Structure function and parton distribution parameterisations

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Motivations

1. Structure of proton (and other objects : photon, pion, pomeron)

+ « conventional » pdf's, in terms of quarks and gluons momentum share (*x*)

+ « unintegrated » parton distributions (x *and* k_T)

+ parton correlations (GPD)

+ who is carrying proton spin ? (polarised pdf's)

2. Tests and deeper understanding of QCD (write the Lagragian // understand QCD)

+ historical : scaling – quark parton model

Q^2 (DGLAP) evolution of structure functions

factorisation theorems // higher order calculations // etc

+ many fundamental open questions :

very high energy (BFKL evolution), diffraction, saturation

soft to hard transition

Motivations (2)

3. Precision measurements of SM processes / parameters

+ Λ_{QCD} , α_S DIS, jets, heavy quark production

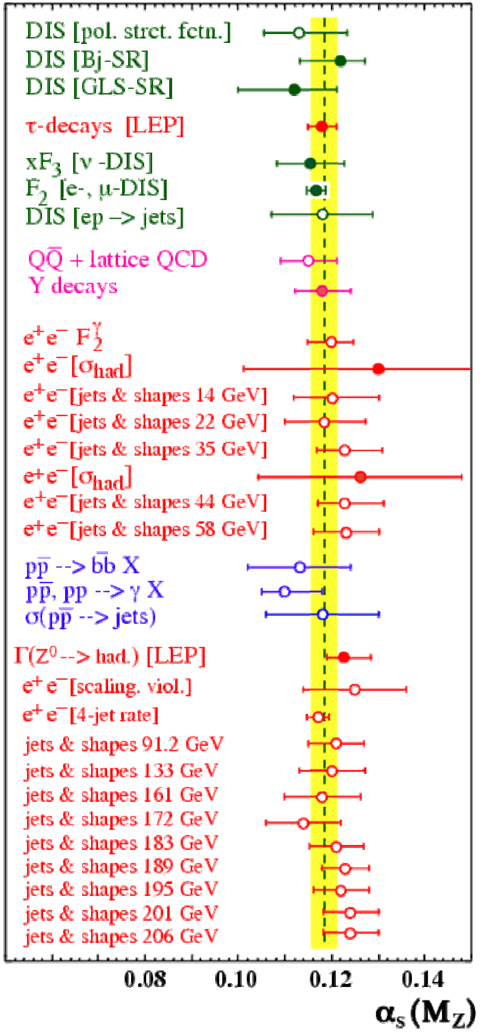
+ Higgs production

4. Input for any BSM studies

+ feasibility studies

+ SM backgrounds to any discovery claim

Basic tool for any physics at LHC !

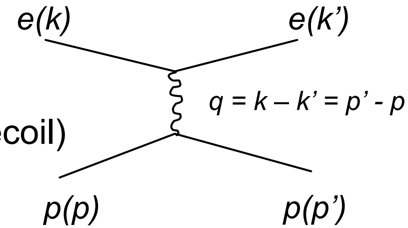


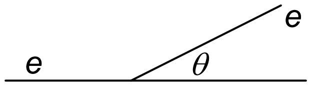
Plan

1. Deep inelastic scattering and structure functions
2. Quark parton model
3. QCD evolution, DGLAP equations
4. Factorisation theorems and parton density functions
5. Parton distribution parameterisations
6. Parton distribution uncertainties
7. Some (of many) uncovered topics
8. Some references

1. Deep inelastic scattering and structure functions

1. Rutherford formula





scattering of spin 0 (α particle) by spin 0 nucleus (no recoil)

$$\frac{d\sigma}{d\Omega} = Z^2 \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \bullet F(q^2) = \int \rho(R) e^{iqR} d^3R$$

form factor : extended target (Fourier transform of charge distribution)

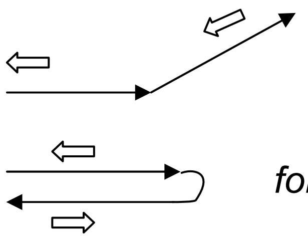
2. Mott formula

scattering of spin 1/2 (electron) by spin 0 nucleus (neglecting electron mass)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \cos^2(\theta/2)$$

Rutherford — electron spin

recoil mass



forbidden

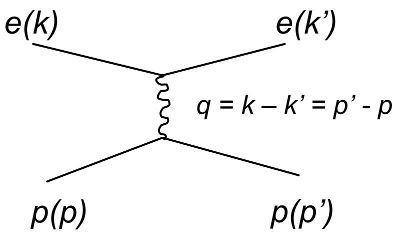
3. Spin 1/2 – spin 1/2 point-like scattering (« e – μ »)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \left[\cos^2(\theta/2) + \frac{Q^2}{2M^2} \sin^2(\theta/2) \right]$$

Mott — magnetic interaction ($\sigma^{\mu\nu}$)

$$= \frac{4\pi\alpha^2}{Q^4} (1 - y + y^2/2) \quad \text{with } Q^2 = -q^2 = 4EE' \sin^2(\theta/2) \qquad y = 1 - E'/E \cos^2(\theta/2)$$

4. Extended spin 1/2 target ; form factors



$$M = \frac{4\pi\alpha^2}{Q^4} J_\mu^{(e)}(q)\, J^{\mu(p)}(q)$$

with $J_\mu^{(e)} = \bar{u}(k')\,\gamma_\mu\, u(k)$

$$J^{\mu(p)} = \bar{u}(p')\left[F_1(q^2)\,\gamma^\mu + i\frac{\kappa}{2M}F_2(q^2)\,q_\nu\,\sigma^{\mu\nu}\right]u(p)$$

covariance : $\gamma^\mu \quad q^\mu \quad q_\nu\,\sigma^{\mu\nu} \quad \oplus$ current conservation : $\partial_\mu J^{\mu(p)}(x) = 0 \Rightarrow q_\mu J^{\mu(p)}(q) = 0$

$F_1(q^2) \quad F_2(q^2)$: 1 invariant variable (+ trivial azimuthal angle) + CM energy $\sqrt{s}$

In practice, combine F_1 and F_2 → G_E and G_M « form factors »

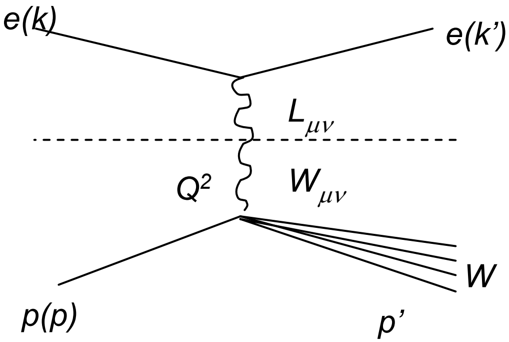
→*Rosenbluth formula* for *e p* elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}(Mott)\cdot\left[\frac{G_E^2 + (Q^2/4M^2)\,G_M^2}{1+Q^2/4M^2} + \frac{Q^2}{4M^2}2\,G_M^2\,tg^2(\theta/2)\right]$$

Experimentally : $G_E \simeq G_M \simeq \frac{1}{(Q^2+0.71^2)^2}$ (dipole parameterisation)

i.e. $1/Q^8$ compared to point-like target !

