Institut de Statistique et de Recherche Opérationnelle

ULB

Banque Nationale de Belgique
Nationale Bank van België

Self-Training in Time Series Analysis

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Acknowledgements

- Beside the authors, Laurent Seinlet and Stefano Ugolini have also participated to the project
- We thank them as well as
  - ✓ Hassane Njimi and Marc Hallin - *ULB, Brussels (B)*
  - ✓ Jan Beirlant and An Carbonez - *UCS, KULeuven (B)*
  - ✓ André Klein - *University of Amsterdam (NL)*

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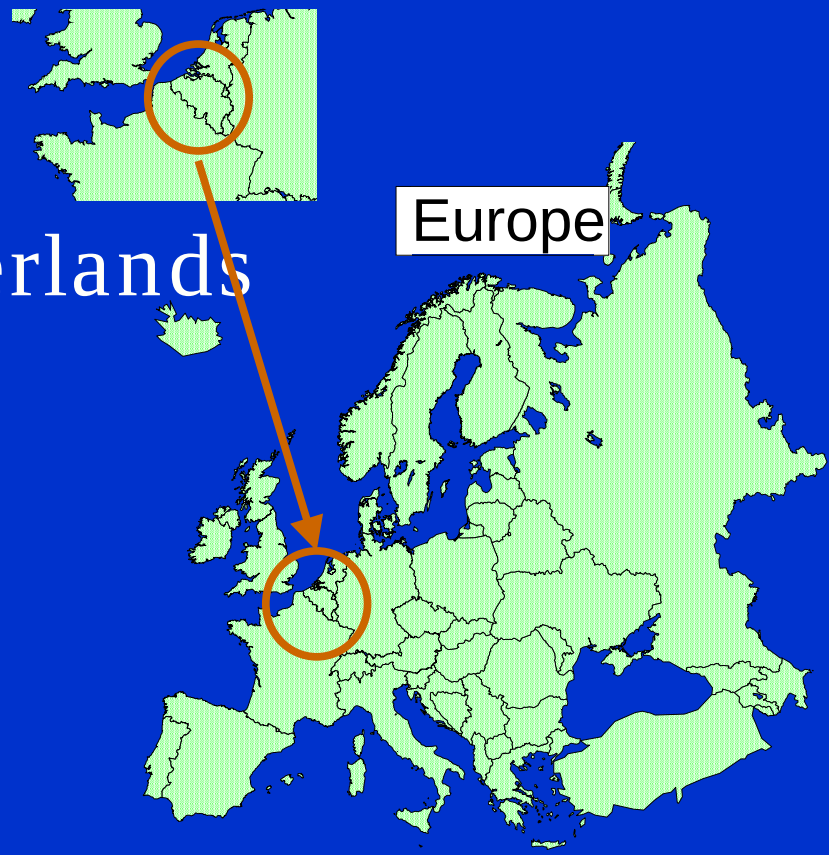
Belgium

- European country of 30000 km^2 and 10 millions inhabitants
- Between France, Germany, Luxembourg and the Netherlands
- Constitutional Kingdom
- 3 languages : Dutch, French, German
- 3 regions : Flanders, Wallonia, Brussels
- Location of several European institutions including the Commission and the Parliament



National Bank of Belgium

Central bank,
but with more and more statistical activities

Financial statistics

General economic statistics

- national and regional accounts,
- international commerce,
- business cycle surveys

diffusion of these statistics

Priority towards quality of the statistics

General Statistical Department

- Audience with scattered ages (20-60)
- French speaking and Dutch speaking
- From secondary to university education, mainly with an economic background
- The large majority has not a strong mathematical orientation
- staff > 250

Training Needs

- ⇨ Provide a general statistical culture to the staff of the department
- ⇨ No development of a statistical tool but
  - ✓ understand what is used
  - ✓ take the best solution to the problem on hand
- ⇨ Two courses : basic statistics (60 h) and time series analysis (30 h)
- ⇨ Self-training with a tutor provided by the Bank rather than training cycles
- ⇨ Practical orientation without too many computations

Constraints

- Languages :
  - ✓ French and Dutch (possibly English for software)
- Tools :
  - ✓ office suite + software packages to be delivered
- Recommendations from Eurostat :
  - ✓ for seasonal adjustment of time series :
    X12 (Bureau of the Census) and
    TRAMO-SEATS ([1] V. Gómez et A. Maravall, Bank of Spain)

Basic course in statistics

⇨Contents : 13 chapters in 5 parts

Part I	Descriptive Statistics
Part II	Inductive Statistics
Part III	Linear Models
Part IV	Non-parametric Statistics
Part V	Index Numbers

Time Series Analysis

Contents : 13 chapters

Chapter 0

Introduction to
- ✓using Excel for time series analysis
- ✓using Time Series Expert and Demetra
- ✓using assessment tests

Chapters 1 to 4

1. Concepts and definitions
2. Simple linear regression
3. Growth curves
4. Smoothing with moving averages

Chapter 5

5. Methods of seasonal decomposition

Time Series Analysis
continued

Chapter 6	6. Exponential smoothing
Chapter 7	7. Multiple linear regression
	Time series models
Chapters 8 à 11	8. Autocorrelation and forecast errors 9. ARIMA models 10. Box and Jenkins method 11. Regression with autocorrelated errors
	Advanced methods of seasonal decomposition
Chapter 12	12. X-12-ARIMA method
Chapter 13	13. TRAMO/SEATS method

References

⇨Course 1

✓[3] Moore, D. S. et McCabe, G. P. (1998).
Introduction to the Practice of Statistics ,
3rd edition, Freeman, New York. (EN + NL)

✓Wonnacott, Thomas. H. et Wonnacott, Ronald. J (1991).
Statistique, Economie – Gestion – Science – Médecine,
4e édition, Economica, Paris. (EN + FR)

⇨Course 2

✓[2] Mélard, G. (1990)
Méthodes de prévision à court terme,
Editions de l'Université de Bruxelles, Bruxelles,
et Editions Ellipses, Paris.

Submission for course 2

- ⇨ Interactive and multimedia learning system
- ⇨ Integration of voice, image, computational treatment and video
- ⇨ Different pedagogical activities going from learning to self-assessment
- ⇨ Digital media : intranet and/or CD-ROM
- ⇨ Course developed starting with [2] (with 2 new chapters for X-12-ARIMA and TRAMO/SEATS)

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Characteristics of the course (1/2)

⇨ They depend heavily on the audience and the self-learning situation :

- ✓ simple presentation of the material
- ✓ low use of mathematical formulae
- ✓ organization as an interactive multimedia slide show (100-250 slides per chapter)
- ✓ large number of questions and exercises
- ✓ detailed solutions
- ✓ . . .

Characteristics of the course (2/2)

- ✓ many simulations
- ✓ computing is avoided (software packages used : Microsoft Excel, Time Series Expert & Demetra)
- ✓ frequent recall of the structure
- ✓ frequent reminders of the material
- ✓ navigation by hyperlinks
- ✓ access to instructions and files for the exercises, as well as to the other chapters of the course
- ✓ self-assessment

⇨ We also discuss the advanced part of chapter 13 on the TRAMO/SEATS method

Table of contents

Navigation within the course

- ⇨ We start from the menu
- ⇨ Adobe Acrobat Reader is launched :

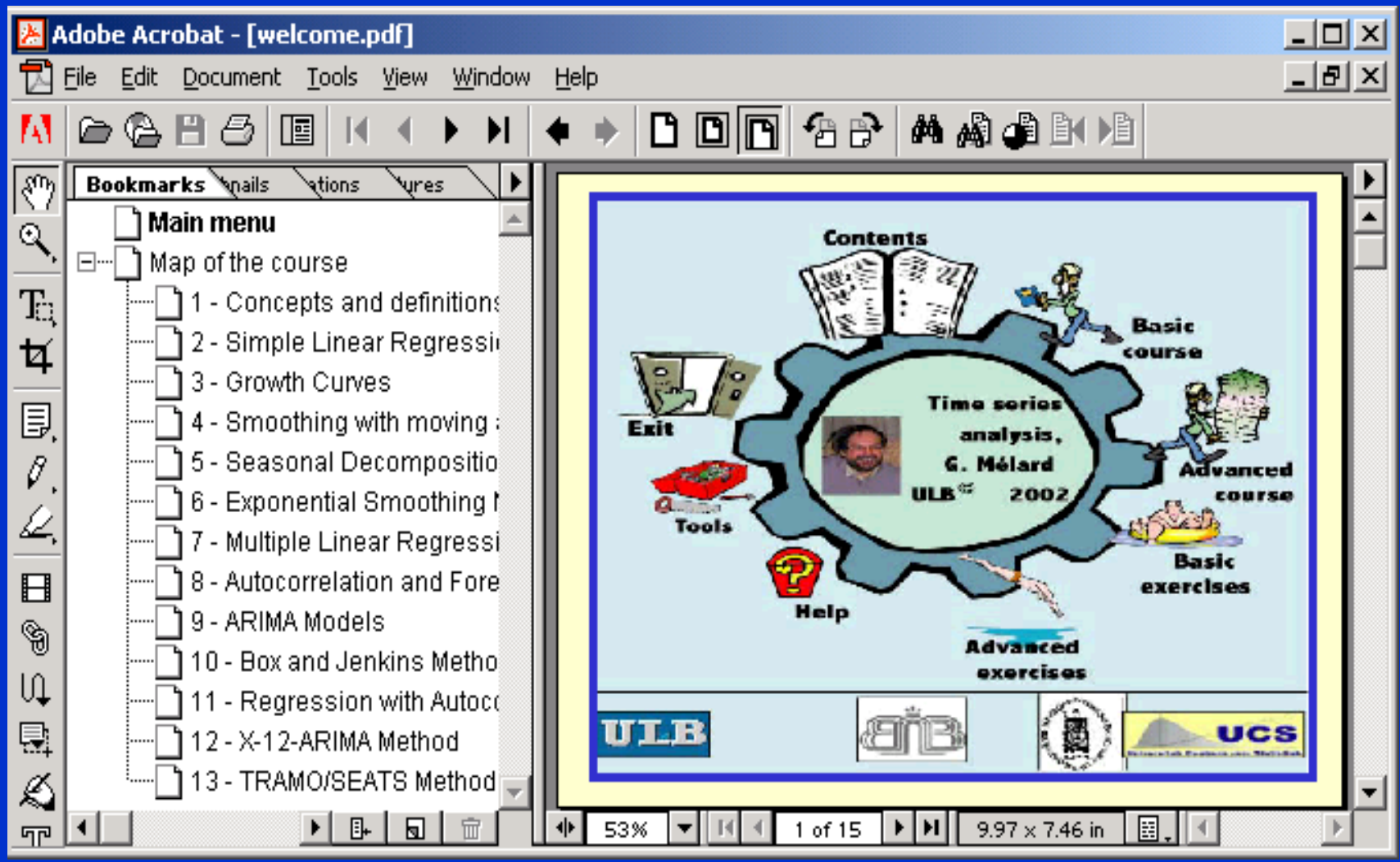
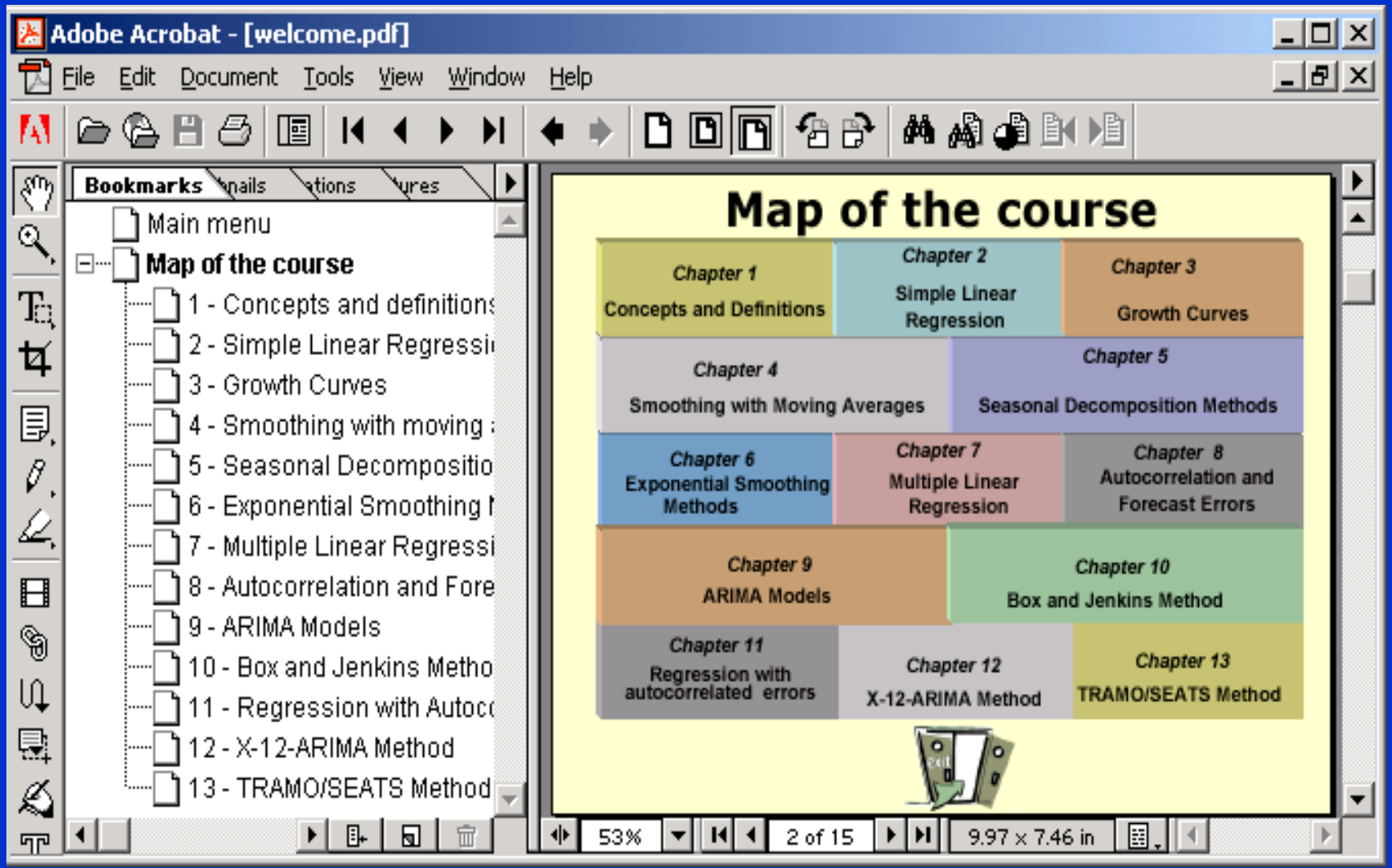


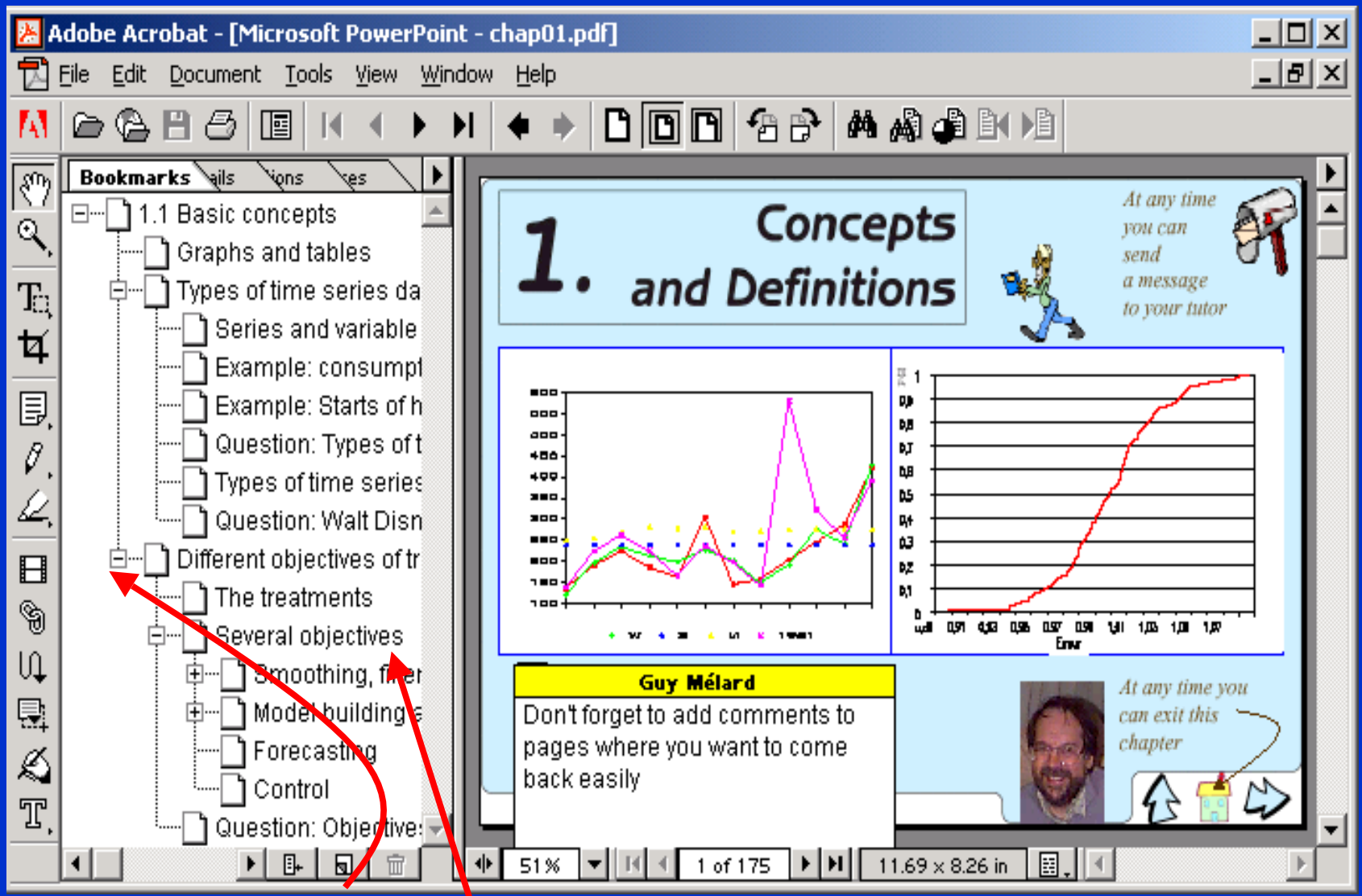
Table of contents of the basic course

⇨Let us look at the map of the course :



