness and isostatically adjusted bedrock.

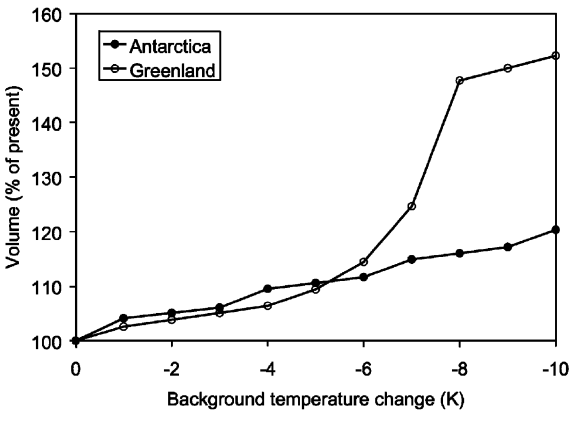

Fig. 6. Steady-state ice-sheet volumes under a colder climate for Antarctic (solid circles) and Greenland (open circles) ice sheets.

Volume increase of the West Antarctic ice sheet is largely due to geometric conditions, resulting in a substantial advance of the grounding line over the continental shelf, which is not captured by the GRANTISM model. The direction of the response signal is, however, correct.

4.3. Transient response

Since the GRANTISM simulates ice sheets in a time-dependent way, a comparison is made for the rate of formation of the ice sheets under different climatic conditions. The curves for the Greenland ice sheet (Fig. 7) can be compared to the results of Letreguilly et al. (1991) and the curves for the Antarctic ice sheet (Fig. 8) to those of Huybrechts (1993). Ice sheet growth with the GRANTISM model is in agreement with both results. Some minor differences are that (i) for $T_f = +4\,\mathrm{K}$ the Greenland ice sheet does not develop with GRANTISM and (ii) that the ice volumes for $T_f = +3\,\mathrm{K}$ and $T_f = +2\,\mathrm{K}$ are underestimated compared to the model of Letreguilly et al. (1991). Growth curves for the Antarctic ice sheet are in general agreement with the model results of Huybrechts (1993).

The rate of formation of the ice sheet also depends on the initial size of the accumulation area of the ice free bedrock, and hence on the topography. For $T_f = 0$ and $-10\,\mathrm{K}$ the Greenland ice sheet builds up faster than for $T_f = 1\,\mathrm{K}$ or $2\,\mathrm{K}$. In the latter cases, the accumulation area at the start is restricted to high elevated zones of the eastern mountains, so that the initial ice cap is very small. It only grows due to the mass balance-elevation feedback, where raising the overall surface elevation (by

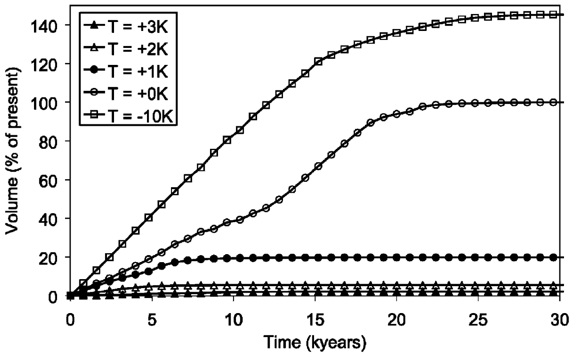

Fig. 7. Evolution of Greenland ice-sheet volume in time starting from no ice conditions and an isostatically adjusted bedrock for different scenarios in background temperature.

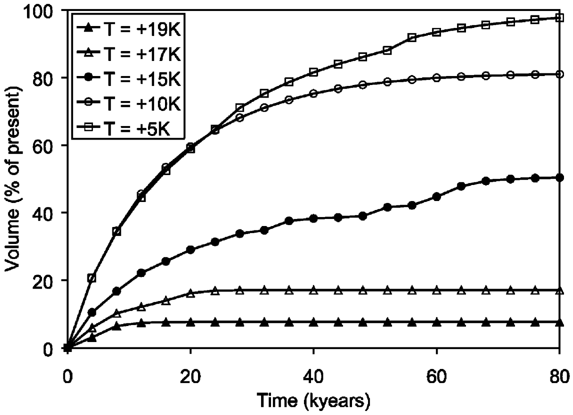

Fig. 8. Evolution of Antarctic ice-sheet volume in time starting from no ice conditions and an isostatically adjusted bedrock for different scenarios in background temperature.

developing an ice cap) increases the accumulation area. Such process is however slow, and depicted in both Figs. 7 and 8.

5. Summary and conclusions

This paper describes a flowline model (GRANTISM) through the Greenland and (East) Antarctic ice sheets that is capable of simulating with relatively good confidence the reaction of both ice sheets to changes in the climatic signal, for a background temperature range of -10 to $+25$ K compared to the present conditions. Compared to model experiments carried out with three-dimensional thermomechanical ice-sheet models of both ice sheets (Letreguilly et al., 1991; Huybrechts, 1993), the GRANTISM model behaves in a similar way for steady-state simulations as well as for the time-dependent response of the ice masses to changes in background temperature.

The novelty of the GRANTISM model lies in its simplicity, thereby taking care of most features and feedbacks present in nature and captured in most advanced models, such as those mentioned above. In view of such simplicity, dynamics of marine ice sheets are not properly accounted for, which limits the Antarctic model to a representation of the East Antarctic ice sheet. GRANTISM is based on conservation laws of mass and momentum, and relies on Glen's flow law to relate stresses within the ice mass to strain rates. Many processes are properly included and accounted for, such as basal sliding, isostatic bedrock adjustment, and ice-thickness evolution in time. Other processes are taken into account in a simplified way by parameterization, such as the ice-temperature coupling, surface accumulation and ablation, and surface temperature. The ice-thickness equation is solved with a an over-implicit numerical scheme (Hindmarsh, 2001), which makes the model run considerably faster. The fast computation make the model suitable as a classroom model. Therefore, a user-friendly interface was designed and the GRANTISM model was implemented within a common spreadsheet program (Excel™).

The Excel™ model is freely available for non-commercial use and can be downloaded on the website: http://www.vub.ac.be/DGGF/grantism

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