5. Deep inelastic scattering



$$M \sim \left[J_\mu^{(e)}(q)\, J^{\mu(p)}(q) \right] + cc \quad = L_{\mu\nu}\, W^{\mu\nu}$$

$$L_{\mu\nu} = 2k_\mu k'_\nu + 2k'_\mu k_\nu - Q^2\, g_{\mu\nu} \qquad \text{(for em interactions, } \gamma \text{ exchange)}$$

$$W^{\mu\nu} = -W_1\, g^{\mu\nu} + W_2\, p^\mu\, p^\nu + W_4\, q^\mu\, q^\nu + W_5\, (p^\mu\, q^\nu + q^\mu\, p^\nu)$$

current conservation $q_\nu\, W^{\mu\nu} = q_\mu\, W^{\mu\nu} = 0 \quad \Rightarrow$ only 2 of the 4 W functions contribute : W_1 and W_2

proton dissociation → one additional invariant *W* in addition to Q^2 (and *s)*

→ $W_{1,2}(Q^2, W)$ or any combination, in particular $\nu = p.q\,/\,M$ or $x = Q^2\,/\,2\,p.q$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \left[W_2(\nu,Q^2)\cos^2(\theta/2) + 2W_1(\nu,Q^2)\sin^2(\theta/2) \right]$$

$W_{1,2}(Q^2, W)$: <u>*physical observables*</u> (measured quantities)

DIS cross section

$$F_1(x,Q^2) = MW_1 \qquad F_2(x,Q^2) = \nu W_2$$

$$\frac{d^2\sigma}{dxdy} = \frac{d^2\sigma}{dxdQ^2}\, xs = \frac{4\pi\alpha^2}{Q^4}\, s\left[(1-y)\,F_2(x,Q^2) + \frac{y^2}{2}\, 2xF_1(x,Q^2)\right]$$

em interaction : NC γ exchange

$$= \ldots\left[\ldots \pm \frac{G_F^2}{8\pi^2}\frac{Q^4}{(1+Q^2/M^2)}\, y(1-y/2)\; xF_3(x,Q^2)\right]$$

weak interaction : CC W exchange

"polarised cross sections" (with different y dependences)

$$\sigma_T = \frac{1}{2}(\sigma^+ + \sigma^-) = \frac{4\pi\alpha^2}{s}\, 2\, F_1 \qquad \sigma_L = \sigma^0 = \frac{4\pi\alpha^2}{sx}(F_2 - 2\, xF_1) \qquad F_L = \frac{1}{2x}(F_2 - 2xF_1)$$

NB At high Q^2 (HERA), in NC also significant contributions of Z exchange + γ–Z interference

In the following, we shall concentrate on the structure function behaviour,
but don't forget the $1/Q^4$ factor in the cross section !

For completeness

Kinematical variables definitions and relations

$$s = (p+k)^2 \qquad Q^2 = -q^2 \qquad \nu = p.q/2M \qquad x = Q^2/2M\nu \qquad y = \frac{p.q}{p.k}$$

$$W^2 = Q^2(1/x - 1) + M^2 \simeq Q^2/x \simeq y\,s \qquad Q^2 \simeq x\,y\,s \qquad 0 \le x \le 1 \quad 0 \le y \le 1$$

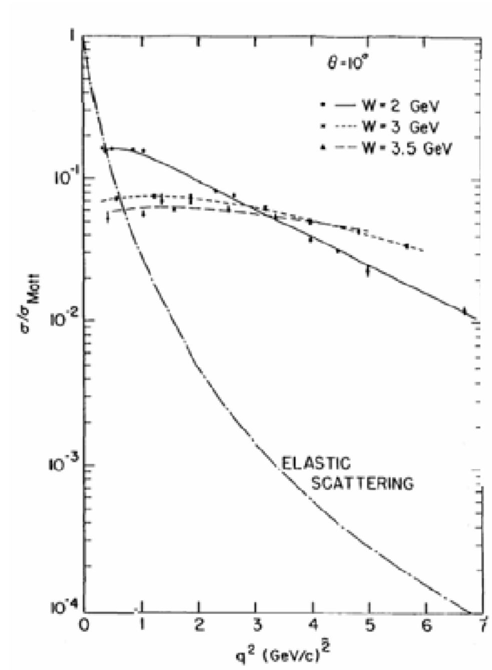
Laboratory frame $\nu = E - E'$ $\quad y = \frac{\nu}{E}$ $\quad Q^2 = 4EE'\sin^2(\theta/2)$ $\quad y = 1 - E'/E\cos^2(\theta/2)$

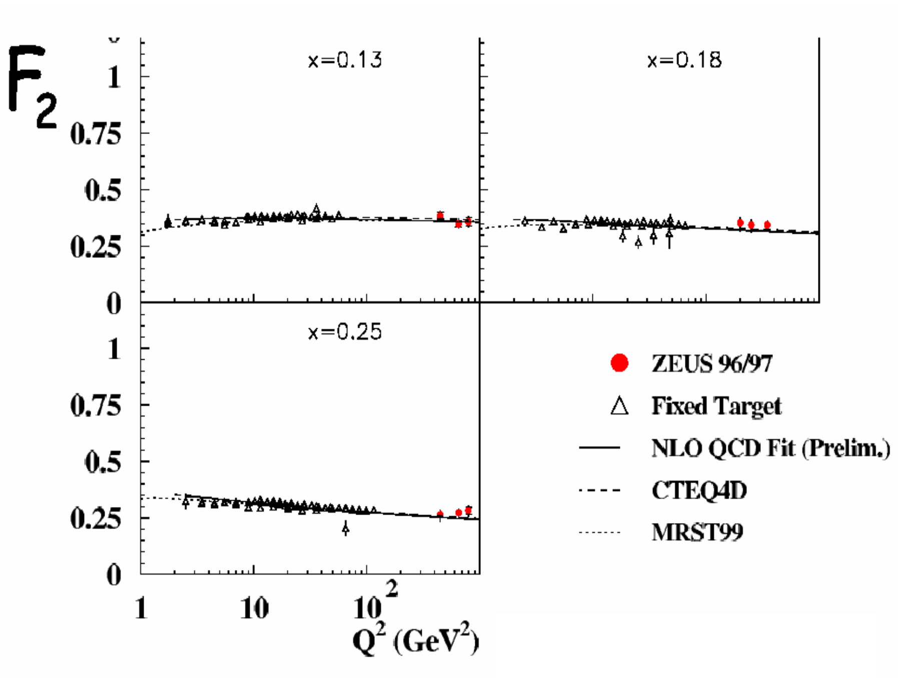
CM frame $1 - y = \frac{1}{2}(1+\cos\theta^*)$ $\rightarrow$ controls helicity : $y = 1 \leftrightarrow$ backward scattering, forbidden for long. photon

2. Quark parton model

Scaling

ep scattering SLAC 1969, for sufficient energy and Q^2 :
observation of « scaling »
i.e. no strong Q^2 dependence of cross section
(except for common $1/Q^4$)





Point-like partons

Compare

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left[\cos^2(\theta/2) + \frac{Q^2}{2M^2}\sin^2(\theta/2)\right]\delta(\nu - \frac{Q^2}{2M}) \qquad (1)$$ elastic scattering on point-like spin 1/2 target

$$= \frac{4\alpha^2 E'^2}{Q^4}\left[W_2(\nu,Q^2)\cos^2(\theta/2) + 2W_1(\nu,Q^2)\sin^2(\theta/2)\right] \qquad (2)$$ deep inelastic scattering

If DIS is in fact elastic scattering on spin 1/2 pointlike « partons » with charge e, momentum p, mass m, then one has (using $\delta(x/a) = a\,\delta(x)$)

$$\nu W_2(\nu,Q^2) = e^2\,\delta(1-\frac{Q^2}{2m\nu}) \qquad 2mW_1(\nu,Q^2) = e^2\frac{Q^2}{2m\nu}\,\delta(1-\frac{Q^2}{2m\nu}) \qquad (3)$$

→ scattering on *partons* explains *scaling*,

i.e. the fact that the structure functions $W_{1,2}$ depend on one variable only :