⇨We can also reach the list of exercises

A chapter



Development of a branch

Access to a page

A page of the course

Message to the tutor

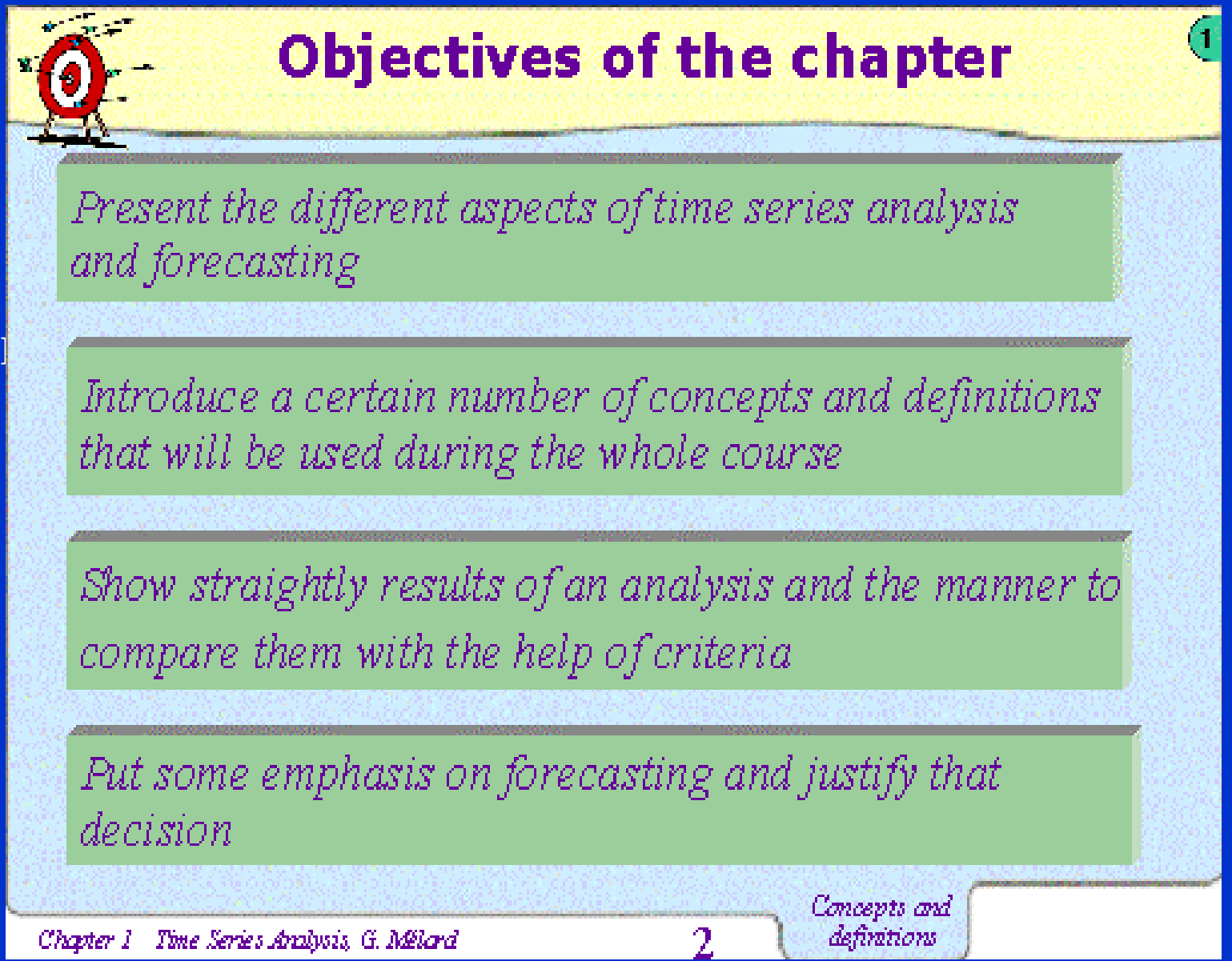
Annotation of the learner

Oral explanations

Fast move

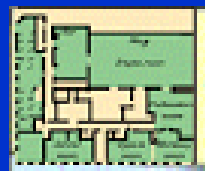
Objectives of a chapter

The second page of the chapter



The contents of a chapter

The table of contents is progressi-vely developed

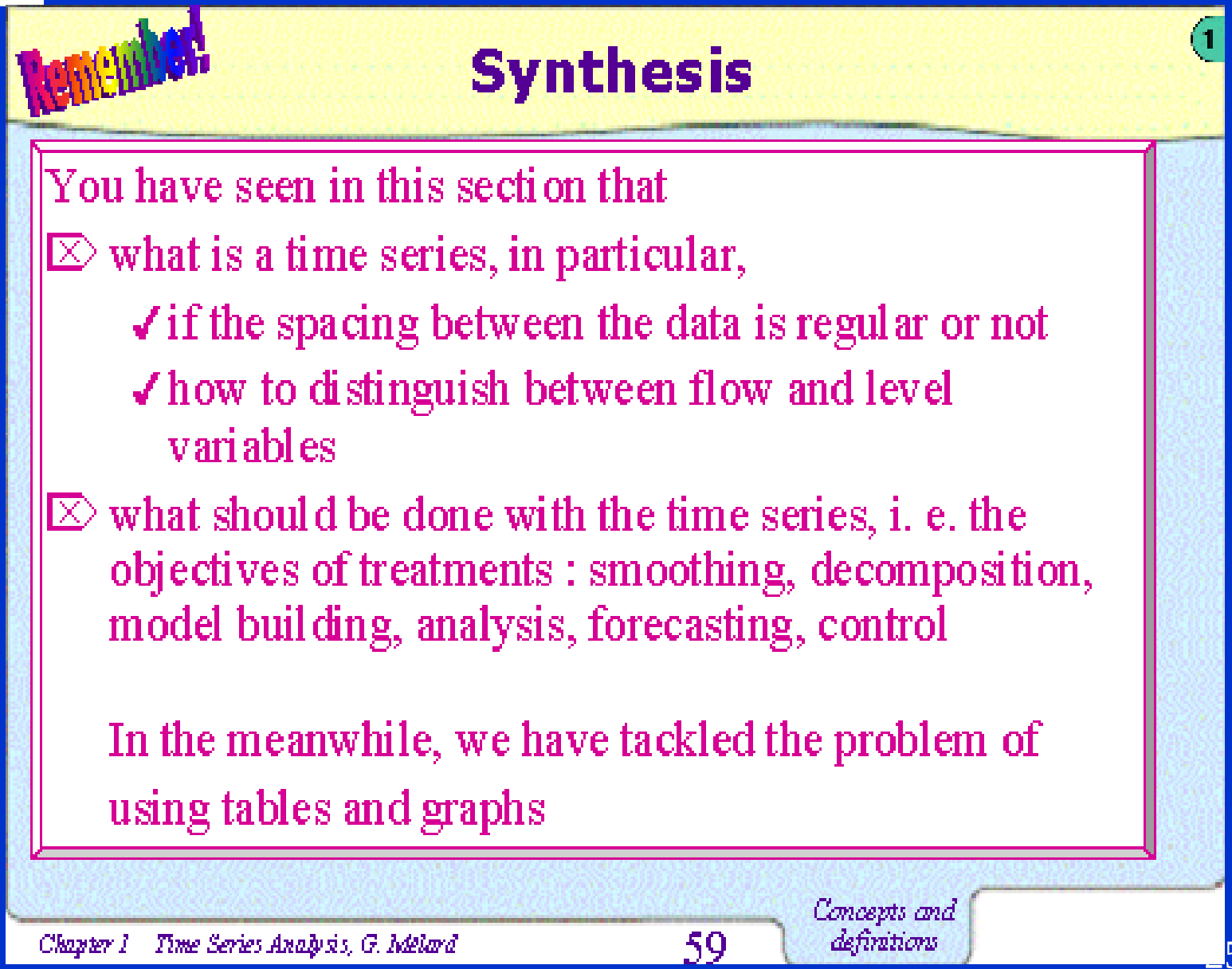


Types of time series data

1

The sections

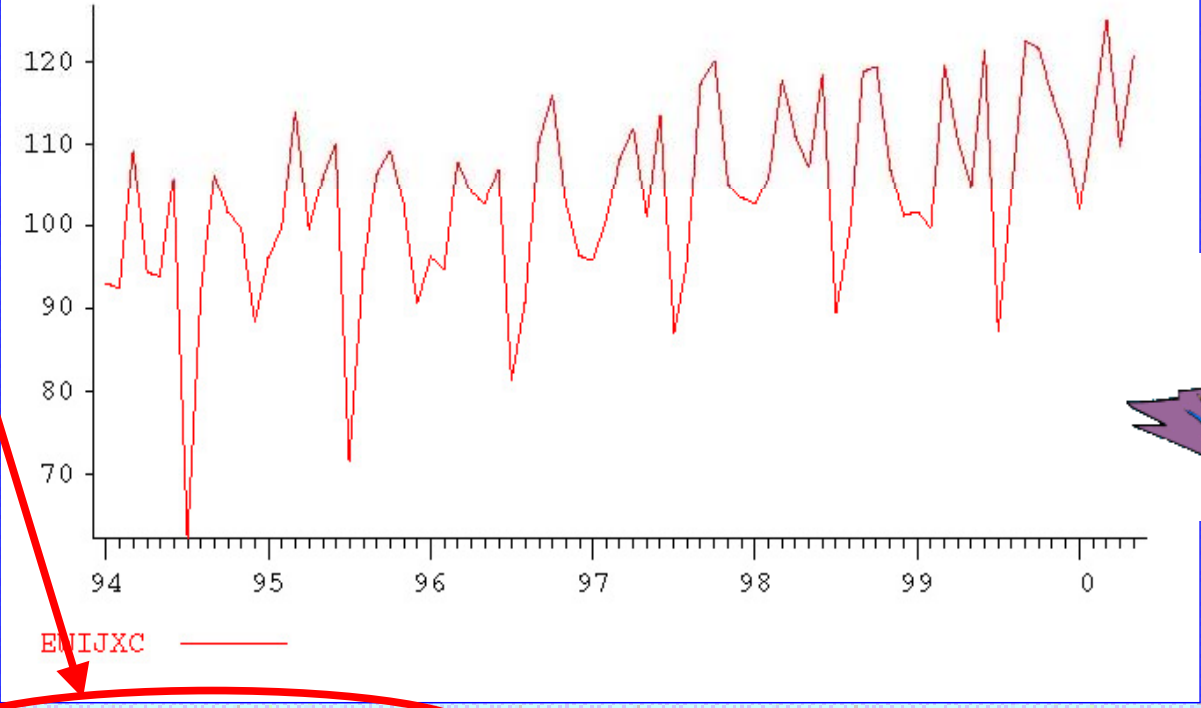
At the end of each section there is a synthesis



The questions

A space for the answer can be found on a paper document.

Question: Belgian industrial production (1994-2000)



Write your answer on page 1

"We consider the index of the Belgian industrial production, for the whole industry (base year 1995=100). On the basis of a plot and of wise sense, do you think that the series is annual, quarterly, something else?"

Remark. Like all the questions, the answer will be given in the main document which corresponds to the present chapter.

Conclusions of the chapter

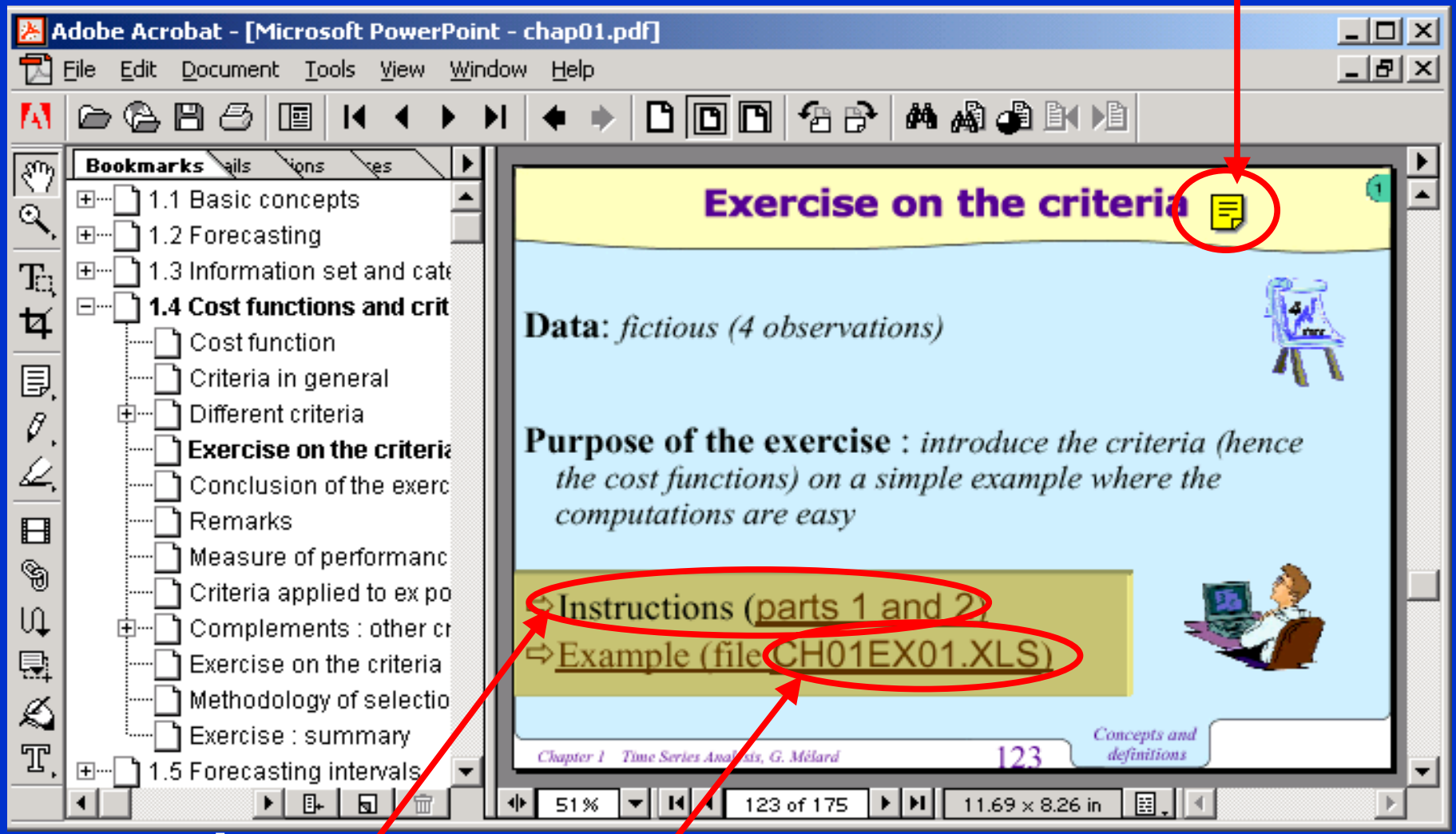
They are always vocally com-mented

Conclusion: concepts and definitions

- Information set versus methods → *what methods?*
- Cost function
- Ex post and ex ante forecasts → *how to detect difficulties?*
- Criteria (MSE, MAE, MAPE, ...)
- Methodology of selection → *how to select methods?*
- → *what to do if the forecast is not good?*
- Use of of the distribution of forecast errors → *additive versus multiplicative model?*
- Forecast intervals

The main part : the exercises

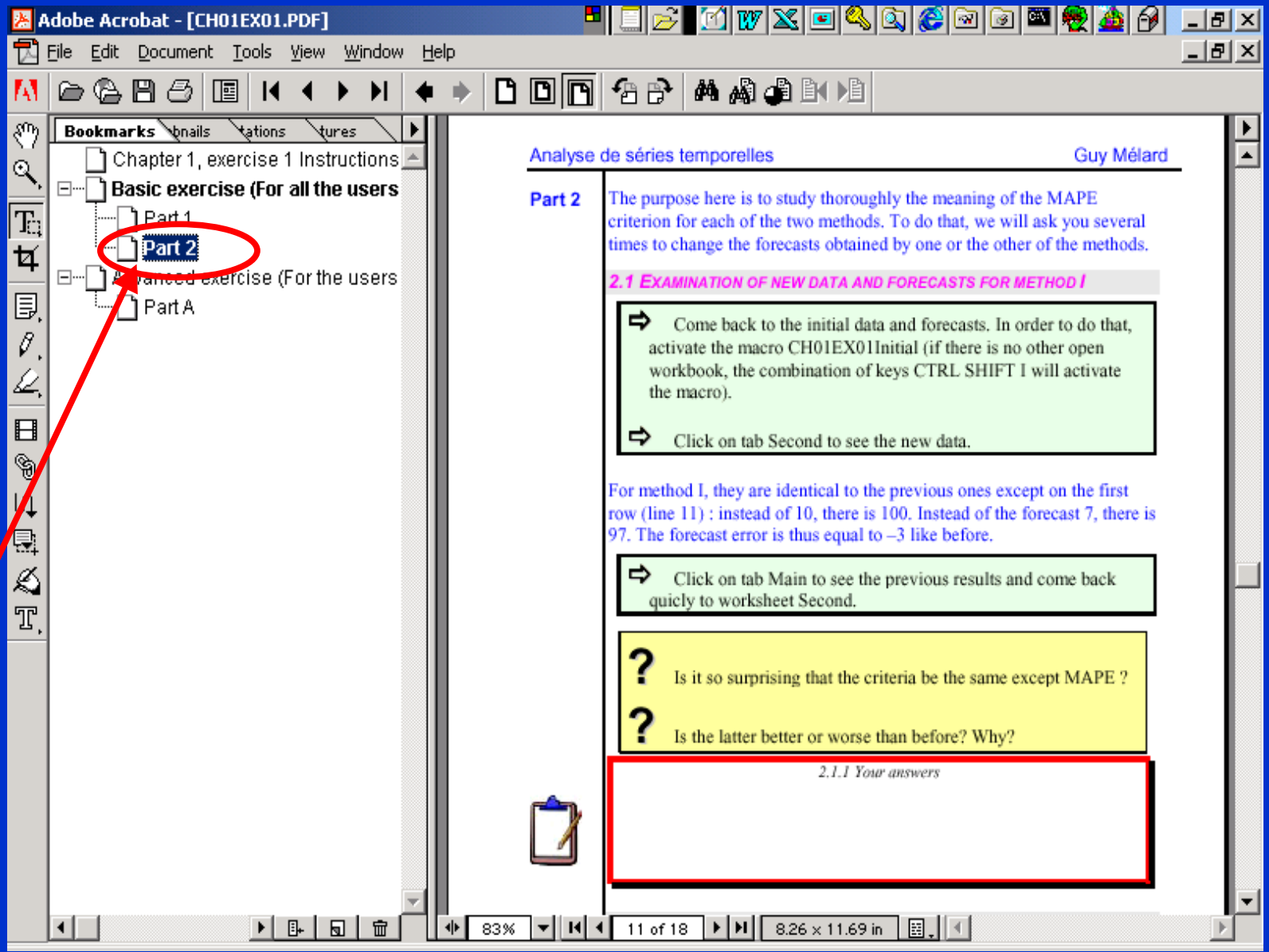
Annotation put here by the learner



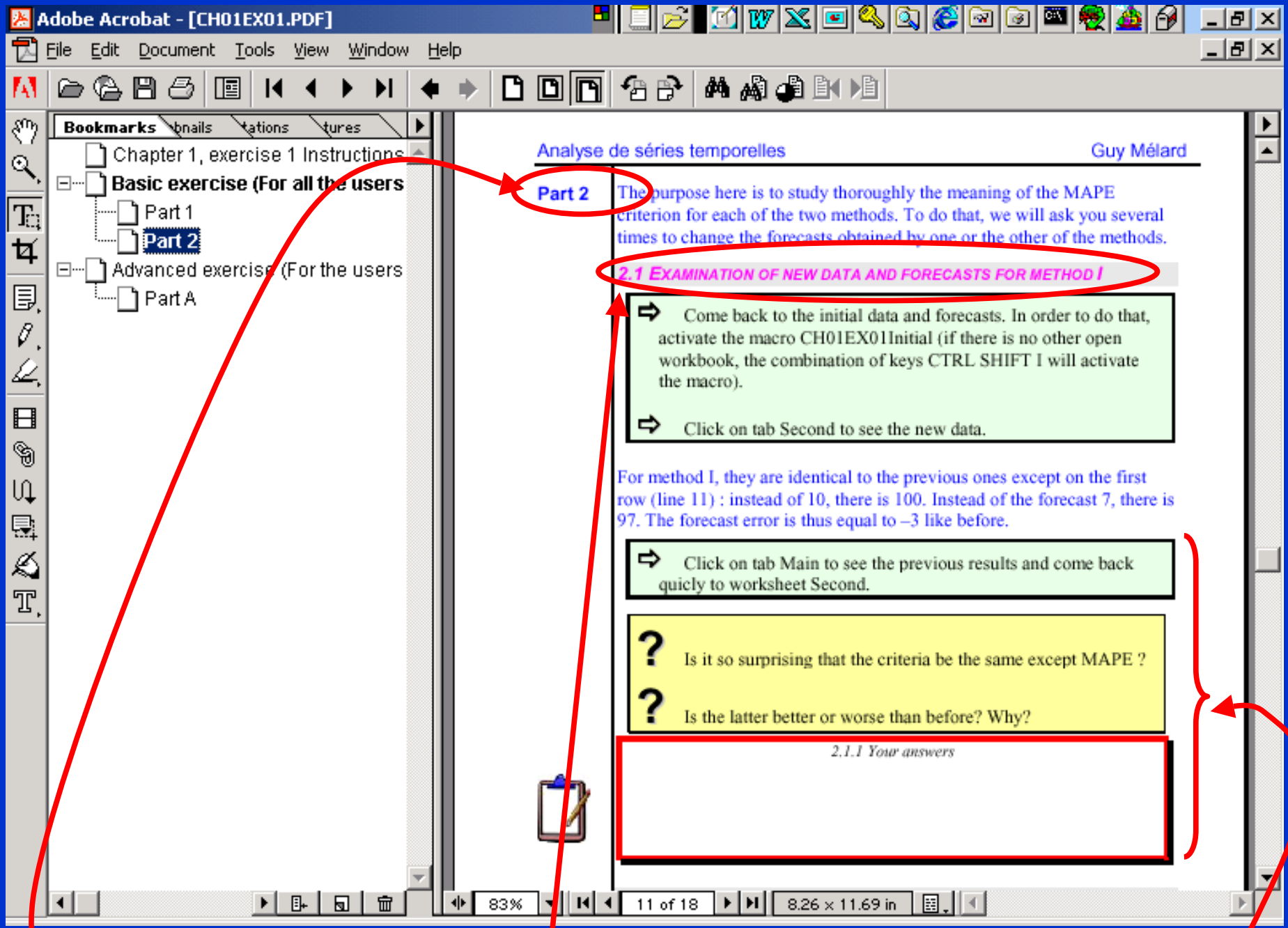
Access to the instructions, also delivered on paper

The exercise is directly run, here in Excel

Instructions for the exercises



Navigation within the instructions



part

section

instructions, questions, space for the answer

⇨ Click on tab Main to see the previous results and come back quicly to worksheet Second.

Instructions

? Is it so surprising that the criteria be the same except MAPE ?

? Is the latter better or worse than before? Why?

Questions

2.1.1 Your answers

Space for the answer

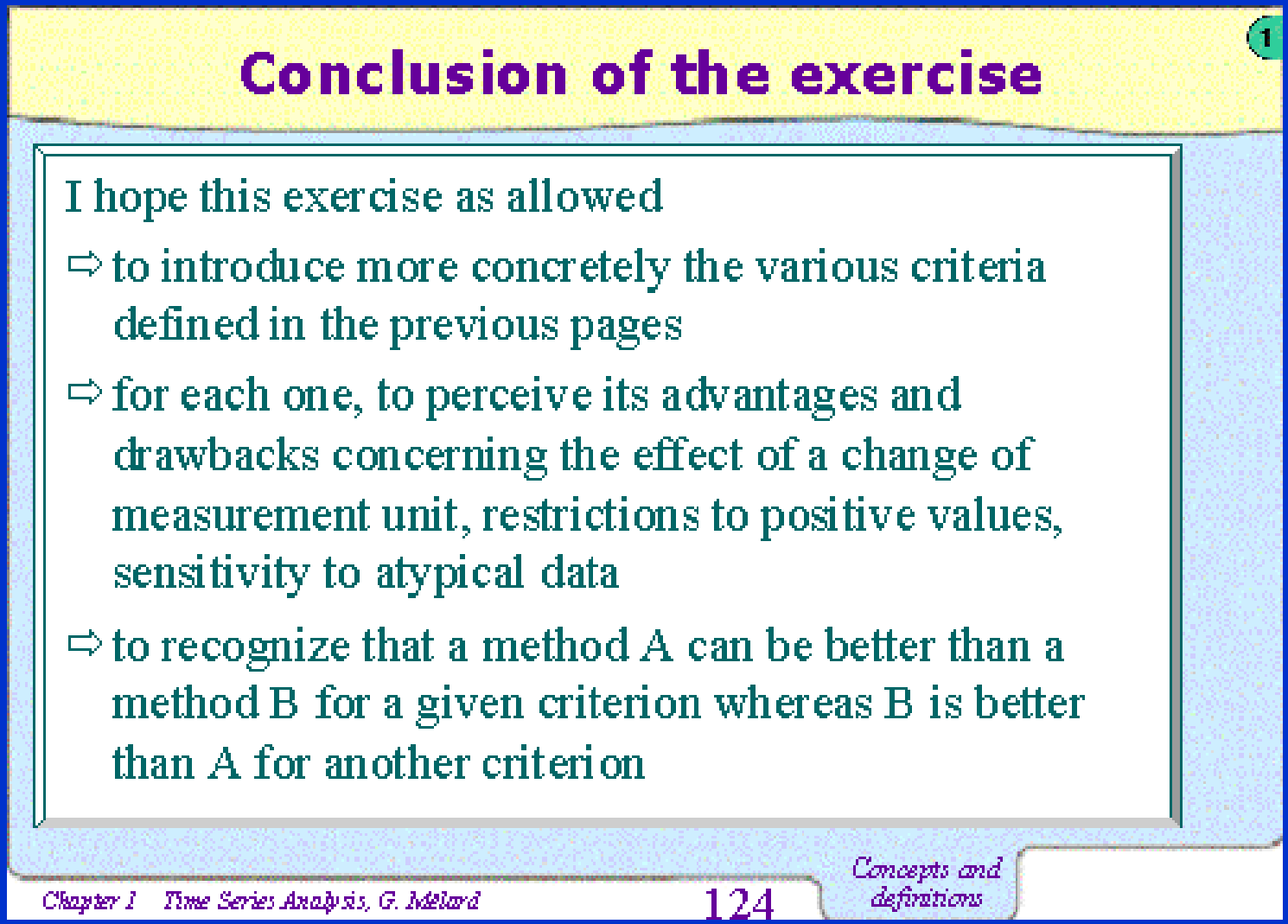
EXCERPTS FROM THE MAIN DOCUMENT OF THE CHAPTER

Part 2

2.1.1 The criteria have not changed because the errors have not changed. The exception is MAPE which is smaller because one of the data has been increased, hence the corresponding ratio in MAPE is smaller.

Conclusions of the exercise

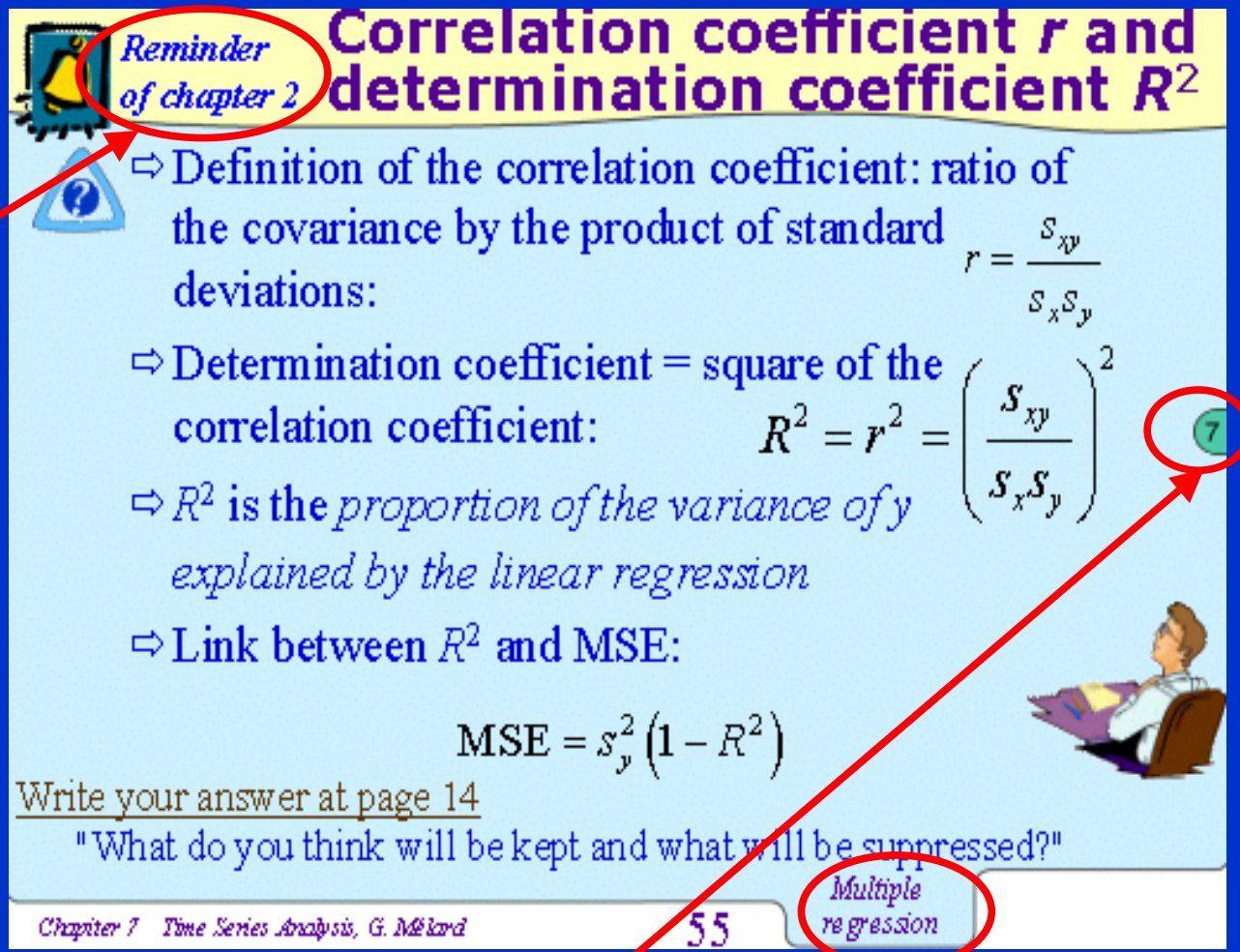
The main results of the last exercises are recalled



This allows to a learner already at a more advanced level to have a **sketch** of the contents of the exercises

Reminder of the material

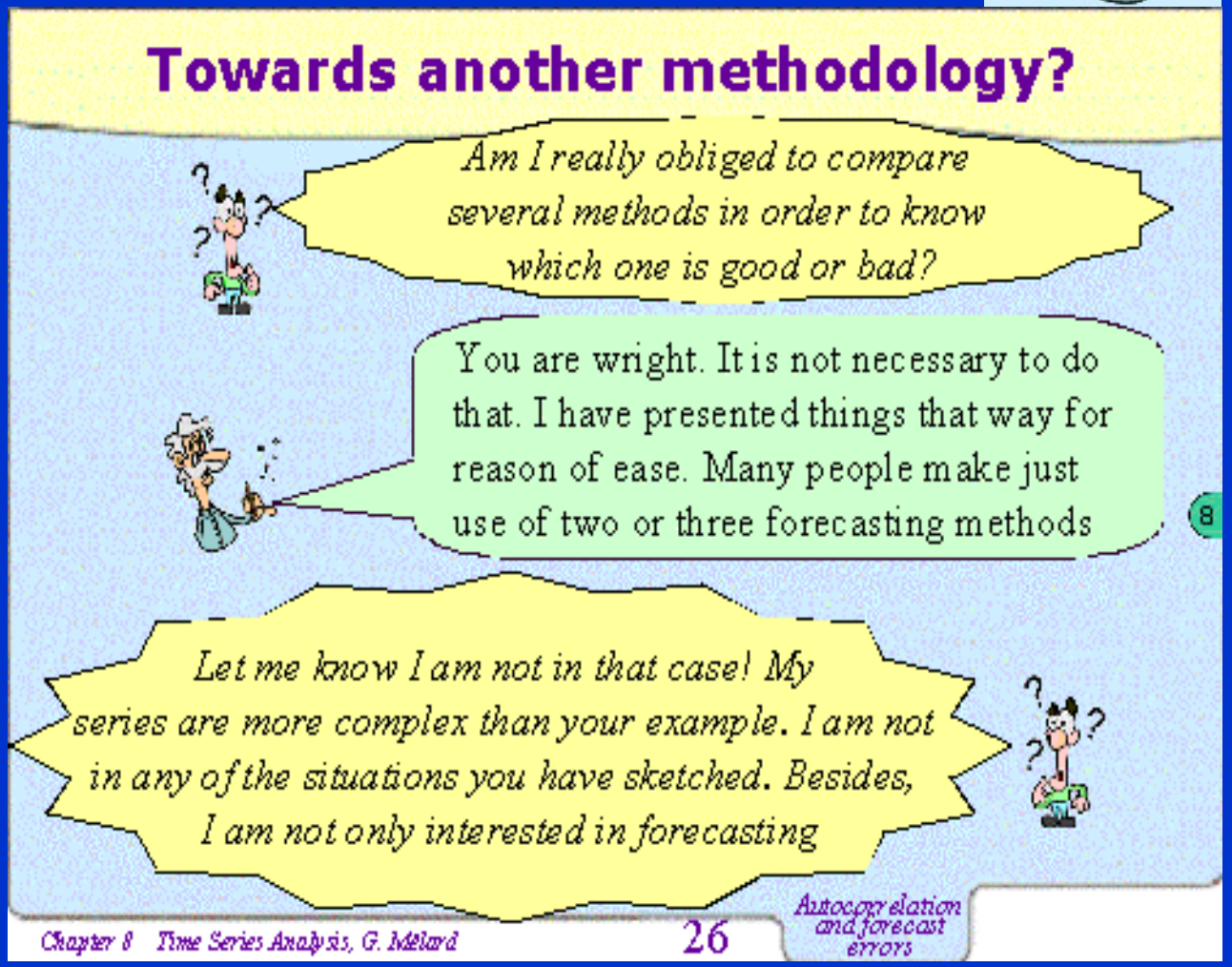
These **reminders** allow to situate the subject and to recall the terminology and the notations



Each page of the course has a **number** of the chapter and its **title** (abbreviated here)

The virtual learner and the virtual trainer

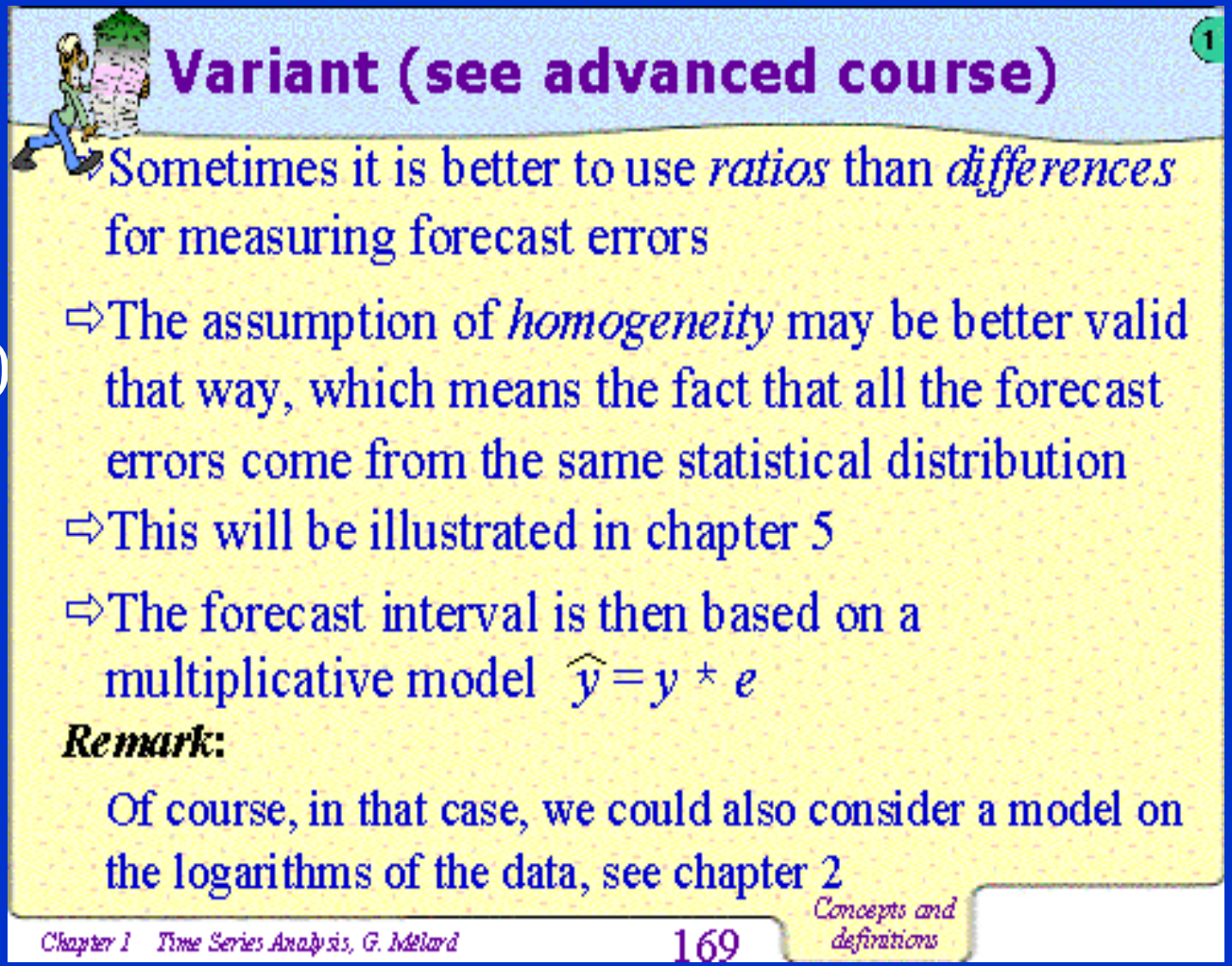
The class metaphor is used from time to time to lower the tension and improve the transitions