$x = Q^2/2m\nu$ – or equivalently W or ν –

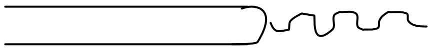
and not separately on Q^2 *and* ν

Interpretation of the x variable

Let the hit quark carry be parallel to the proton and carry

the fraction ξ of the proton momentum p

In the Breit frame, i.e. where photon is purely space-like :



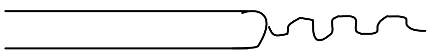
$$\text{initial state inv. mass } = m_q^2 + (\xi p)^2 - (\xi p + q)^2$$
$$= m_q^2 + (\xi p)^2 - (\xi p)^2 - 2\xi p.q - q^2 = m_q^2 \text{ (final state quark)}$$

$$\Rightarrow \xi = \frac{Q^2}{2p.q} = x$$

x is, in the Breit frame, the momentum fraction of the proton carried by the struck quark

NB Breit frame is also called the « brick wall » frame :

$$Q^2 = 2\xi p.q \quad \Rightarrow \quad q = -2\,\xi p$$



More generally : x = fraction of proton momentum carried by the quark in *IFM* (infinite momentum frame), where masses and transerse momenta can be neglected

Parton distribution functions

Incoherent scattering on constituent partons, « frozen » in the proton by time dilatation (NB also long. contraction) :

parton-parton interaction time ~ γ / R_p $\ll$ high energy γ p interaction time

hence :

$$\sigma(e\,p) = \sum_i \int dx\, f_i(x)\, \sigma(e\, q_i) \quad (4)$$

where $f_i(x)$ = probability to find in the proton parton of species *i* carrying momentum fraction *x* (in IMF)

NB : f_i = valence + sea

Using $p_i = x\, P_p$ and thus formally $m = xM$ (= 0 in IMF !), putting in (4) the $W_{1,2}$ structure functions (3) and integrating over the δ function, only an *x* dependence remains at high energy, high Q^2 (*DIS regime*)

$$\nu W_2(\nu, Q^2) = e^2\, \delta\left(1 - \frac{Q^2}{2m\nu}\right) \quad \rightarrow \quad F_2(x) = \sum_i e_i^2\, x\, f_i(x)$$

$$M W_1(\nu, Q^2) = e^2\, \frac{Q^2}{2m\nu}\, \delta\left(1 - \frac{Q^2}{2m\nu}\right) \quad \rightarrow \quad F_1(x) = \frac{1}{2x} F_2(x)$$

} QPM

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{Q^4}\, s\left[1 + (1-y)^2\right] \sum_i e_i^2\, x\, f_i(x) \qquad \text{QPM}$$

Note that in QPM $F_2 = 2xF_1$ (Callan Gross relation)

→ measurement of $F_L = 0$ indicates that partons are massless spin 1/2 objects → identified with quarks

(Note also if quark spin were 0, $\sigma_T = 0$ – cf. Mott formula)

Sum rules, first pdf measurements

$$\sum_i \int dx\, x\, f_i(x) = 1$$ momentum conservation

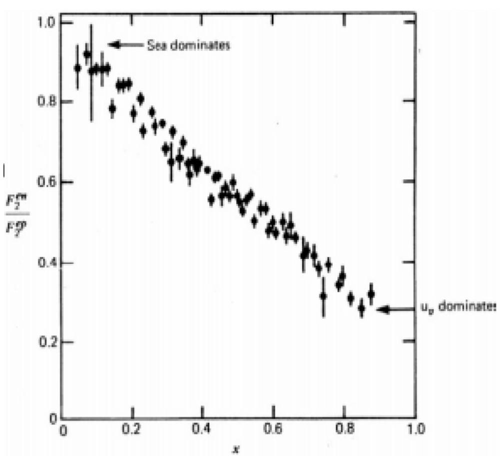
$$\int_0^1 dx\,[u(x)-\bar{u}(x)] = 2 \quad \int_0^1 dx\,[d(x)-\bar{d}(x)] = 1$$ valence quarks $$\int_0^1 dx\,[s(x)-\bar{s}(x)] = 0$$ sea - idem for c,b,t

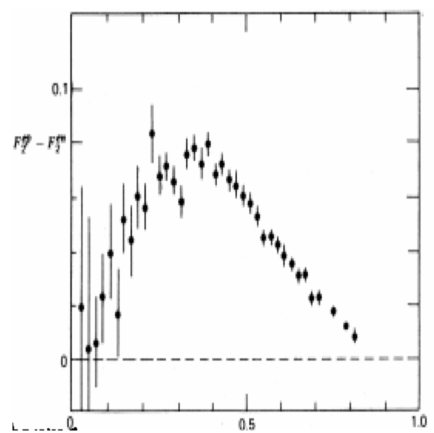
$$\frac{1}{x}F_2(x) = \sum_q e_q^2\, x\, f_q(x)$$ proton $\leftrightarrow$ neutron : $u(x) \leftrightarrow d(x)$ (isospin)

$$\frac{1}{x}F_2^{ep}(x) = \frac{4}{9}u_V + \frac{1}{9}d_V + \left(\sum_{SEA} e_q^2\, S(x)\right) \qquad \frac{1}{x}F_2^{en}(x) = \frac{1}{9}u_V + \frac{4}{9}d_V + \left(\sum_{SEA} e_q^2\, S(x)\right)$$

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \xrightarrow[x\to 0]{} 1 \qquad \frac{F_2^{en}(x)}{F_2^{ep}(x)} \xrightarrow[x\to 1]{} \frac{u_V + 4d_V}{4u_V + d_V}$$

$$\frac{1}{x}\left(F_2^{ep}(x) - F_2^{en}(x)\right) = \frac{1}{3}\left(u_V(x) - d_V(x)\right)$$





Using $\int_0^1 dx\, F_2^{ep}(x)$ and $\int_0^1 dx\, F_2^{en}(x)$ $\rightarrow$ gluons $\simeq$ 0.46 proton momentum

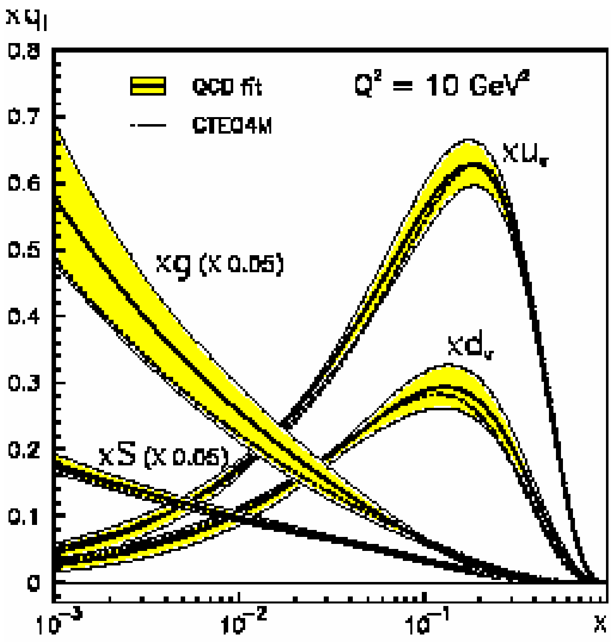
- Fixed target electron and muon scattering on hydrogen and nuclei → F_2 for p and n
- neutrino scattering → F_2 and xF_3

$$F_2^{\nu+\bar{\nu}} = x\sum_q (q(x)+\bar{q}(x)) \qquad F_2^{\nu-\bar{\nu}} = x\sum_d d^V(x) - x\sum_u u^V(x)$$

$$F_3^{\nu+\bar{\nu}} = \sum_q (q(x)-\bar{q}(x)) = \sum_q q^V(x) \qquad F_3^{\nu-\bar{\nu}} = \sum_d (d(x)+\bar{d}(x)) - \sum_u (u(x)+\bar{u}(x))$$

→ first determinations of pdf's

NB Remember that structure functions are observables, but pdf's are « theoretical » quantities !