The course can be followed at two levels, basic and advanced. For two chapters (6 and 13), there are specific presentations for the advanced course

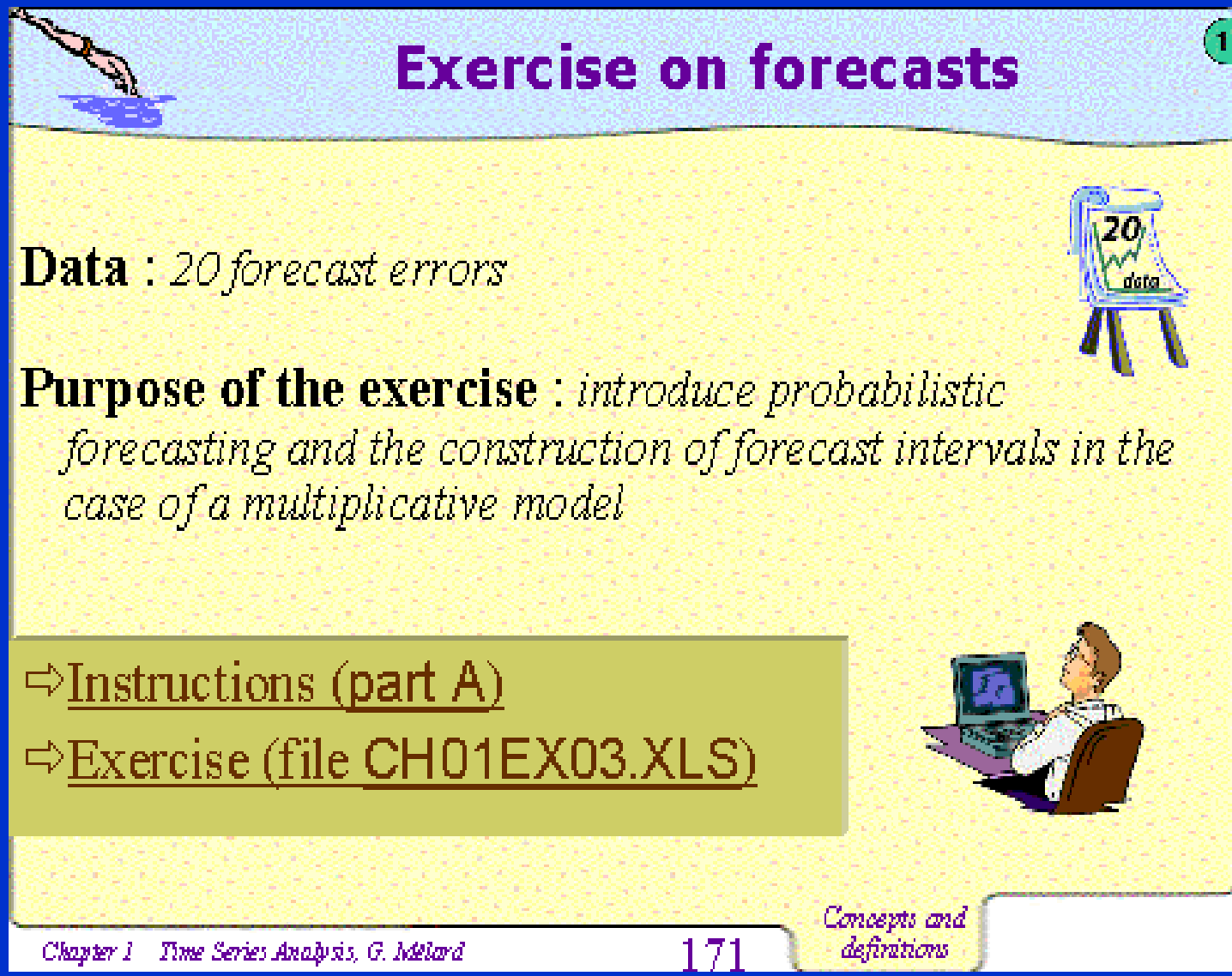
The advanced course

Some parts of the presentation (identified by a yellow background) are reserved to those learners wishing to achieve a more advanced training in the domain of time series analysis



The exercises of the advanced course

The exercises of the advanced course are indentified (with a yellow background again ; parts are identicated using letters instead of numbers)



Back to the basic course

When several pages are devoted to the advanced course, where to go back to the basic course is indicated.

Global test of randomness (2/4)

- The test statistic bears the name of Box and Pierce:

$$Q = T\sum_{k=1}^{K} r_k^2 \qquad \chi = \text{greak letter chi}$$

- If the process is white noise and if T is large enough, the distribution of Q approximately follows a χ^2 law (chi-square) with K degrees of freedom
- We should therefore reject the hypothesis that the first K autocorrelations of the process are equal to 0 if the Q statistic is greater than the quantile of order 0.95 of the χ^2_K law
- Instead of Box-Pierce, we can use the Ljung-Box test

For the basic course → *page 180*

For the basic course → *page 180*

Table of contents

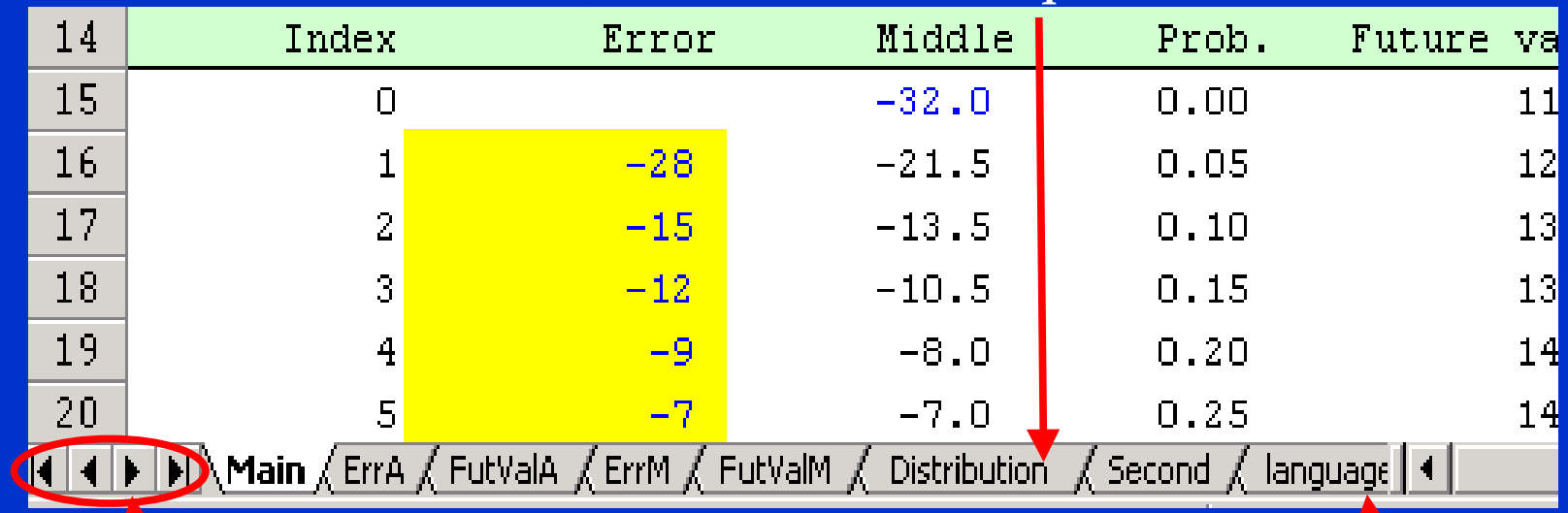
Exercises

⇨ Nearly all of them make use of a *software package*:

- ✓ **Excel** for many exercises in chapters 1 to 8, plus 13, including:
  - linear and nonlinear regression, exponential smoothing
  - seasonal decomposition on artificial series
  - details on the first steps of the Census X-11 method
  - spectral analysis and optimal filtering
- ✓ **Time Series Expert** for some exercises of chapters 2-11, mainly on real time series
- ✓ **Demetra** for chapters 12 (X-12 ARIMA) and 13 (TRAMO/SEATS)

Excel workbooks

To access a worksheet, click on the requested tab



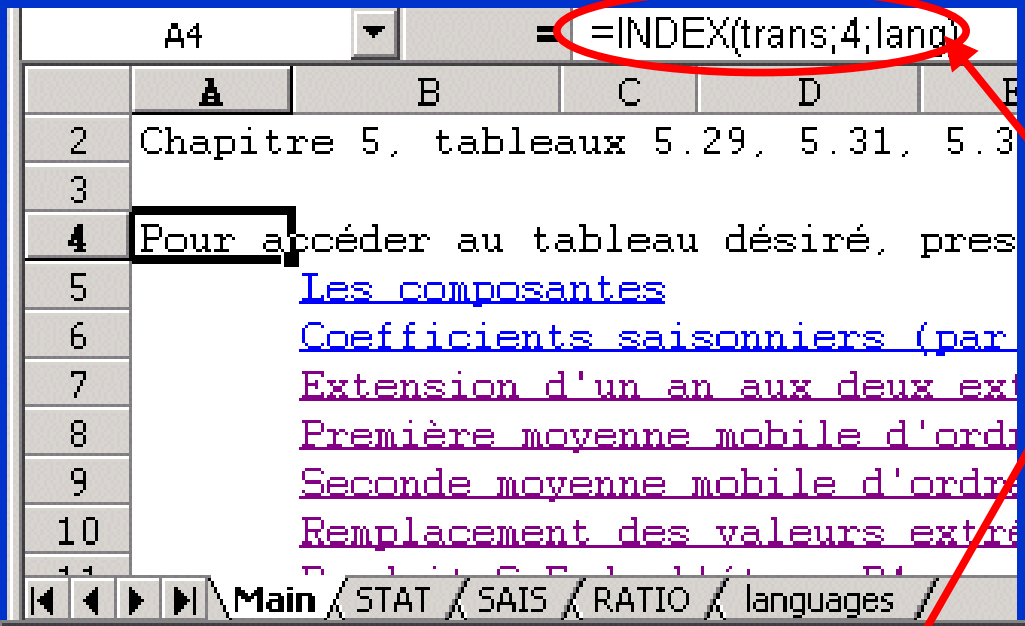
14	Index	Error	Middle	Prob.	Future va
15	0		-32.0	0.00	11
16	1	-28	-21.5	0.05	12
17	2	-15	-13.5	0.10	13
18	3	-12	-10.5	0.15	13
19	4	-9	-8.0	0.20	14
20	5	-7	-7.0	0.25	14

Main | ErrA | FutValA | ErrM | FutValM | Distribution | Second | language

Make use of the video buttons to access tabs.

Language worksheet (see below)

Language

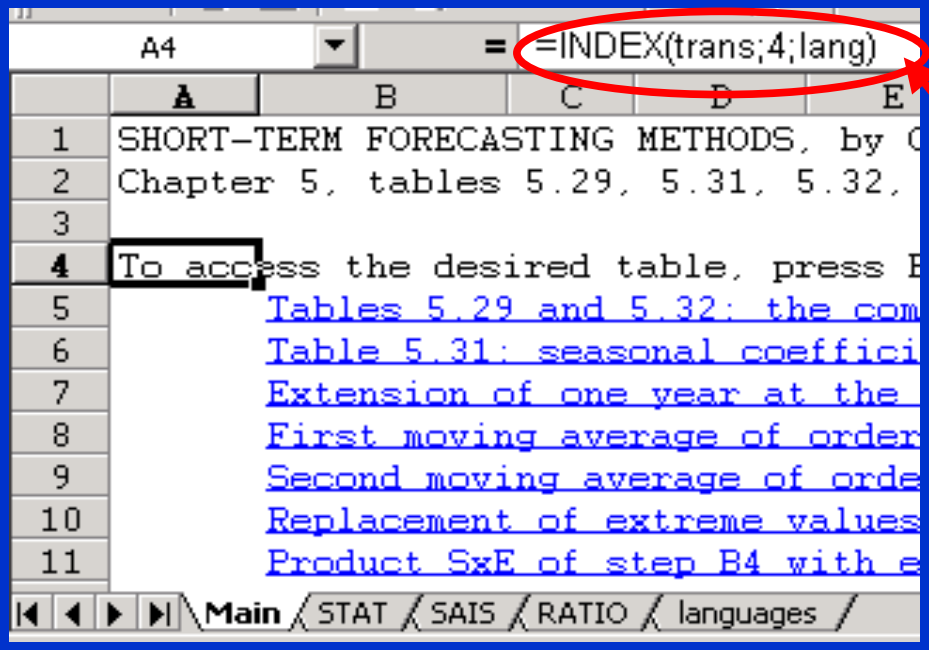


Workbooks are multilingual. All the texts are formulae which refer to a cell in the "languages" worksheet **here in French**

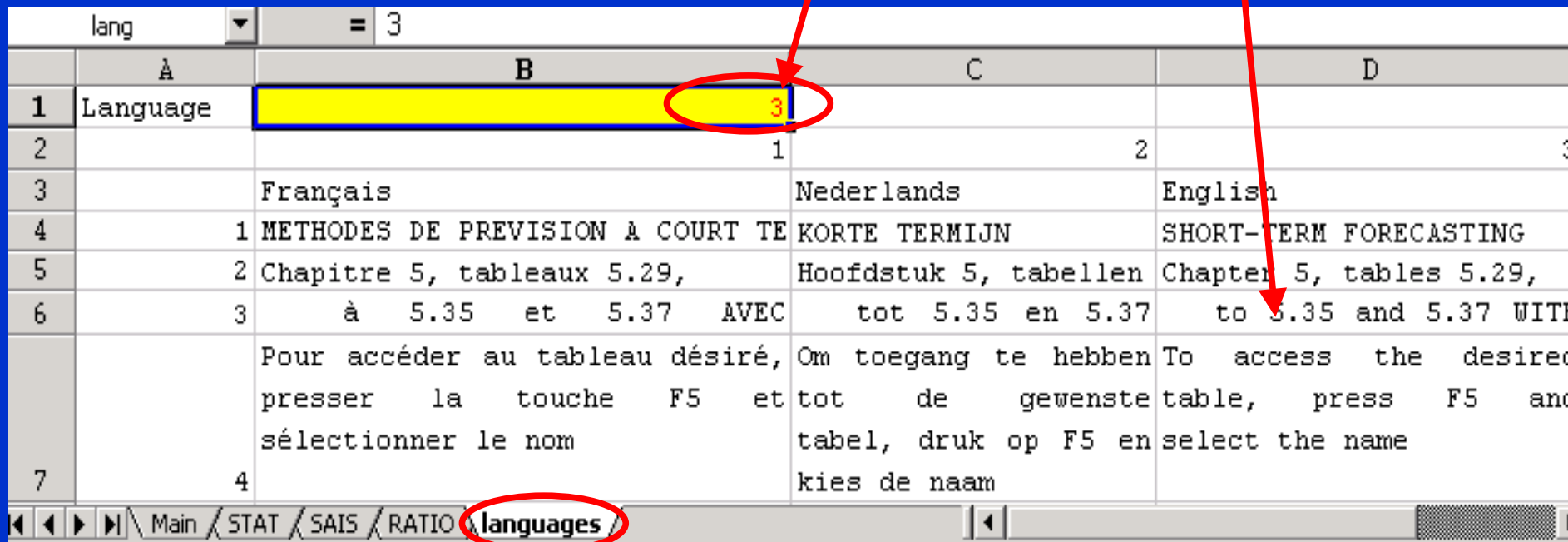
	A	B	C	D
1	Language	1		
2		1	2	3
3		Français	Nederlands	English
4	1	METHODES DE PREVISION A	KORTE TERMIJN	SHORT-TERM
5	2	Chapitre 5, tableaux	Hoofdstuk 5,	Chapter 5, tables
6	3	à 5.35 et 5.37 AVEC	tot 5.35 en 5.37	to 5.35 and 5.37
7	4	Pour accéder au tableau désiré, presser la touche F5	Om toegang te hebben tot de gewenste tabel, druk op F5 en	To access the desired table, press F5 and select
8	5	Les composantes	De componenten	The components

Main / STAT / SAIS / RATIO / languages

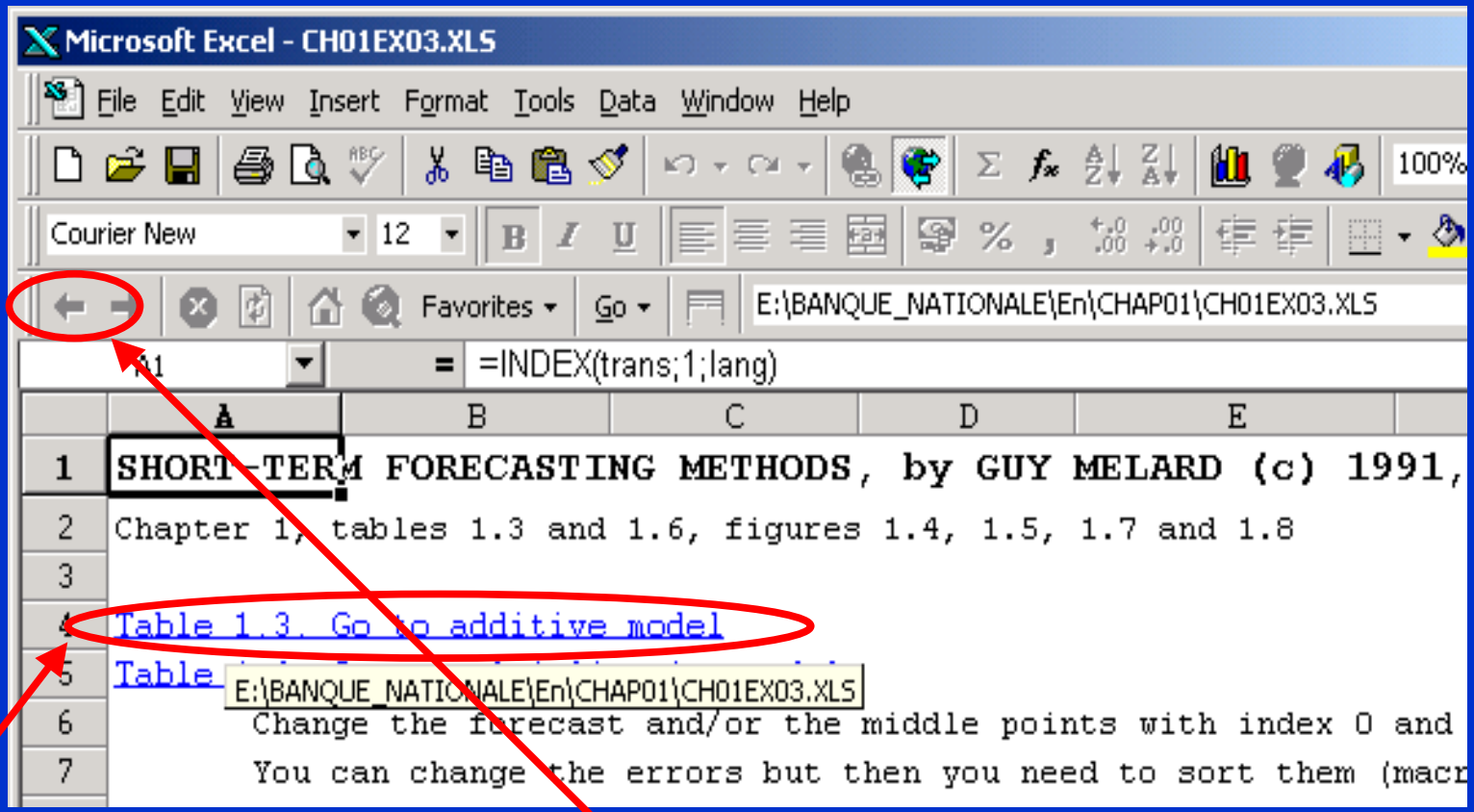
Language



Workbooks are multilingual.
All the texts are
formulae which refer to a cell
in the "languages" worksheet
here in English



To move in a workbook

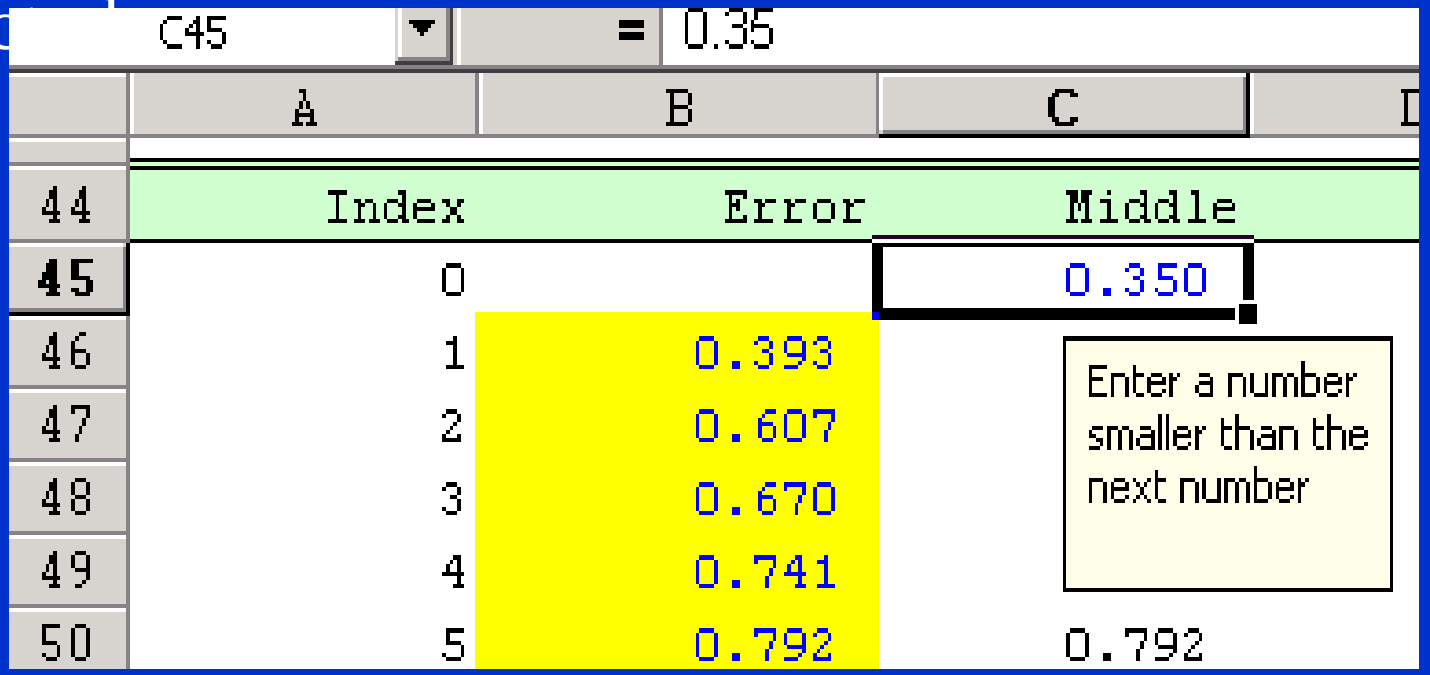


The simplest way: using predefined hypertext links.

- These fields are colored (blue or brown) and underlined
- Simply click on the link to reach the place
- Coming back can be done with the Back arrow on the Web toolbar

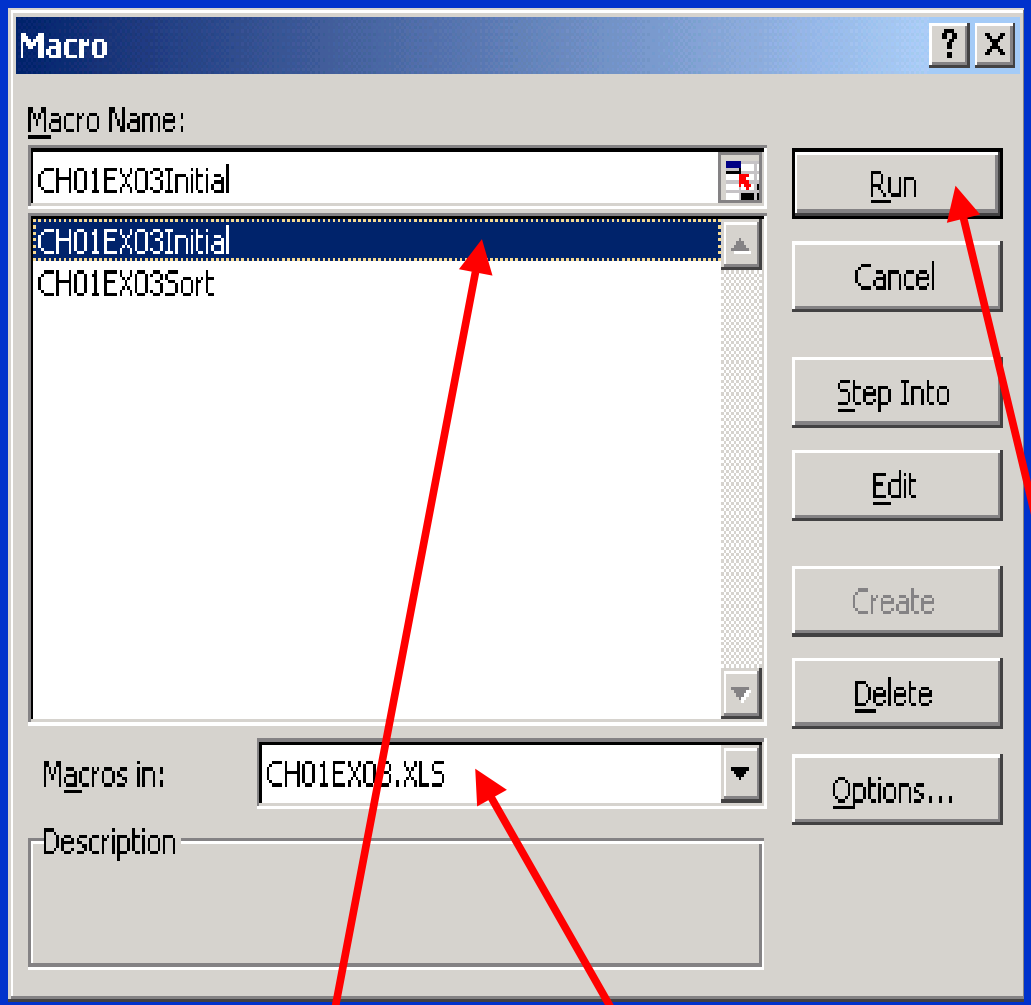
Data entry

Most data and formulae are already entered.
Therefore the sheets are protected except fields where parameters can be changed with a message indicating what is expec



There are sometimes constraints on the number being entered. The keyboard is also used to generate **random data** for simulation purposes (recalculation key F9)

Call a macro



Macros are used in order to:

- restore initial data after a modification
- perform animations

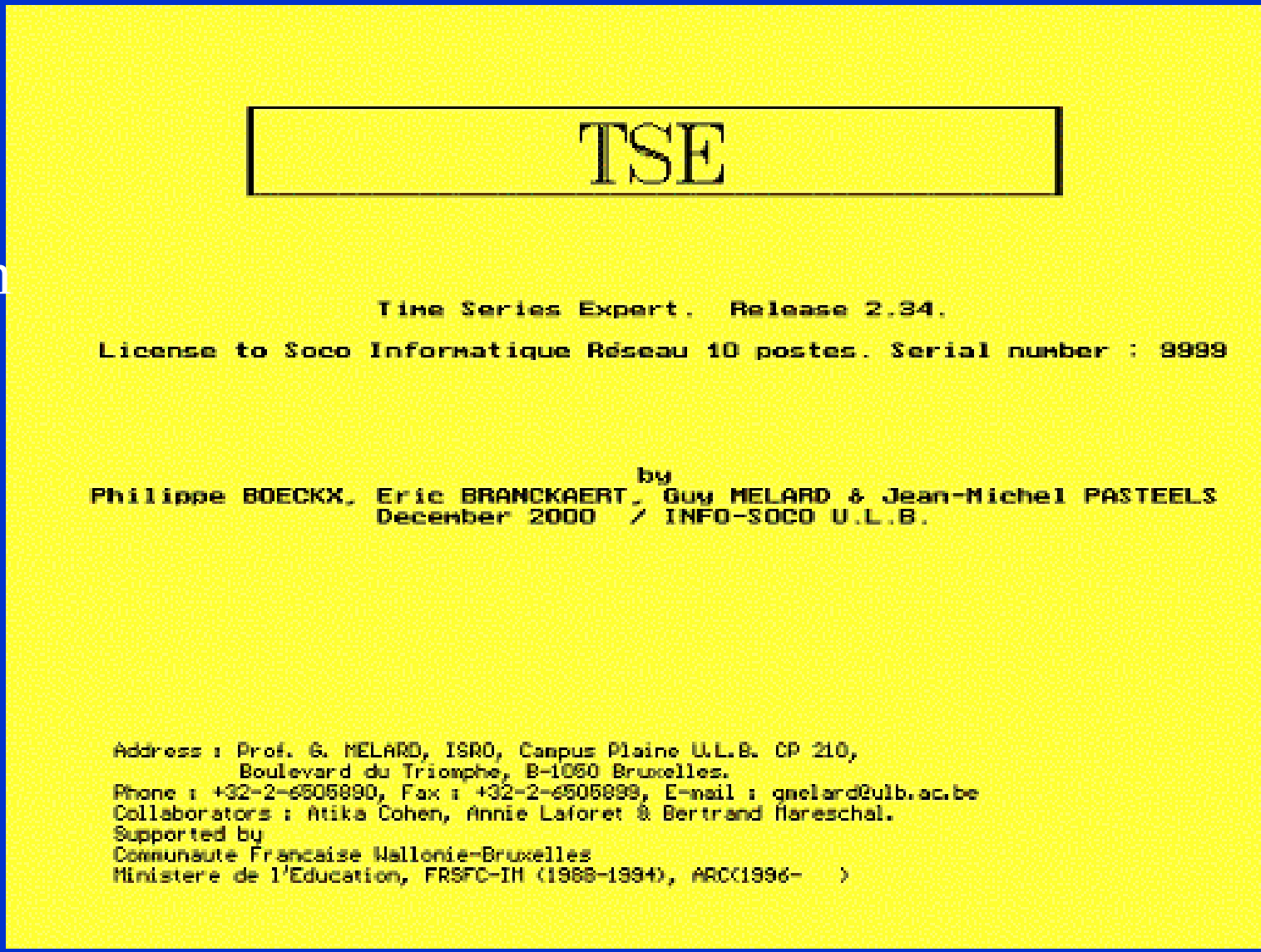
Other capabilities of Excel are also used:

- scenarios
- tables of hypotheses
- matrix calculation
- Solver module
- complex numbers

- Check if the name of the workbook appears.
- Select the name of the macro and click on Run.

Time Series Expert

We use an improved demonstration version of the home-made package, based on the one freely available on the Internet.



How to use it

First select the folder then load a prepackaged *problem* entirely prepared for the exercise

TSE 2.4 Plus

File Data Expertise Methods Configuration Options

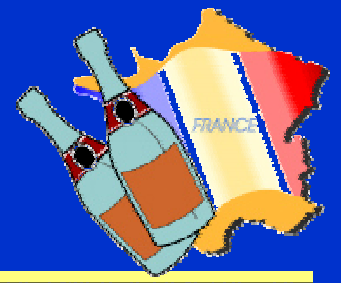
Changing data directory
Dos shell
Report (View, Print, Save)
Problem
Statistic (View, Print, Save)
Export
Import
cHanging data drive
Quit

Save
Load
Clear

User directory	E:\BANQUE~1\DATA\CHAP02\CH02EX01
Problem	PROB
	Method : None
Estimation sample	1 - 400 [1 - 400]
Forecasts to	400 [400]

⇨ File ⇨ Problem ⇨ Load ⇨ Holiday

Contents of a problem

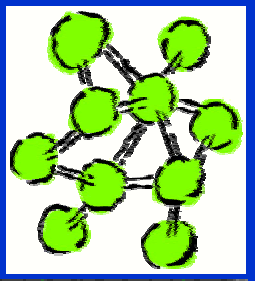


A problem includes the data series (CHAMPC), its dates, the method to be used, here exponential smoothing) and the options, for example under which name the forecasts will be stored, in this example : FORSES

```
                    Exponential Smoothing
Level & Trend                          : Level
Seasonality                            : None (add Level & Trend)
Intervention                           : Select Interventions
Power transformation (Box-Cox)         : 1.0
Parameter/deterministic seasonality    : Select Initial Values/Options
Save residuals                         : Yes on name: RESSES
Save forecasts or predictions          : Yes on Name: FORSES
Save fitted values                     : Yes on Name: FITSES
Leadtime for fit                       :   1
Forecast interval probability (%)      : 80
 {S} to save / {ENTER} to validate / {SPACE} to Change / {ESC} to Esca

      User directory | E:\BANQUE~1\DATA\CHAP06\CH06EX05
             Problem |    CHSES
  Dependent variable |      CHAMPC     Method :  Exponential smoothing
   Estimation sample | 1962.01 - 1969.12          [1962.01 - 1995.04]
       Forecasts to |           1970.09                     [1995.04]
```

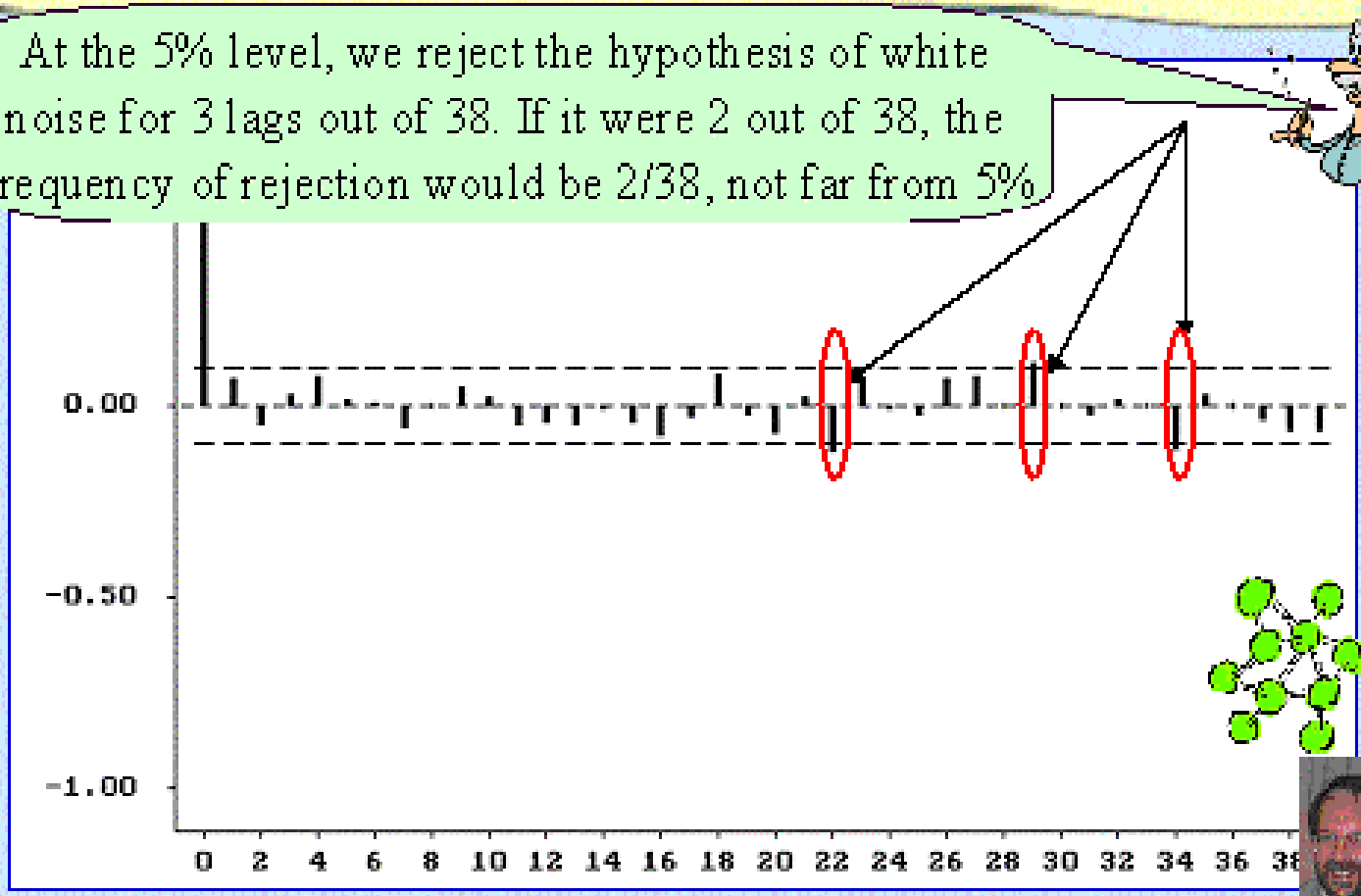
Graphical aspects



The accent is put on graphical aspects, for
- data plots
- scatter diagrams
- autocor-relations