3. QCD evolution

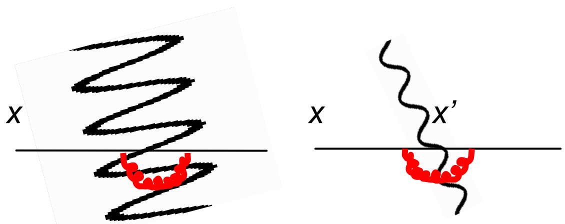
DGLAP equations

Scaling violations

Q^2 evolution of structure functions

photon resolution improves with Q^2

→ disentangles virtual gluon emission



low Q^2
probes x

high Q^2
probes x'<x

As Q^2 increases,

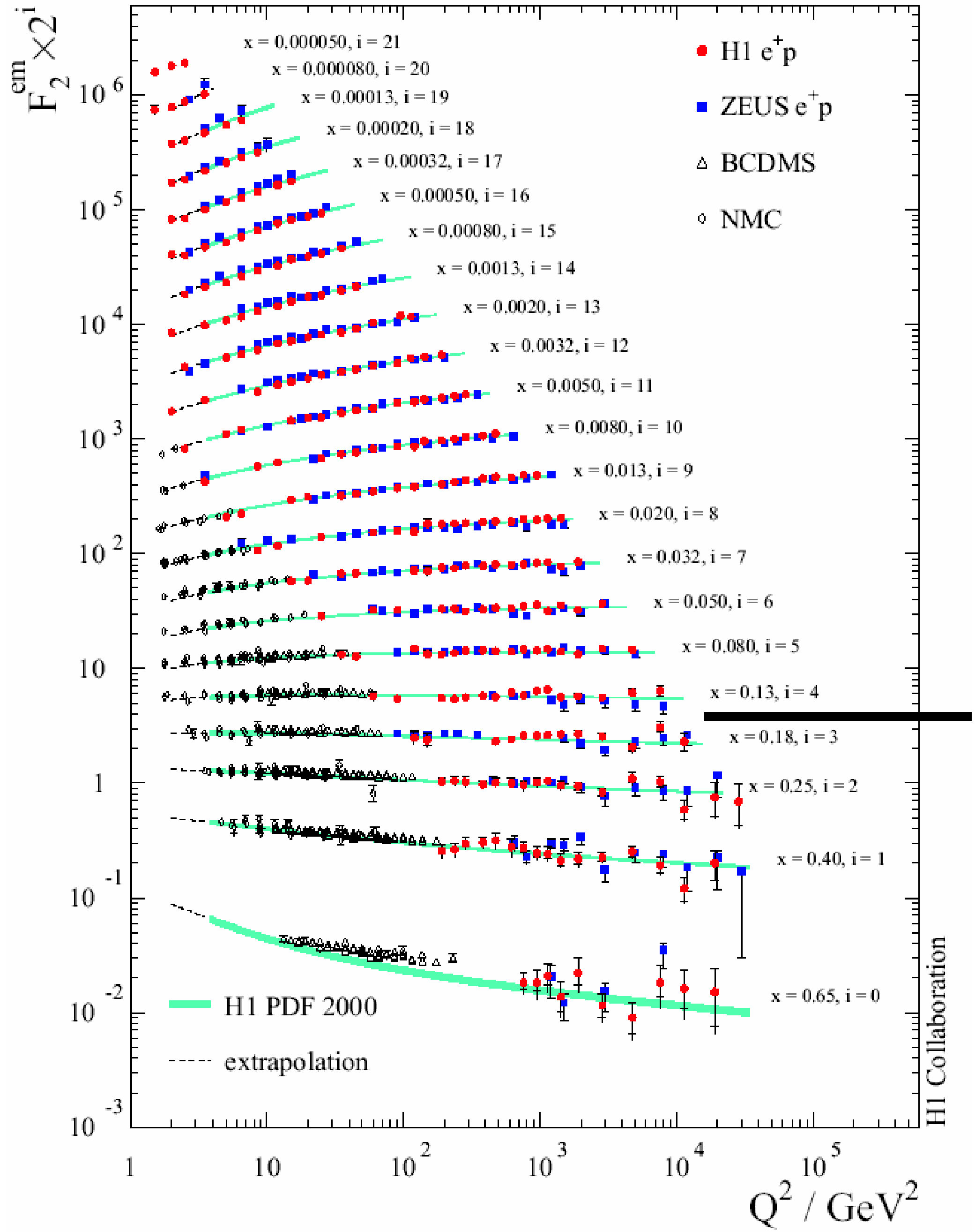
quark content decreases at large x (valence)

and increases at low x

also : at low x, the gluon content and the sea increase

(low x since due to bremsstrahlung → soft)

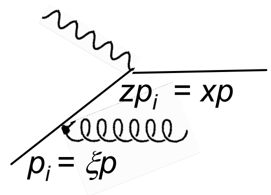
parton distribution function evolutions

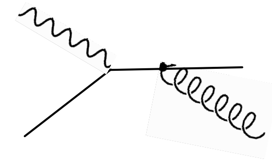


« structure of the quark »

Gluon emission by the quark :

a quark « structure » shows up





NB 1. we consider *hard* gluon emission, over timescale comparable to interaction time → large p_T, well separated jets ←→ soft gluon emission during hadronisation (see later)

2. « before » and « after » are frame dependent - the second diagram for gauge invariance

Take over the SF formalism, with	proton →	quark
	p	$p_i = \xi p$
	$x = Q^2/2p.q$	$z = Q^2/2p_i.q = x/\xi$

Hence $$\frac{1}{x}F_2(x,Q^2) = 2F_1(x,Q^2) = \left.\frac{\sigma_T(x,Q^2)}{\sigma_0}\right|_{\gamma^*} = \sum_i \int_0^1 dz \int_0^1 d\xi\, f_i(\xi)\, \delta(x-\xi z)\left.\frac{\hat{\sigma}_T(z,Q^2)}{\hat{\sigma}_0}\right|_{\gamma^* quark}$$

where $\sigma_0 = \frac{4\pi\,\alpha_S^2(Q^2)}{s}$ and similarly for $\hat{\sigma}_0$ with $\hat{s} = \xi s$

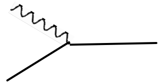
$f_i(\xi)$ is the probability to find in the proton a (« primary ») quark with momentum fraction ξ,

$\hat{\sigma}_T(z,Q^2)$ is the photon-quark transverse cross section, for a (« secondary ») quark of momentum fraction z;

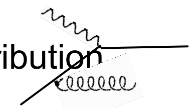
ξ and z can vary from 0 to 1, but $x = \xi z$ is fixed (hence the δ function)

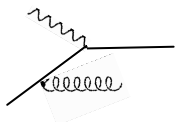
After integration on z : $$2F_1(x,Q^2) = \sum_i \int_0^1 \frac{d\xi}{\xi}\, f_i(\xi)\,\frac{\hat{\sigma}_T(x/\xi,Q^2)}{\hat{\sigma}_0}$$

quark evolution equation

At first order : $\gamma^* q \rightarrow q$  where $z = x / \xi = 1$

Hence $2F_1(x,Q^2) = \sum_i \int_0^1 \frac{d\xi}{\xi} f_i(\xi) \frac{\hat{\sigma}_T(x/\xi,Q^2)}{\hat{\sigma}_0} \rightarrow \sum_i e_i^2 \int_0^1 \frac{d\xi}{\xi} f_i(\xi)\, \delta(1-\frac{x}{\xi}) = \sum_i e_i^2 f_i(x)$