The correlogram of series BLANC

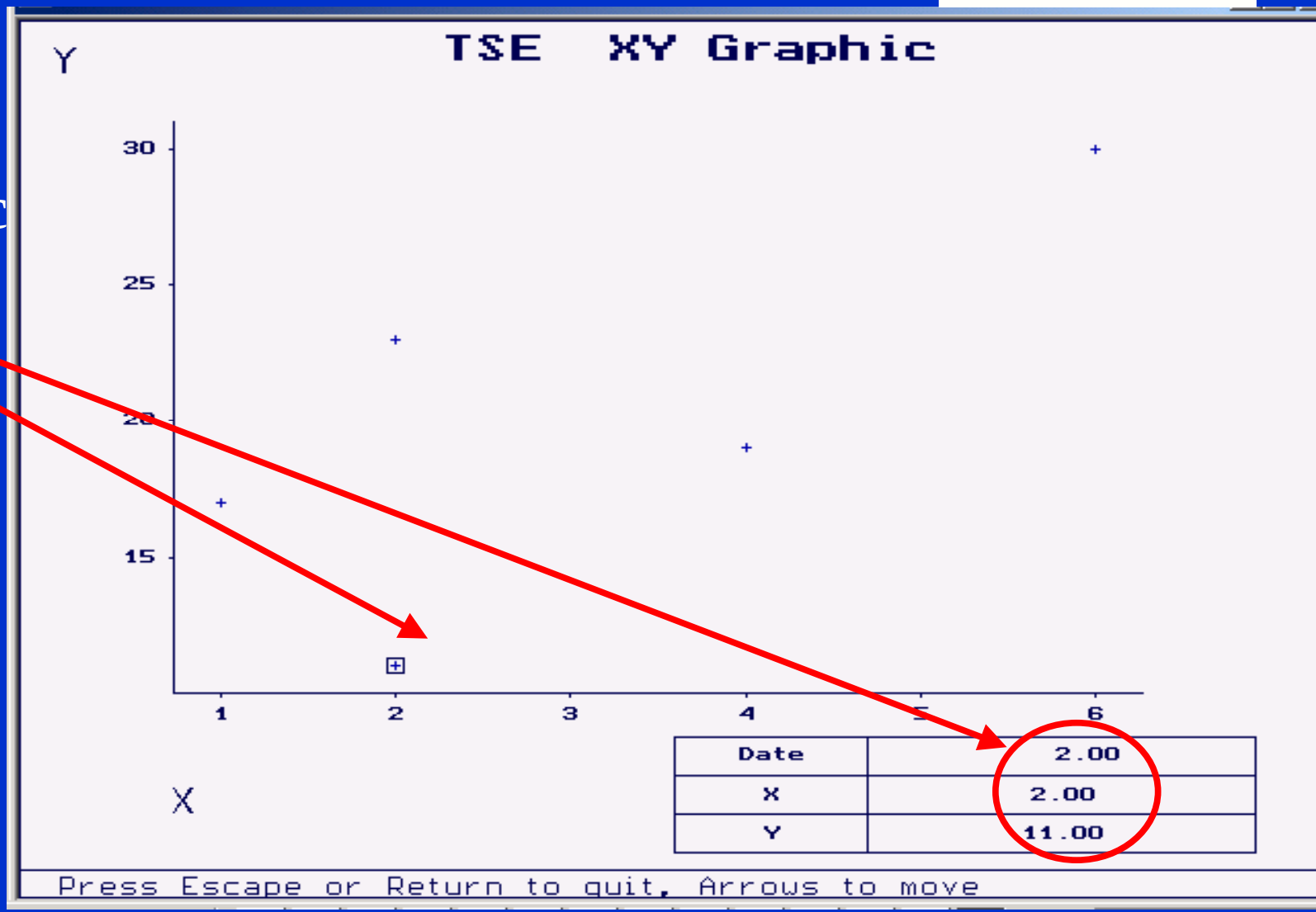
At the 5% level, we reject the hypothesis of white noise for 3 lags out of 38. If it were 2 out of 38, the frequency of rejection would be 2/38, not far from 5%



Chapter 8 Time Series Analysis, G. Midland 120 Autocorrelation and forecast errors

Graphical aspects

Graphs are used in a dynamic and interactive way



DEMETRA

DEMETRA 2.0

Copyright © European Communities, 1999-2001

DEMETRA Version 2.0 (Service Pack 1) (Oct 7 2001)

developed for Eurostat mainly by:
Jens Dossé and Servais Hoffmann

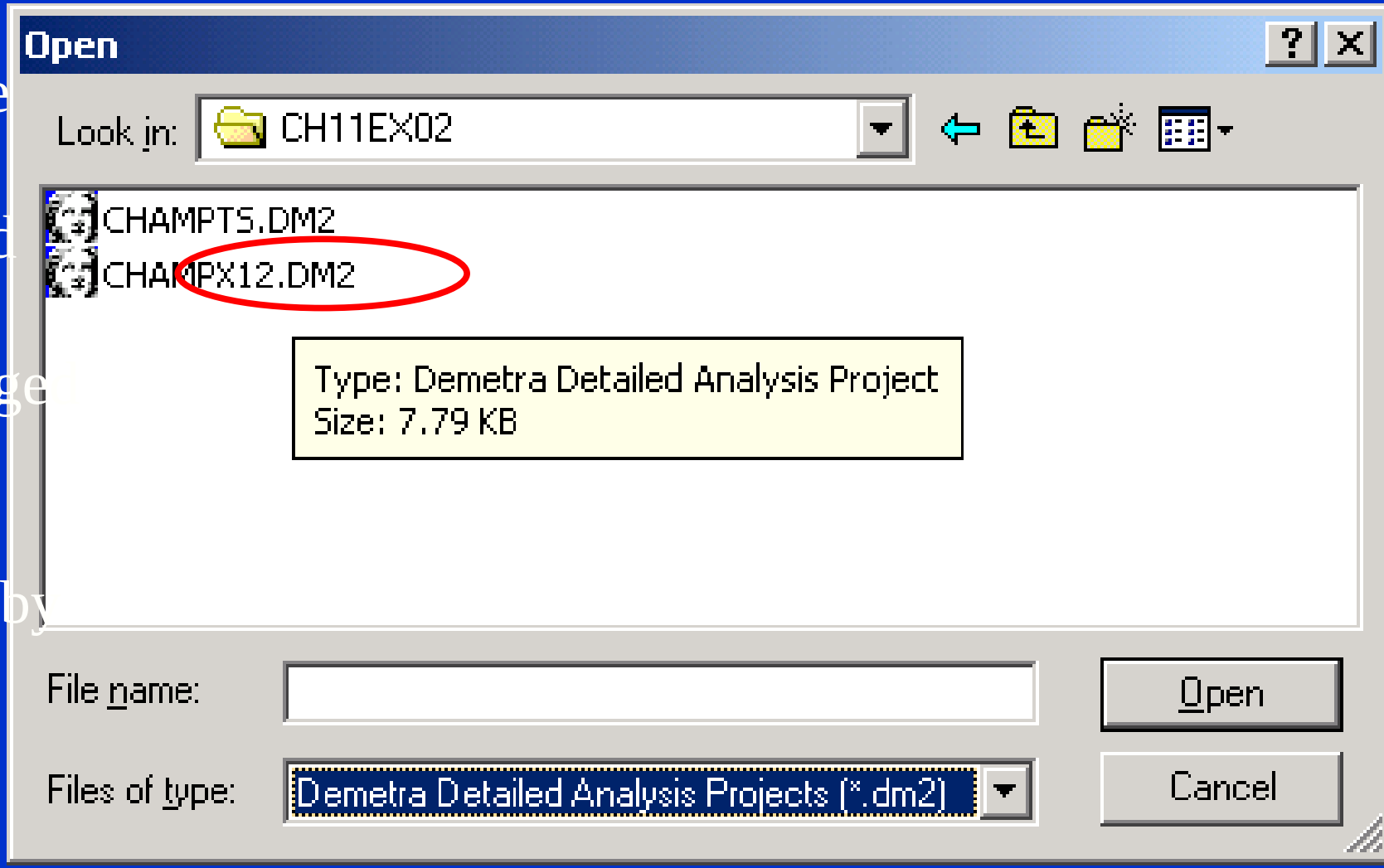
with help from:
Raoul Depoutot, Pierre Kelsen and Christophe Planas

based on:
TRAMO & SEATS by Víctor Gómez, Agustín Maravall and Gianluca Caporello
X-12-ARIMA by the US Bureau of the Census with David Findley, Brian Monsell, William R. Bell, Marc C. Otto, Bor-Chung Chen and Catherine Hood

Developed by the European Communities (EuroStat), as an interface to both X-12 ARIMA (Bureau of the Census) and TRAMO/SEATS (V. Gómez et A. Maravall, Bank of Spain)

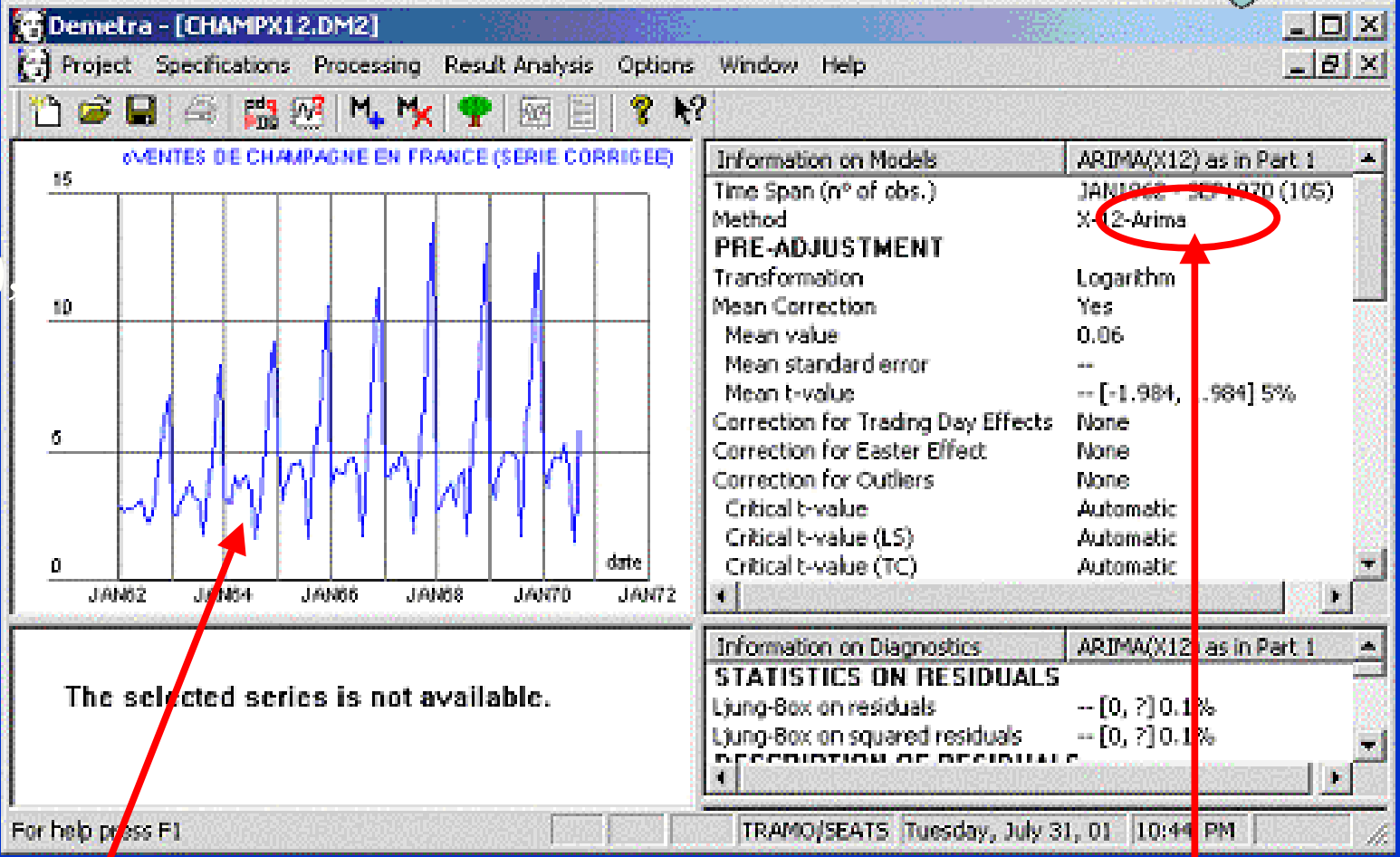
How to use it

Select the folder
Then load the prepackaged *project* entirely prepared by advance



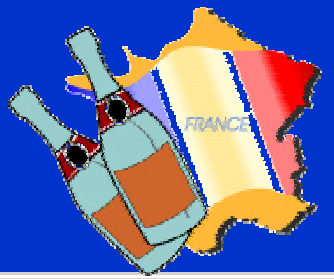
Contents of a project

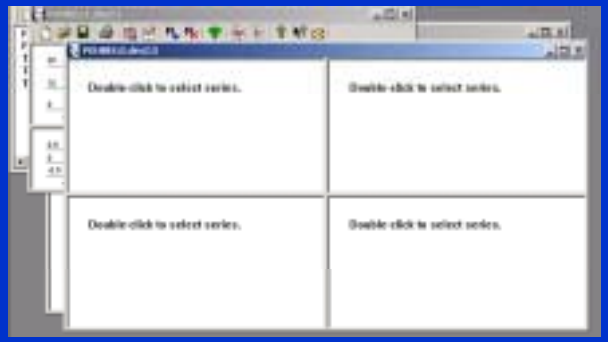
A project refers to a series (**CHAMPC**), its dates, the method to be used and options, for example the kind of parameters.



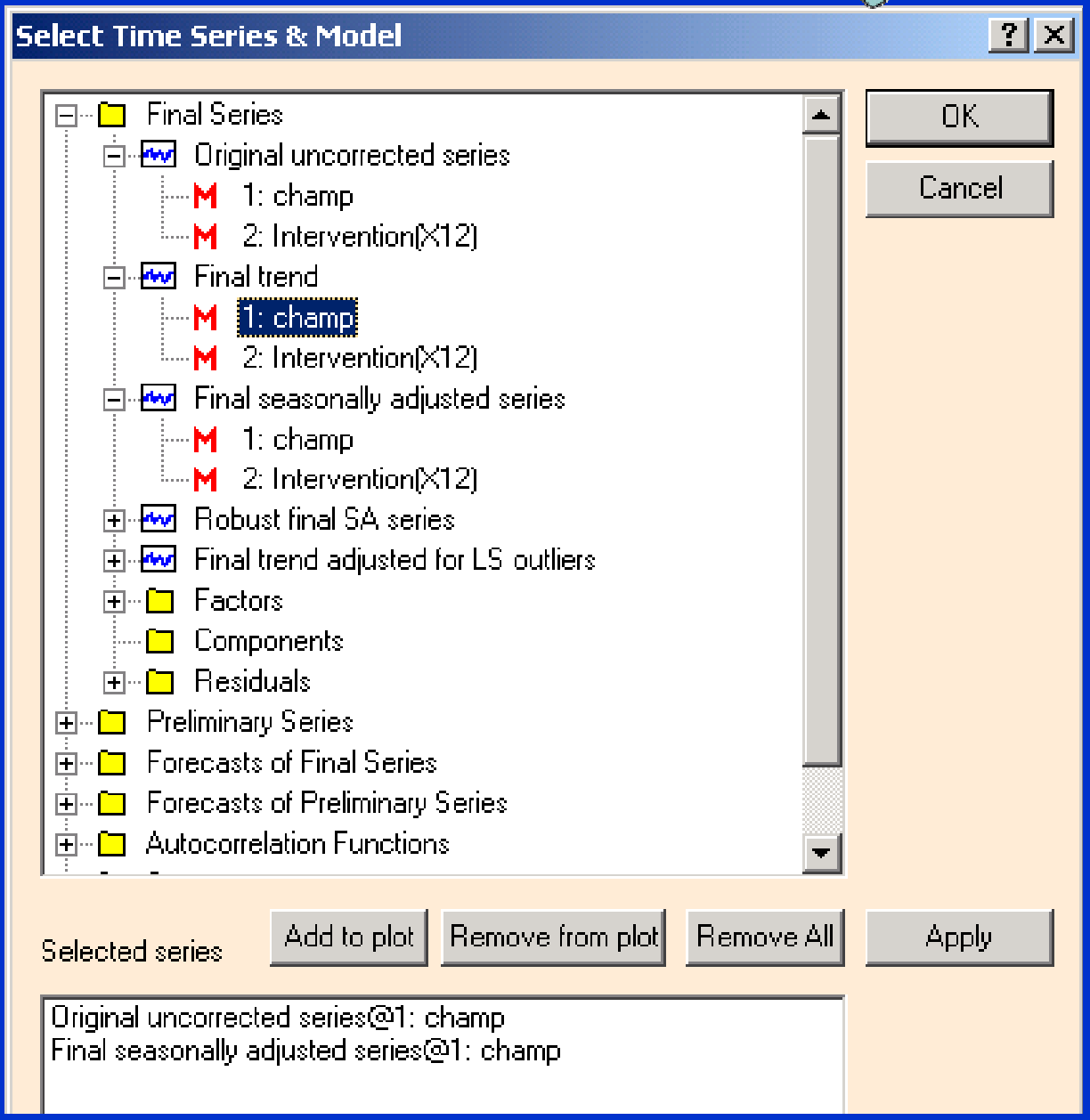
We see the data plot, the models being tried, here with name **ARIMA(X12)** and the method to be used, here **X-12-ARIMA**.

Graphical aspects



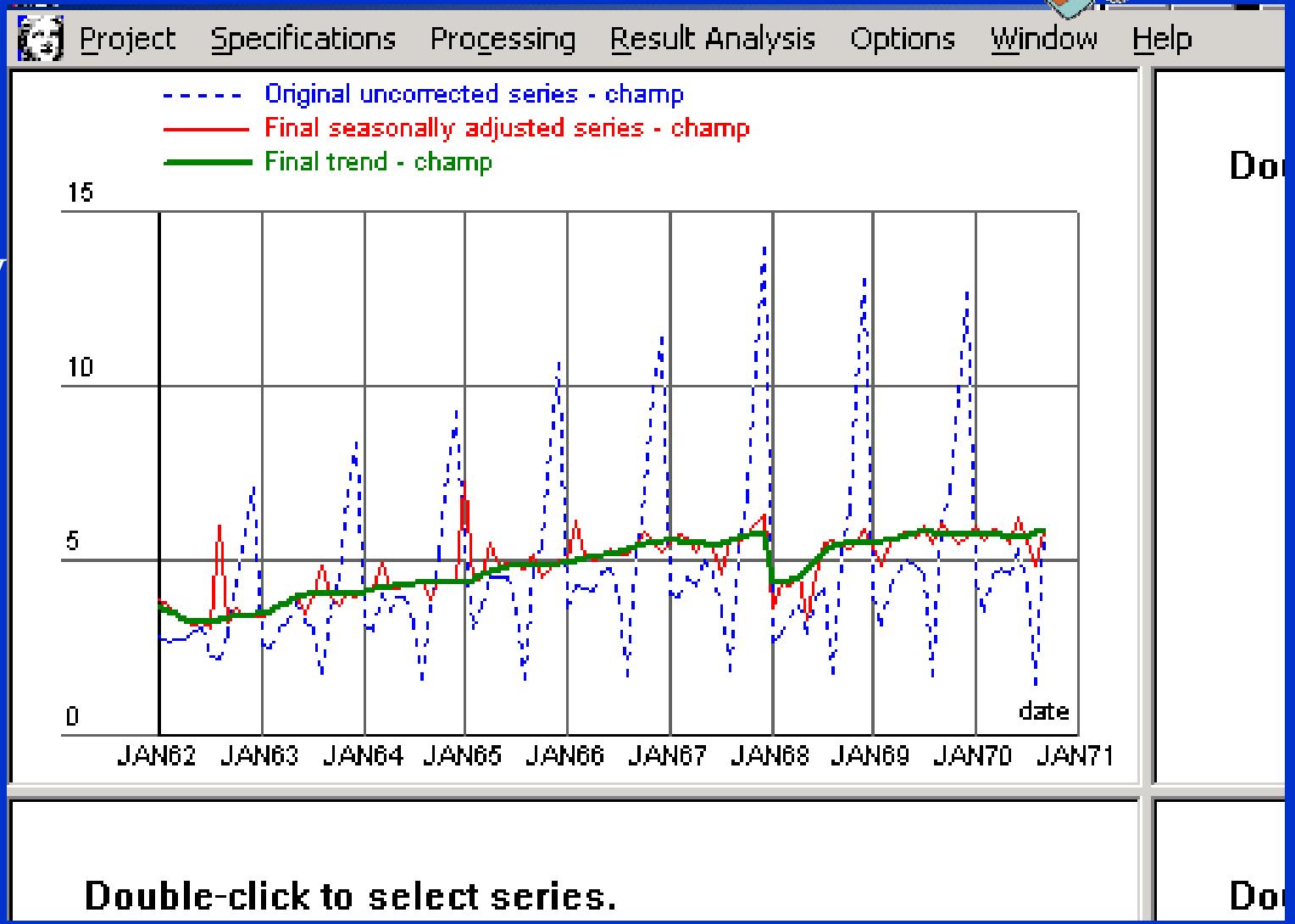


The accent is put on graphical aspects, plots w.r.t. time, autocorrelations and spectra



Graphical aspects

Graphs are used in a relatively interactive way



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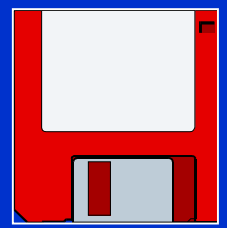
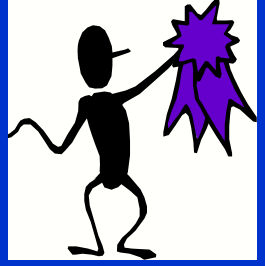
Main paper document

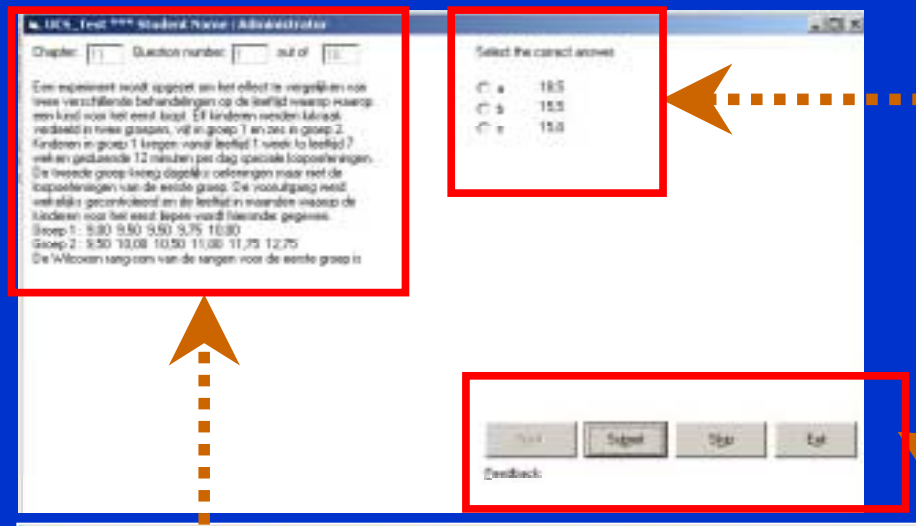
- ⇨Only in paper form
- ⇨Available separately for each chapter
- ⇨Contents
  - ✓space for answering questions raised in the presentation
  - ✓expected answers to these questions
  - ✓corrected exercises
  - ✓instructions for additional exercises
  - ✓corrected additional exercises
  - ✓references
  - ✓in an annex: a paper copy of the presentation and of the the instructions for the exercises

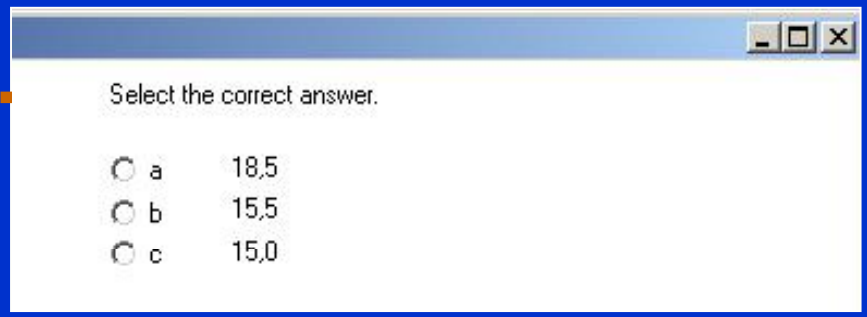




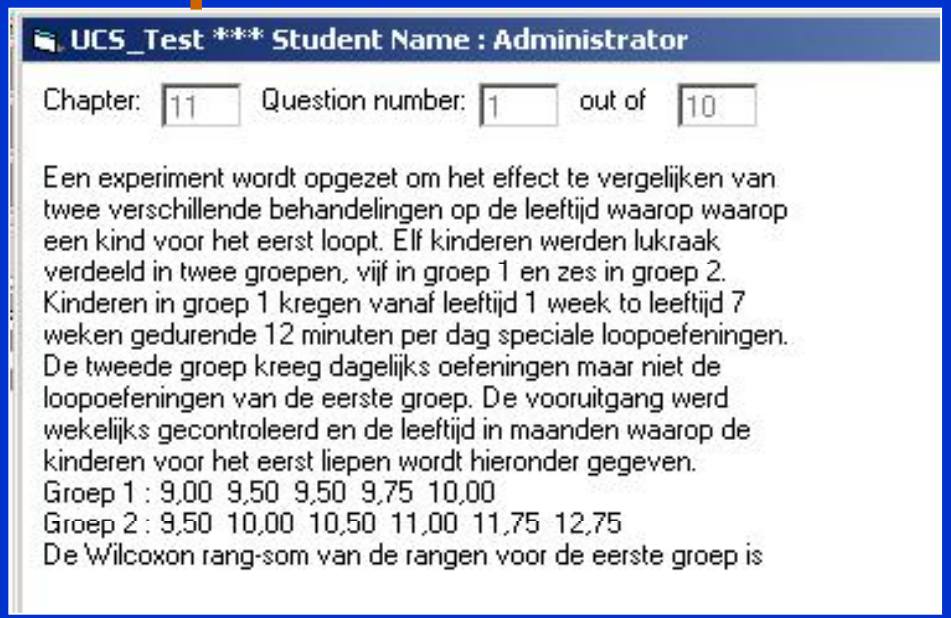
Assessment

Universitair Centrum voor Statistiek





(software package made by UCS, KULeuven)



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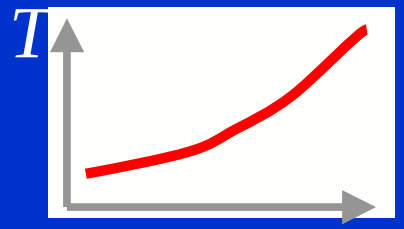
Excerpts of the advanced course

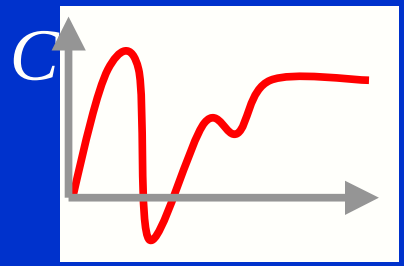
- ⇨We show here some excerpts of the course, based mainly on the advanced course
- ⇨Chapter 13 : TRAMO/SEATS method
- ⇨This is a seasonal adjustment method: how to obtain the seasonally adjusted series

General presentation

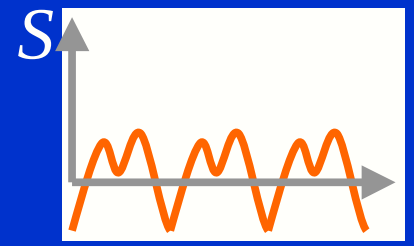
⇨ trend (T)



⇨ business cycle (C)

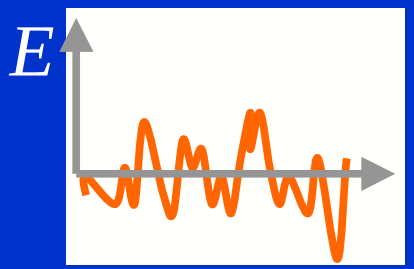


⇨ seasonal component (S)

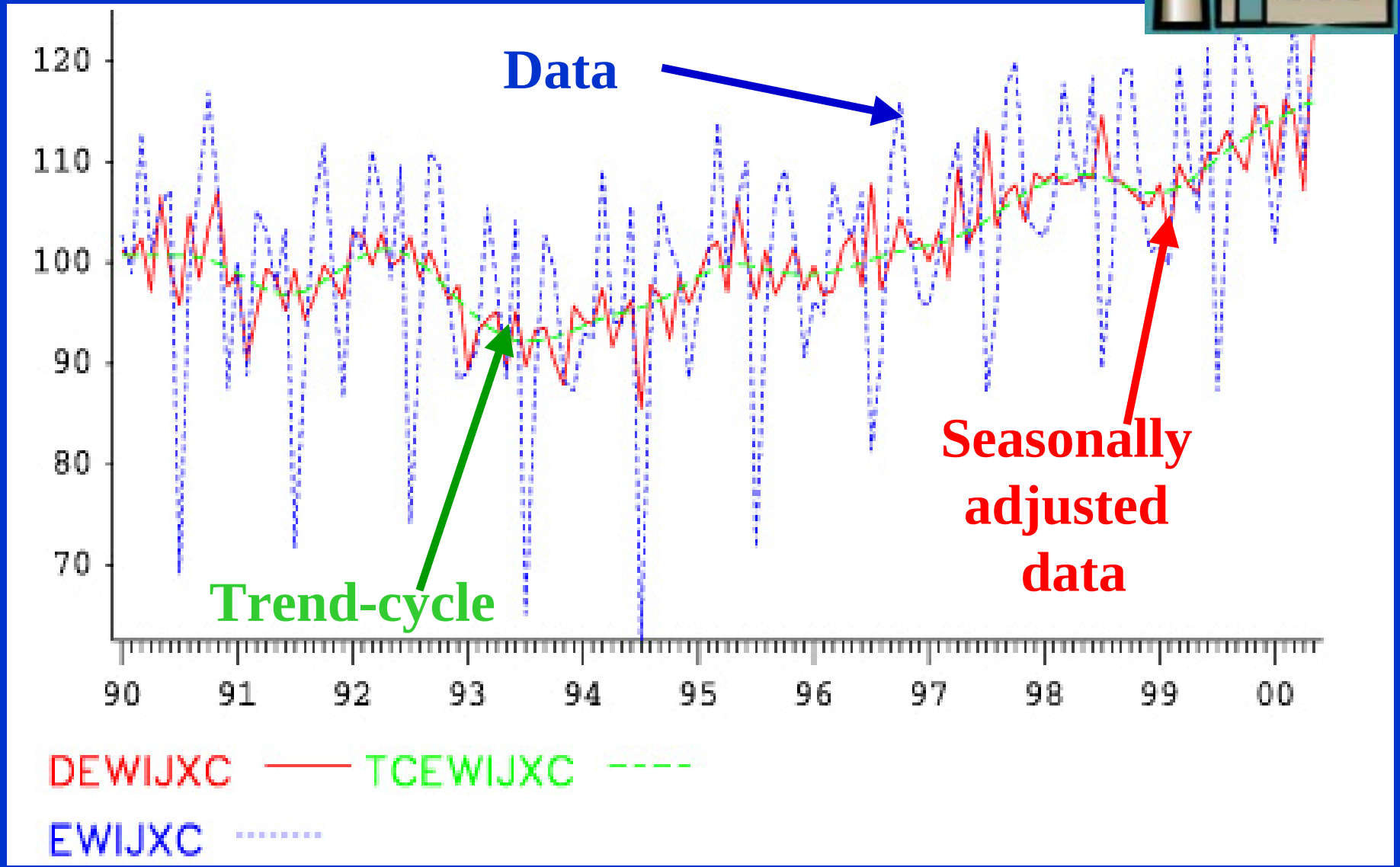


Here,
3 years
are shown

⇨ irregular component (E)



Example : Industrial production of Belgi



Advanced methods of seasonal decomposition

⇨Census X-11 (1957, 1967, 1988)

⇨X-11-ARIMA (Statistics Canada) :

replaces artificial extensions of the moving averages by using forecasts of the data obtained by an ARIMA model ***(see chapters 9 and 10)***

⇨Census X-12-ARIMA : (Bureau of the Census)

the same + ARIMA regression for the treatment of corrections (outliers, trading day, ...)
(see chapter 12)

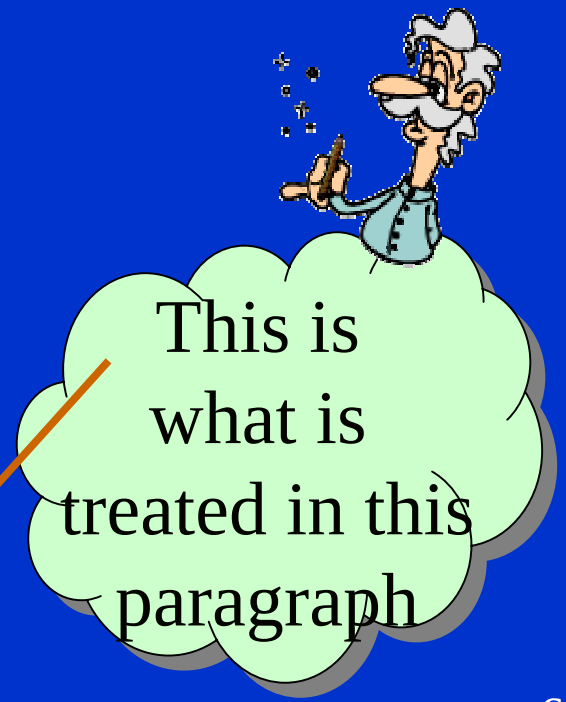
⇨TRAMO/SEATS (Bank of Spain) :

based on ARIMA models and signal extraction
(see chapter 13)

Flow chart of the 2 programs

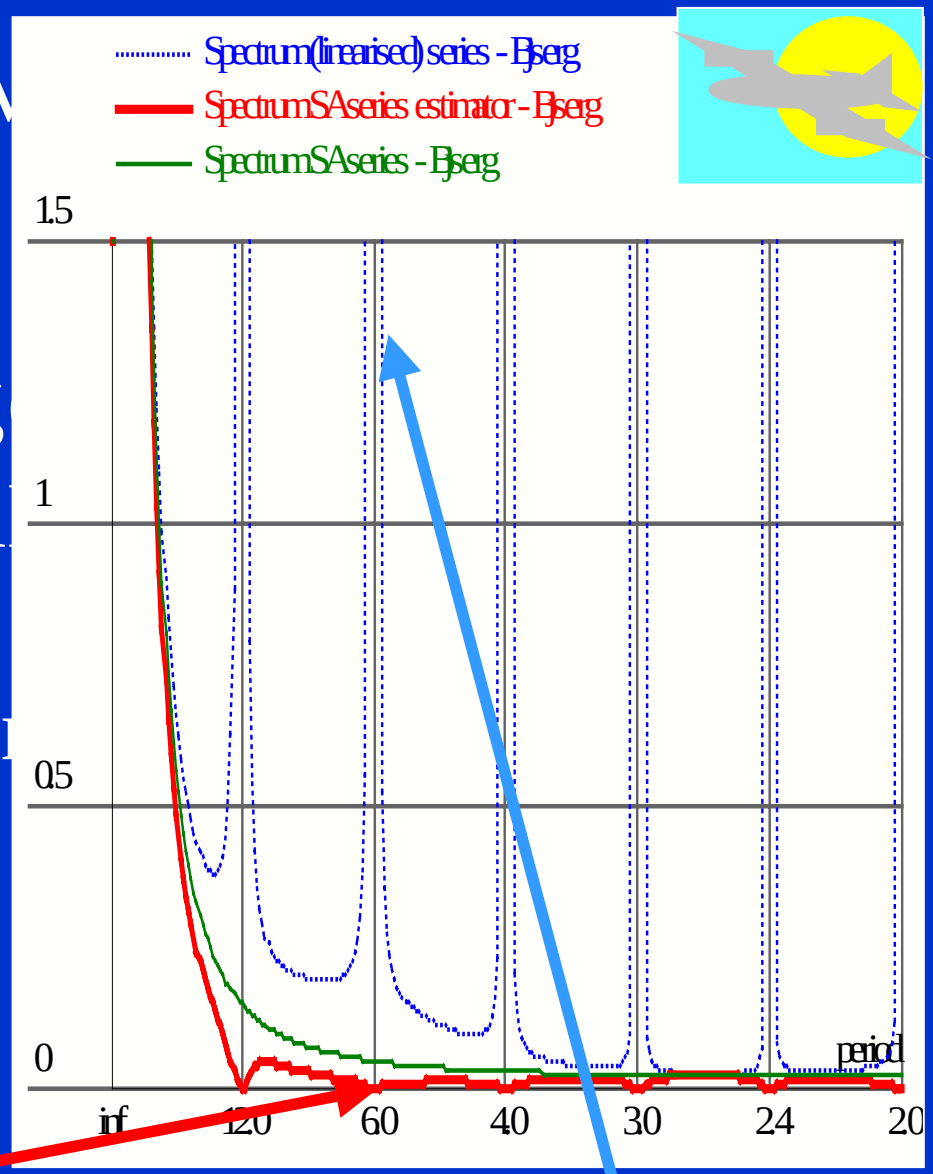
X-12-ARIMA
TRAMO/SEATS

- **Preliminary adjustment**
  - **Trading day correction**
  - **Easter correction**
  - **Detection and correction of outliers**
  - **Treatment of missing data**
- **ARIMA models**
- **Seasonal decomposition**