At next order, the photon quark cross section contains a $\gamma^* q \rightarrow q\, g$ contribution  (and others)

with for  $\frac{d\hat{\sigma}}{dp_T^2} \simeq e_q^2\, \hat{\sigma}_0 \frac{1}{p_T^2} \frac{\alpha_s(Q^2)}{2\pi} P_{qq}(z)$ where $P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$

$P_{qq}(z)$ is the probability of a quark emitting a gluon and reducing its momentum by the factor z : « splitting function »

Thus $\hat{\sigma}(\gamma^* q \rightarrow qg) = \int_{\mu_F^2}^{s^2/4} dp_T^2 \frac{d\hat{\sigma}}{dp_T^2} \simeq e_q^2\, \hat{\sigma}_0 \frac{\alpha_s(Q^2)}{2\pi} P_{qq}(z) \log\frac{Q^2}{\mu_F^2}$ μ_F = cut off for $p_T \rightarrow 0$ (see below)

and $\frac{1}{x} F_2(x,Q^2) = \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} q(\xi)\left(\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(Q^2)}{2\pi} P_{qq}(\frac{x}{\xi}) \log\frac{Q^2}{\mu_F^2}\right)$ logarithmic scaling violation

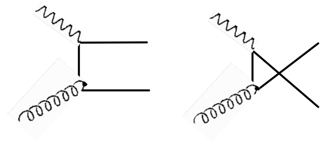
$= \sum_q e_q^2 \left[q(x) + \Delta q(x,Q^2) \right]$ log. dependence formally absorbed in quark density redefinition

Hence integro-differential ***evolution equation*** for quark distribution :

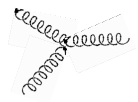
$$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} q(\xi,Q^2)\, P_{qq}(\frac{x}{\xi})$$

DGLAP equations

Similarly : quark in gluon $P_{q\,g}$



gluon in gluon $P_{q\,g}$



Notation $$P_{ij} \otimes f_i(x,Q^2) = \int_x^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{x}{\xi}\right) f_i(\xi,Q^2)$$

$$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left[P_{qq} \otimes q(x,Q^2) + P_{qg} \otimes g(x,Q^2)\right]$$

$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left[P_{gq} \otimes q(x,Q^2) + P_{gg} \otimes g(x,Q^2)\right]$$

Remarks

1. DGLAP equations = Renormalisation group equations (RGE)

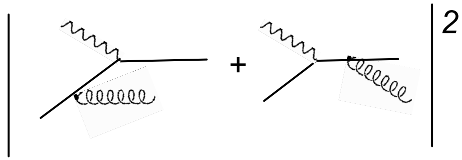
$$q(x,Q^2;\mu_F^2) = q(x) + \frac{\alpha_S(Q^2)}{2\pi}\log\frac{Q^2}{\mu_F^2}\int_x^1 \frac{d\xi}{\xi} P_{qq}\left(\frac{x}{\xi}\right)\; q(\xi)$$

Choice of factorisation scale μ_F is arbitrary → $q(x,Q^2)$ should not depend on μ_F :

$$\frac{dq(x,Q^2;\mu_F^2)}{d\log\mu_F} = 0 \quad \rightarrow \text{ the DGLAP equations}$$

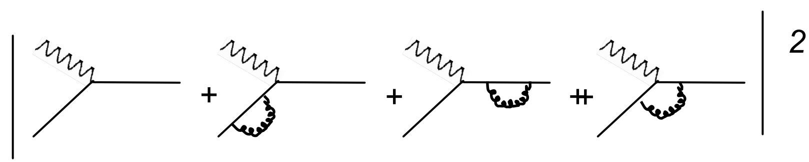
2. Singularities in splitting functions

Remember P_{qq} comes from



and is singular $P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$

But interference of virtual corrections with leading order diagram regularise the singularity in P_{qq}



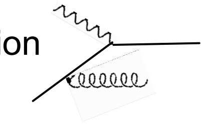
3. Higher orders

NLO and NNLO splitting functions have been calculated. Very complicated !

4. Factorisation theorems and parton density functions

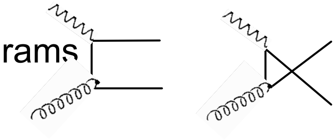
Infrared singularities

Remember logarithmic singularity for quark structure, due to collinear gluon emission



$$\hat{\sigma}(\gamma^* q \to qg) = e_q^2\, \hat{\sigma}_0\, \frac{\alpha_s}{2\pi} P_{qq}(z) \, \log\frac{Q^2}{\mu_F^2} + \int_0^{\mu_F^2} dp_T^2 \, \frac{d\hat{\sigma}}{dp_T^2}$$

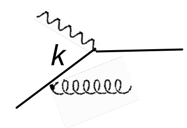
For gluon structure, log (Q/m) singularity due to $\gamma\, g$ fusion diagrams



In general, singularities coming from vanishing gluon mass

Generally speaking, *infrared* singularities due to *soft* and *collinear* configurations (degenerate kinematic situations)

correspond to *on mass shell* intermediate parton, with $k^2 = m^2 \approx 0$



Correspond to *long distances*

light-cone coordinates : $v^\pm = (v^0 \pm v^3)/\sqrt{2}$

$$k^2 = m^2 = 2k^+k^- - k_T^2 \quad (\simeq 0 \text{ if parton on mass shell })$$

$$x \cdot k = x^- k^+ + x^+ k^- - \vec{x}_T \cdot \vec{k}_T$$

$$k^+ = \frac{E + p_z}{\sqrt{2}} \simeq \sqrt{s/2} \quad \text{very large !} \qquad k^- = \frac{k_T^2 + m^2}{2k^+} \simeq \frac{k_T^2 + m^2}{\sqrt{s}} \quad \text{very small !}$$

$k \to x$ space by Fourier transform $(\int d^4k\, e^{ik\cdot x} \ldots)$ with $x^+ \simeq$ time

$$\to x^+ \sim \frac{\sqrt{s}}{k_T^2 + m^2} \quad \text{very large !} \qquad x^- \sim \frac{1}{\sqrt{s}} \quad \text{very small !}$$

QCD factorisation theorems

(to be demonstrated : DIS, jet production, Drell-Yan, prompt photon emission, fragmentation in e^+e^-) :

Infrared (long distance) singularities (due to nearly on mass shell partons)

can be separated from hard (short distance) partonic process (with large off mass shellness)

i.e. infrared singularities can be « factorised out »

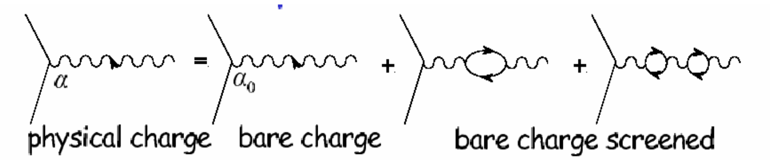
order by order in pQCD (or useless !)

into *universal* parton density functions (or fragmentation functions)

- which must be *measured* (cannot be calculated !)
- at some factorisation *scale* μ_F
- of which the *evolution* from μ_F can be calculated using the P_{ij} coefficient kernels

(DGLAP and in general RGE equations)

Very much like charge and mass are redefined to dispose of familiar *UV* singularities due to loop corrections



« *renormalisation* » is factorisation of UV divergences

« *factorisation* » is renormalisation of soft / collinear divergences

Master formula

$$\sigma^h(x,Q^2) = \sum_{i=q\,\bar{q}\,g} \int_0^1 \frac{d\xi}{\xi} \underbrace{C^i\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\frac{\mu_F^2}{\mu^2},\alpha_S(\mu^2)\right)}_{coefficient\ function} \underbrace{\phi_{i/h}(\xi,\mu_F,\mu^2)}_{pdf}$$

μ renormalisation scale (fixes $\alpha_s(\mu^2)$)

μ_F factorisation scale

ones often takes $\mu_F = \mu$ - can be Q^2 or E_T (jet) etc.

NB complicated cases where 2 scales (e.g. Q^2 and jet E_T ; also when large *log 1/x*)

- the factorisation scale μ_F can be seen as where hard and soft processes separate,
  i.e. maximum off-shelness of partons grouped into pdf $\phi_{i/h}$

- as μ is present in both coeff. fct. and in pdf's,
  a « factorisation scheme » (*MS-bar*, *DIS*) must define (for higher orders) the attribution of the short distance finite contributions (i.e. to coeff. fct. or to pdf's)
  (remember : pdf's are « theoretical » objects)

Parton distribution functions

$$\sigma^h(x,Q^2) = \sum_{i=q\,\bar{q}\,g} \int_0^1 \frac{d\xi}{\xi} \quad C^i\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\frac{\mu_F^2}{\mu^2},\alpha_S(\mu^2)\right) \quad \phi_{i/h}(\xi,\mu_F,\mu^2)$$

- coeff. functions are QCD calculable as power series in α_s,
  infrared safe
  process dependent (NC DIS, CC DIS, jet, etc.)
  independent of initial hadron *h*
- pdf's are specific to *h*
  but process independent (including independent of Q^2)
- pdf evolution kernels (e.g. DGLAP) are
  QCD calculable as power series in α_s
  infrared safe

- compute the process ($e^+ e^-$, DIS, …) cross section at parton level, at a given order of perturbation theory
- compute the « parton structures » $\phi_{i/q}\ \phi_{i/g}$ at the same order (in a given factorisation scheme)
- thus derive the coefficient functions C^i (at same order, in the same scheme) (see fig. NNLO !)
- combine the C^i with the experimental cross section σ^h to derive the non perturbative parton distributions in the hadron $\phi_{i/h}$ (at same order, in the chosen scheme) (i.e. inverse master formula)
- use the evolution kernels to extract the pdf's for a given μ factorisation scale value

5. Parton distribution parameterisations

Parameterising pdf's

- Choose a starting parameterisation for the various parton species (quarks, antiquarks, gluons)

  at a given μ scale (usually $\mu_F = \mu$)

  in a given factorisation scheme (usually *MS-bar*)

  - with a number of parameters sufficiently *large* to describe the data
  - but sufficiently *small* to be really constraint by physics and not artefacts

- Decide upon simplification hypotheses to decrease number of degrees of freedom
  - isospin (*u(x)* in proton = *d(x)* in neutron; *u* sea in proton = *d* sea in neutron, but *u* sea in proton might be different form *u* sea in neutron)
  - x-distributions of quark and antiquark seas : have to be the same in total, but what about *x* dependences ?
  - *s(x)* sea versus *u(x), d(x)* seas

- Choose experimental data
  - theoretically relevant (be sure factorisation applies !)
  - theoretically under control – e.g.

    higher order effects (NLO / LO ; NNLO / NLO)

    treatment of nuclear effects (in extracting neutron pdf's from *eA* and μA scattering)
  - experimentally reliable

    (e.g. phase space extrapolations for HERA charmed meson production)

- … and fit
  - (for errors – see below !)

Main parameterisations

- MRST

  starting scale : $\mu^2 = Q_0^2 = 2$ GeV^2

u quark	$xu(x,Q_0^2) = A_u(1-x)^{\eta_u}(1+\varepsilon_u\sqrt{x}+\gamma_u x)x^{\delta_u}$
d quark	$xd(x,Q_0^2) = A_d(1-x)^{\eta_d}(1+\varepsilon_d\sqrt{x}+\gamma_d x)x^{\delta_d}$
sea	$xS(x,Q_0^2) = A_s(1-x)^{\eta_s}(1+\varepsilon_s\sqrt{x}+\gamma_s x)x^{\delta_s}$
$\Delta q = \bar{u} - \bar{d}$	$x\Delta(x,Q_0^2) = A_\Delta(1-x)^{\eta_\Delta}(1+\gamma_\Delta x+\delta_\Delta x^2)x^{\delta_\Delta}$
gluons	$xg(x,Q_0^2) = A_g(1-x)^{\eta_g}(1+\varepsilon_g\sqrt{x}+\gamma_g x)x^{\delta_g} \quad \left[-A_-(1-x)^{\eta_-}x^{-\delta_-}\right]$
strange sea	$\kappa = \dfrac{s(x)}{\bar{u}(x)+\bar{d}(x)} \simeq 0.4$
sea asymm.	$\Delta s(x) = s(x) - \bar{s}(x)$

- CTEQ
- DIS (H1, ZEUS)

$$(1+\varepsilon_i\sqrt{x}+\gamma_i x) \quad \rightarrow \quad (1+\gamma_i x^{\varepsilon_i})$$

around 20 free parameters (or even more) for some 2000 data points

(A_u and A_d fixed by valence quark counting, A_g fixed by momentum sum rule)

Parameterisations differ in detailed form of parameterisation at starting scale, data sets included, factorisation / renormalisation scale Q_0^2 and scheme, value of $\alpha_s(Q_0^2)$, assumptions on κ, sea asymmetry, possible negative gluon

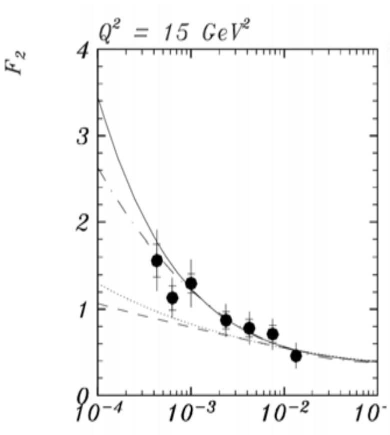
Data sets

DIS (1) fixed target μp, μn BCDMS, NMC, SLAC, E665 $x > 10^{-2}$

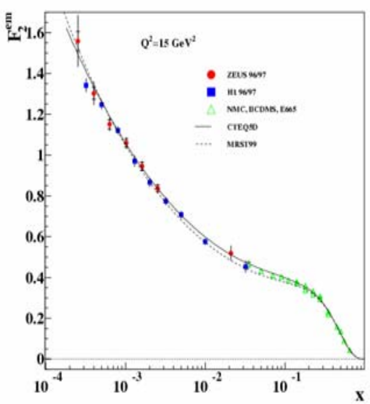
$e^+ p$, $e^- p$ (NC and CC) H1, ZEUS $x > 10^{-5}$ quarks, gluons (through evolution)

$e^+ p$, $e^- p$ CC → u / d at large x (without nuclear target problems)

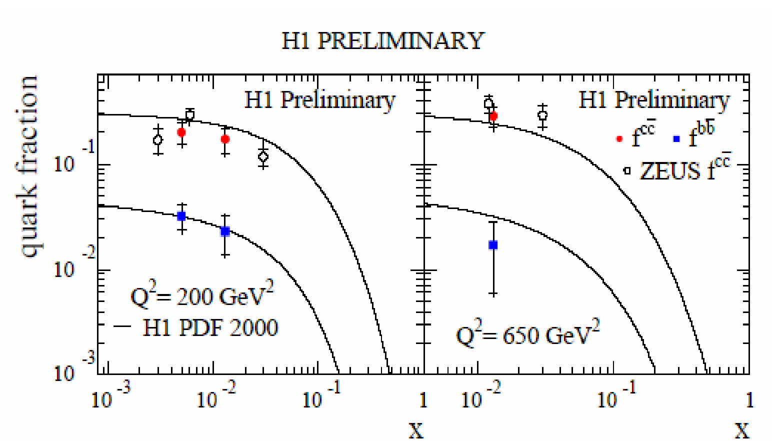
$F^2_{c\bar{c}}$ $F^2_{b\bar{b}}$ → direct access to gluons (photon gluon