Motivations in favor of TRAMO/SEATS

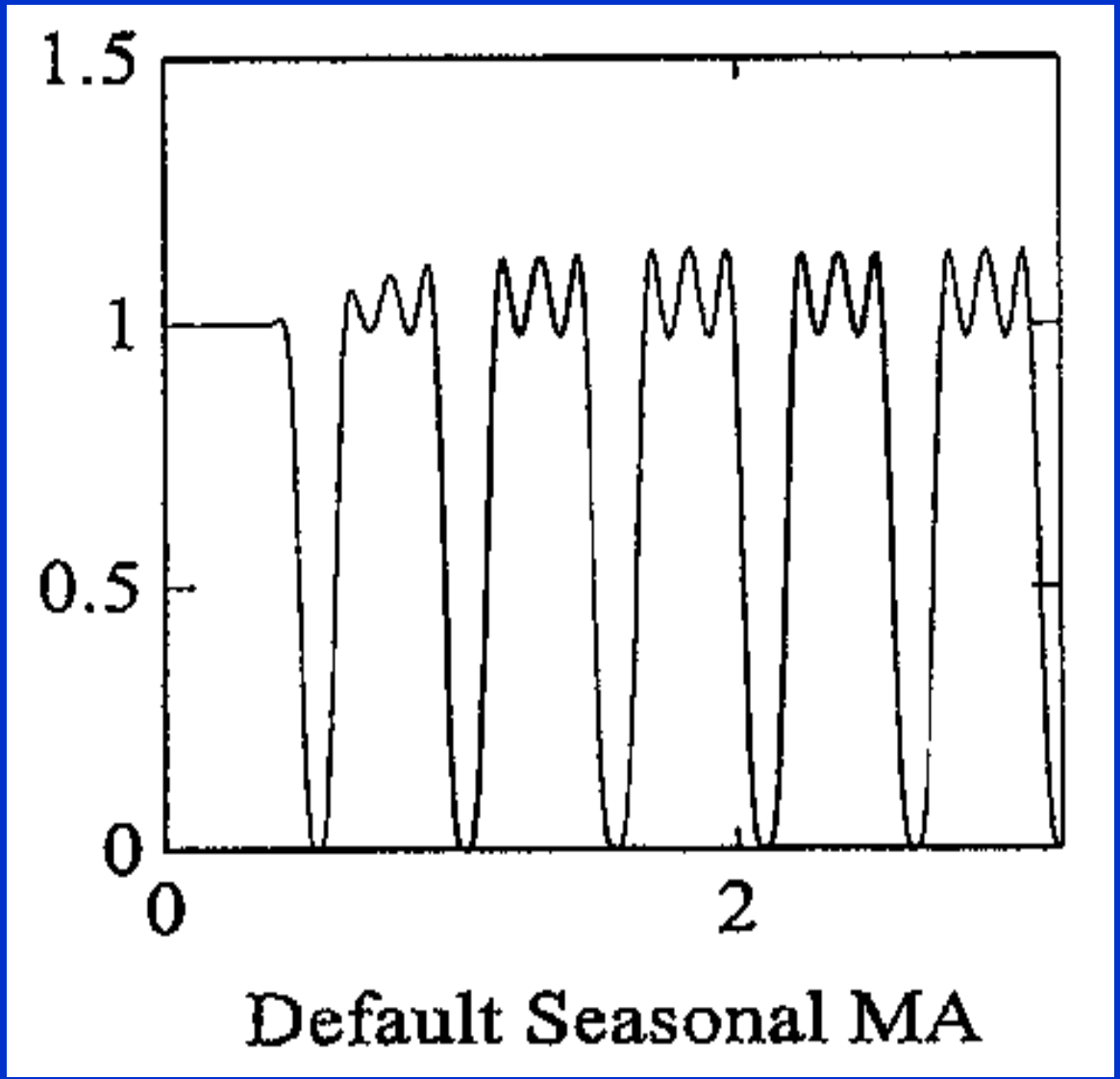
- ⇨ Advantage of X-12-ARIM
  continuity w.r.t. X-11
- ⇨ In X-12-ARIMA: filters
  based on moving averag
- ⇨ For Gómez and Maraval
  - ✓ *spectral analysis*
    reveals some proble
  - ✓ Filters depend
    on the series



Seasonally adjusted series

Original pretreated series

Gain for component S



Monthly series

It can be seen that the default filter completely removes period 12 and the divisors of 12 (6, 4, ...), which is too strong, and that it does affect slightly the other periods in an annoying way

Principles of TRAMO/SEATS (1/3)

- ⇨ First TRAMO is used, then SEATS
- ⇨ TRAMO is aimed for pretreatment (correct the series
  for outliers, trading day effects and other effects) and build a model for the series
  TRAMO = '**T**ime Series **R**egression with **A**RIMA Noise, **M**issing Observations and **O**utliers'
- ⇨ In the sequel, y_t denotes the corrected data at t
- ⇨ SEATS is aimed at decomposing the model obtained by TRAMO in a sum of components and extract them
  SEATS = 'Signal Extraction in ARIMA Time Series'

Principles of TRAMO/SEATS (2/3)

- Here : the case of an **additive** mode of composition
- There can be up to 4 components :
  - ✓ permanent or trend (-cycle) component
  - ✓ temporary or cycle component
  - ✓ seasonal component
  - ✓ irregular component or error
- In the most frequent cases of a **multiplicative** mode of composition, SEATS works with an additive decomposition of the data in **logarithms**, $\log(y_t)$

Principles of TRAMO/SEATS (3/3)

- In the simplest case : $y_t = N_t + S_t$, where
  - ✓ N_t is the non seasonal component
  - ✓ S_t is the seasonal component
- From the ARIMA model for y_t found by TRAMO, SEATS derives ARIMA models (with independent innovations) for N_t and S_t
- SEATS then *builds* series N_t and S_t from the data y_t using the models for y_t, N_t and S_t
- A signal extraction technique is used to that end

The components of SEATS (1/2)

Example (quarterly) ARIMA model : $\nabla\nabla_4 y_t = \theta(B)\, e_t$

- ✓ $\nabla = 1 - B$: the difference is associated to the permanent component or trend (B = lag operator)
- ✓ $\nabla_4 = 1 - B^4 = (1 - B)(1 + B + B^2 + B^3) = (1 - B)\, U_3(B)$, factorization of the seasonal difference
- ✓ We write Permanent c. Seasonal c. Irregular c.

$$y_t = \frac{\theta(B)}{(1-B)(1-B^4)} e_t = \frac{\theta_P(B)}{(1-B)^2} e_t^P + \frac{\theta_S(B)}{U_3(B)} e_t^S + e_t^I$$

- ✓ The variances V^P, V^S and V^I as well as $\theta_P(B)$ and $\theta_S(B)$ are determined by equating the

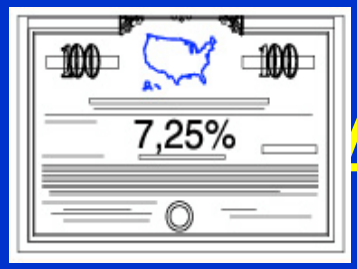
$$\nabla\nabla_4 y_t = \theta(B) e_t = U_3(B)\theta_P(B) e_t^P + \nabla^2 \theta_S(B) e_t^S + \nabla\nabla_4 e_t^I$$

The components of SEATS (2/2)

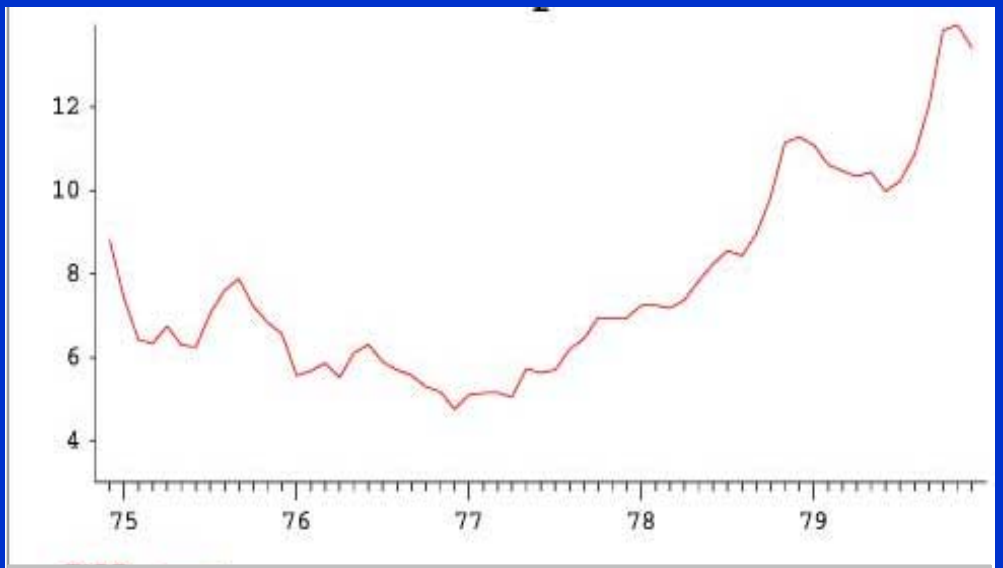
- An *admissible decomposition*, with innovation variances > 0, doesn't always exist
- If such decomposition does exist, others exist
- Look then for a unique *canonical decomposition*
- We have preferred a simple — non seasonal — example with only 2 components : permanent and irregular

$$y_t = \frac{1-\theta B}{1-B} e_t = \frac{\theta_P(B)}{1-B} e_t^P + e_t^I$$

- The consequence of various choices of the polynomial $\theta^P(B)$ can then be studied
- Theory is illustrated on U. S. interest rates on certificates of deposit (CD)



Analysis of the example



The ARIMA model for the series is written :

$\nabla \mathrm{CDIR}_t = e_t + 0{,}495\, e_{t-1}$

$$y_t = \frac{1-\theta B}{1-B} e_t = \frac{1-\theta_P B}{1-B} e_t^P + e_t^I$$

where

$$\nabla y_t = (1-\theta B) e_t$$
$$= (1-\theta_P B) e_t^P + \nabla e_t^I$$

Table of variances of the components

in function of the parameter thetaP

θ_P	V^P	V^I
-1.0	0.1313	0.0153
-0.9	0.1454	0.0149
-0.8	0.1620	0.0136
-0.7	0.1817	0.0112
-0.6	0.2051	0.0070
-0.5	0.2333	0.0007
-0.4	0.2679	-0.0089

Canonical decomposition

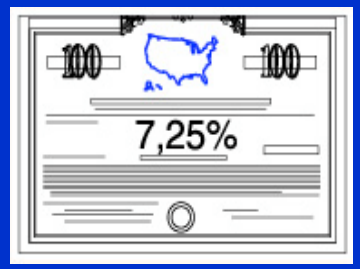
Admissible decompositions

Non admissible decomposition

Signal extraction in SEATS

- ⇨Start from the canonical decomposition of the process
- ⇨Obtain the coefficients of the optimal filters (called Wiener-Kolmogorov)
- ⇨Apply the filters
- ⇨Examine the results
- ⇨Restore the effects and remove the corrections

Exercise : simulation of SEATS

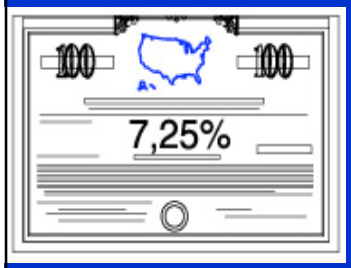


Data: *series of U. S. interest rates for certificates of deposits*

Purpose of the exercise: *illustrate how to obtain the filter for extracting the trend of the series and compare with the main results of TRAMO/SEATS*

⇨ Instructions (part E)

⇨ Exercise (file CH13EX06.XLS)



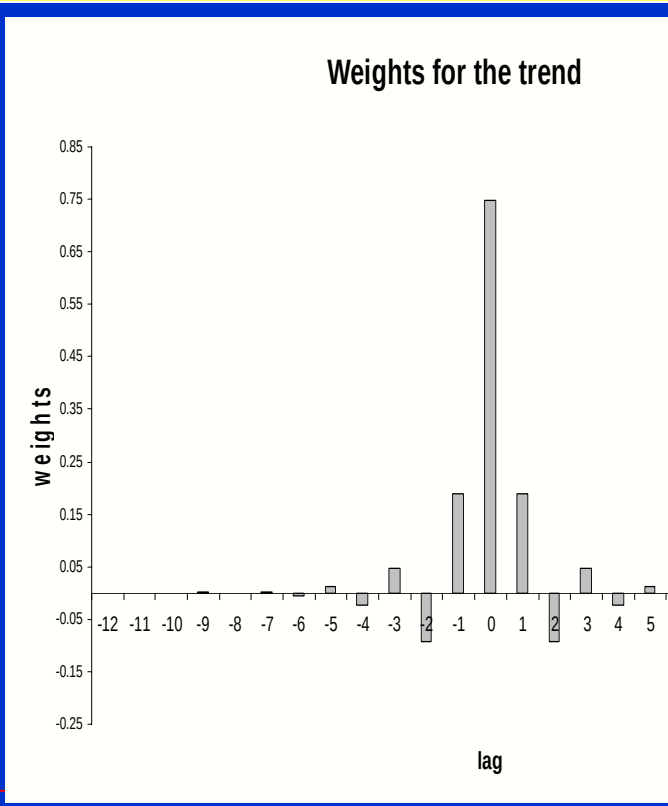
Results (1/2)

⇨ Software output

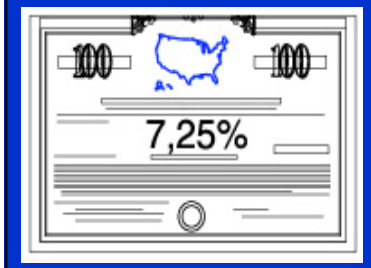
```
WIENER-KOLMOGOROV FILTERS (ONE SIDE)
TREND COMPONENT
 0.7474  0.1888 -0.0934  0.0462 ...
IRREGULAR COMPONENT
 0.2526 -0.1888  0.0934 -0.0462 ...
```

⇨ Contents of CH13EX06

Computation of autocovariances		
	Trend	Irregular
AR1	-0.4947	-0.4947
MA1	-1	1
$Sigma^2$	0.55856	0.06382
Lag	Trend	Irregular
0	0.7474	0.2526
1	0.1888	-0.1888
2	-0.0934	0.0934
3	0.0462	-0.0462

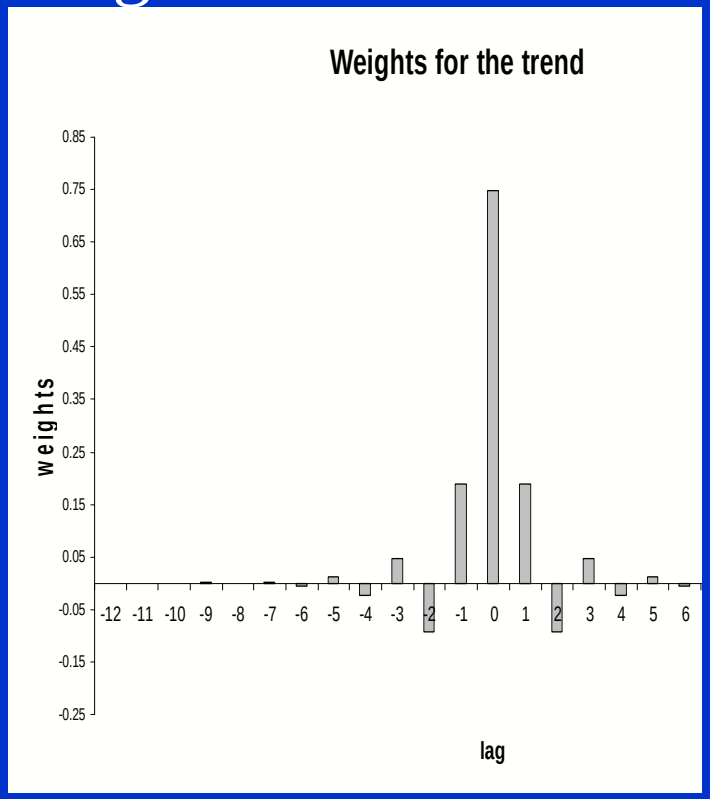


The Wiener-Kolmogorov filters are derived from autocovariances of some ARMA processes based on the model of the series

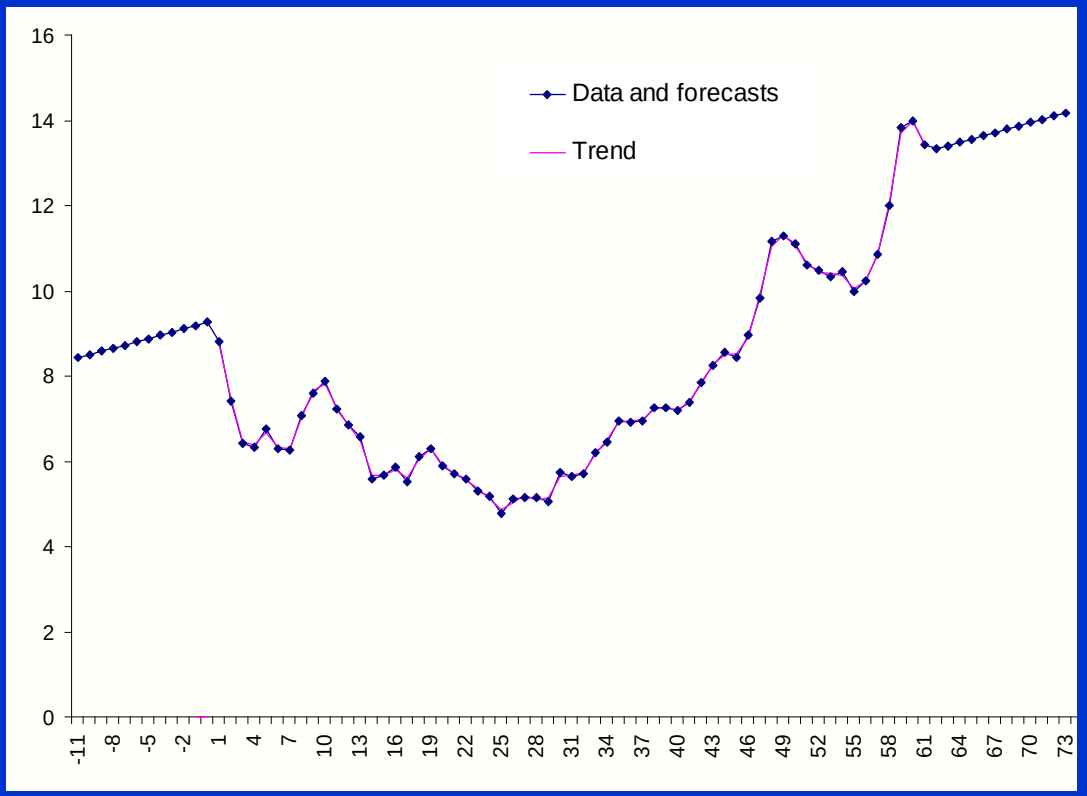


Results (2/2)

For a given component, apply to the data a moving average weighted with these coefficients



The weights of the moving average to extract the trend



The series (extended with forecasts) and the trend component

Everything can be done in the same way for other more comple examples, just with more complex computation

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Conclusions

- ⇨Course on time series analysis built at the requirements of the National Bank of Belgium
- ⇨Difficulty of self-training : keep attention by a combination of pedagogical methods
- ⇨Additional difficulty :
  - ✓clever use of sometimes difficult methods
  - ✓without requiring mathematical skills
- ⇨2 versions : basic and advanced
- ⇨At the advanced level :
  - ✓autocorrelations of MA(1) or AR(1) processes
  - ✓roots of polynomials
  - ✓representation of trading day effects

A more detailed presentation can be organized on request

Availability outside of the framework of the National Bank of Belgium
is under study

Translation in English is under way