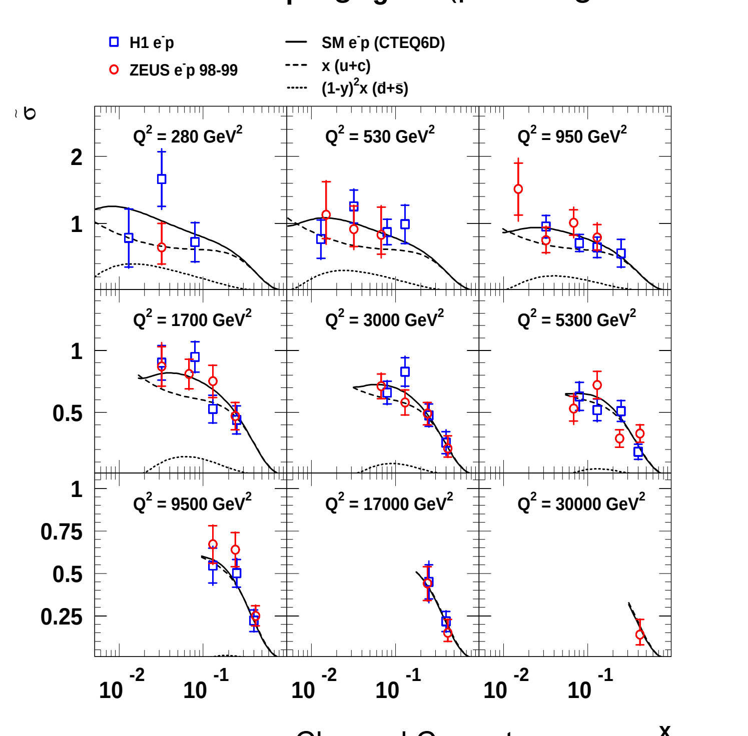


1993, 20 pb^{-1}



2 – 3 % precision





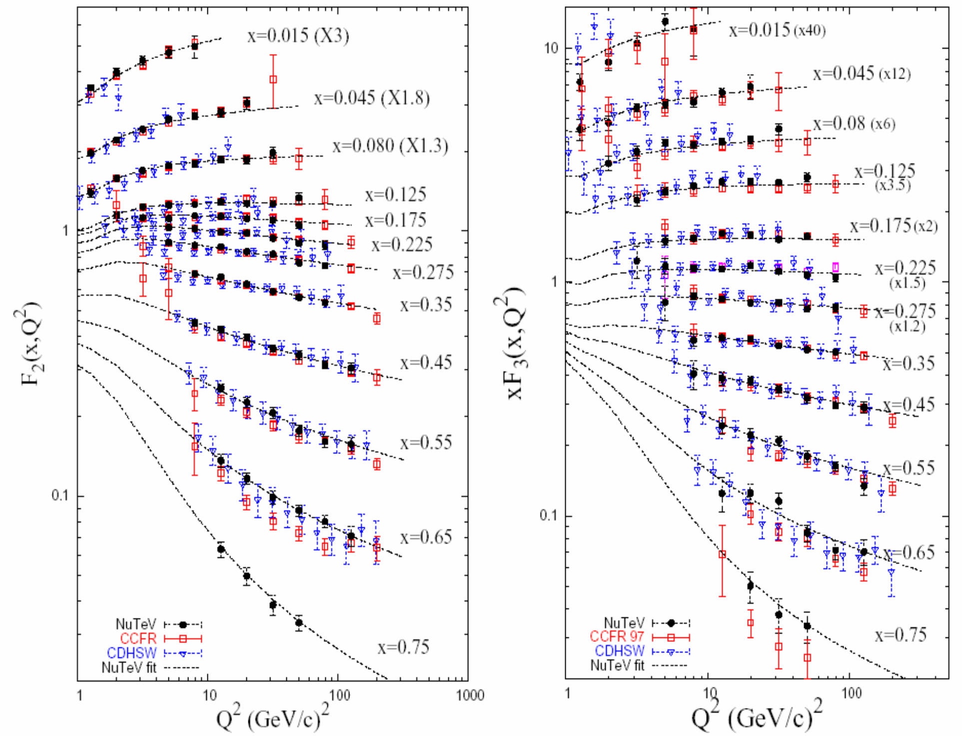
Charged Current

Data sets (2)

DIS (2) νp νn $\bar{\nu} p$ $\bar{\nu} n$

CCFR $x > 10^{-2}$: total quarks, valence

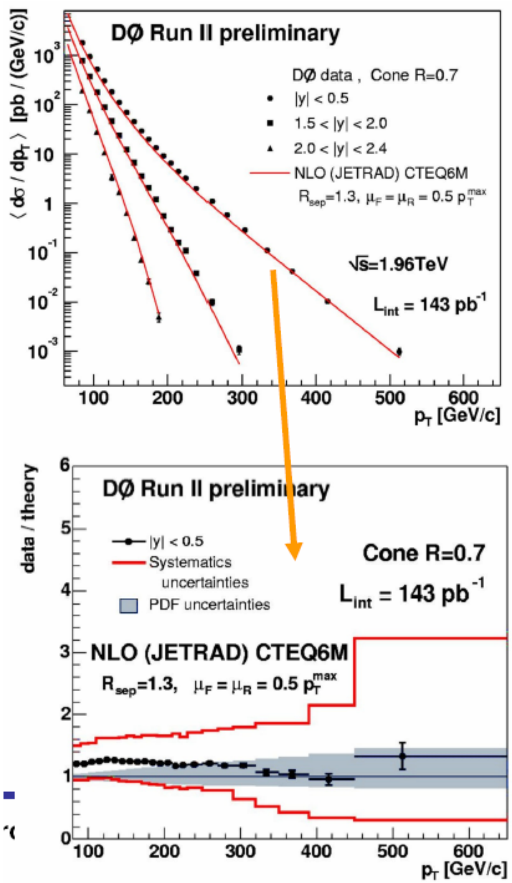
NuTeV + strange sea (dimuon events from CC charm prod.)

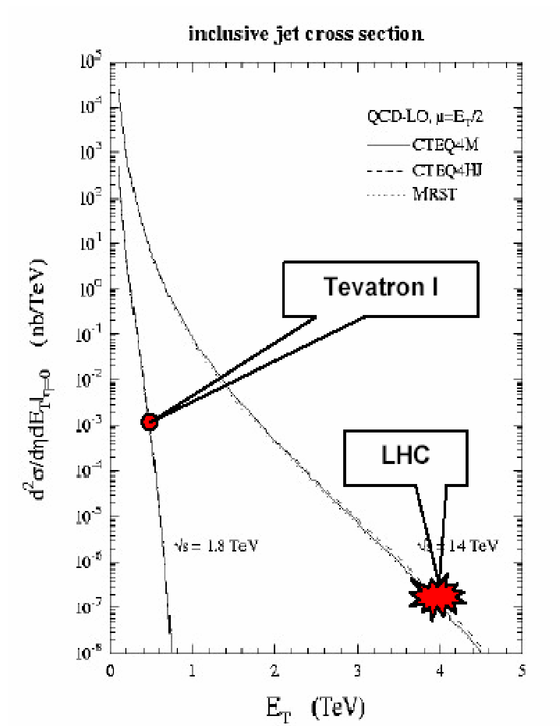


Data sets (3)

Jets Tevatron collider CDF, D0 → constraints on high x gluon

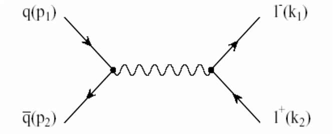
Jets in DIS at HERA ZEUS

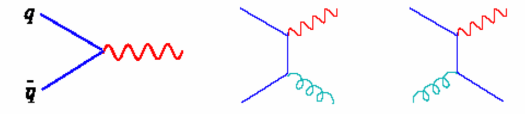




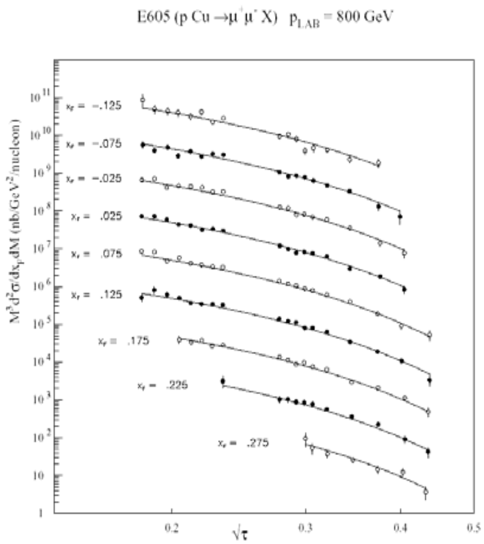
Data sets (4)

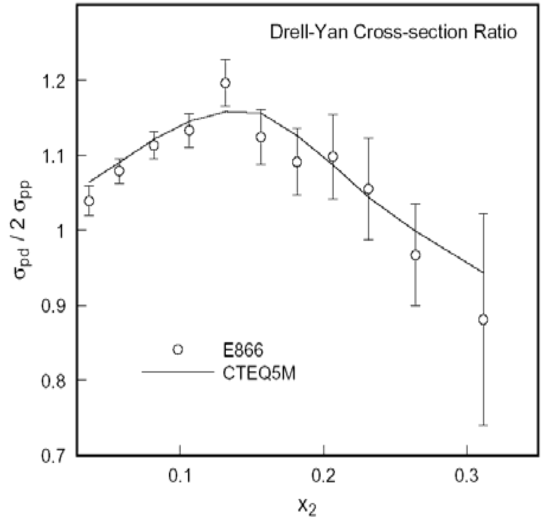
Drell-Yan (muon pair production) : Fermilab, *p* and *n* → *u*, *d* valence; $\bar{u}$, $\bar{d}$





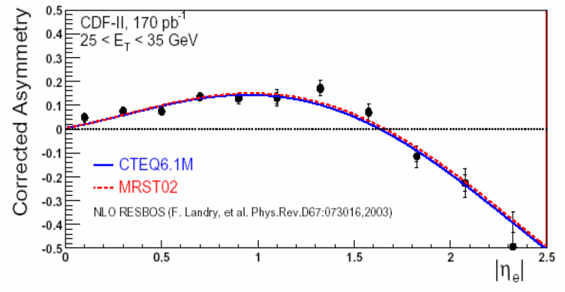
Large *K*-factor (= NLO / LO) → convergence ? factorisation true ? now understood : $\alpha(\mu\mu)$ not small

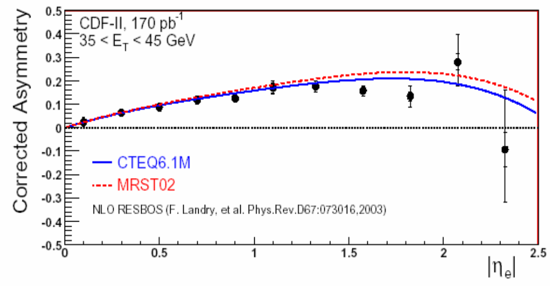




W asymmetry (CDF)

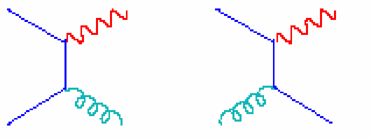
u/d ratio at high x



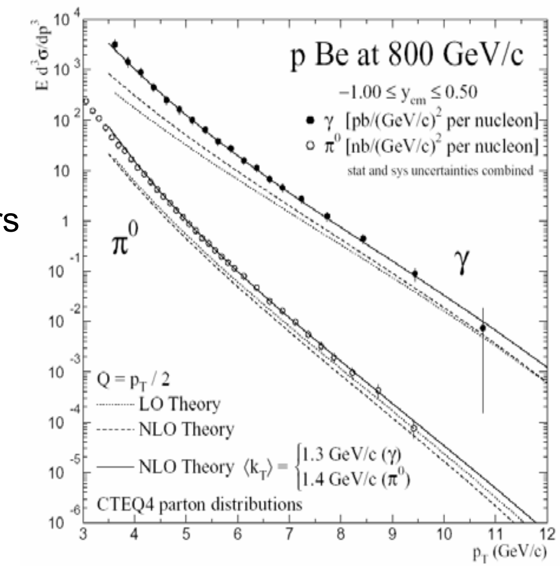


Data sets (5)

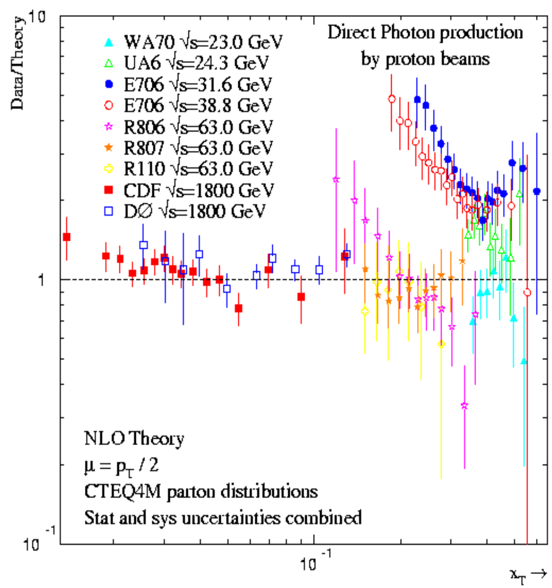
Prompt photon production



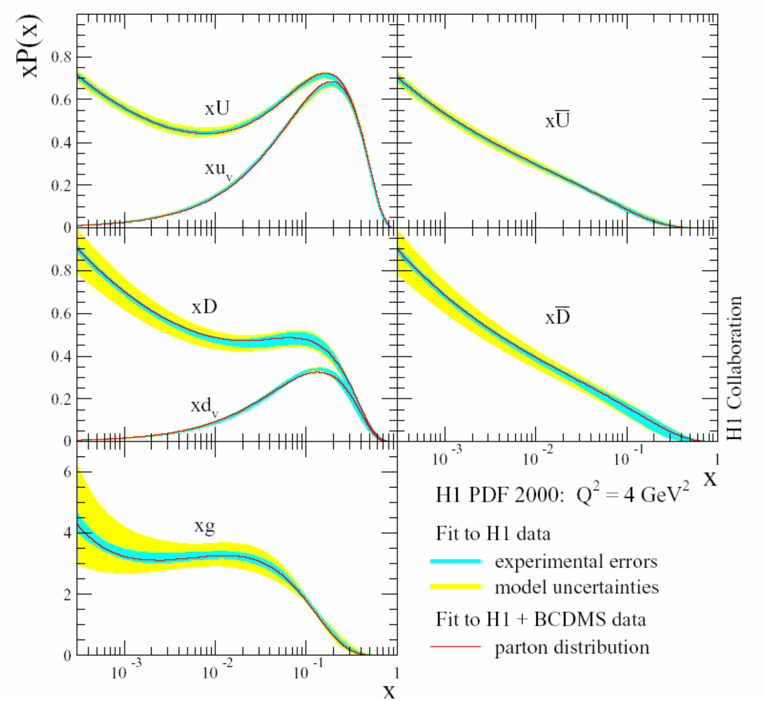
Sensitive to primordial k_T of quarks inside nucleon (i.e. higher orders

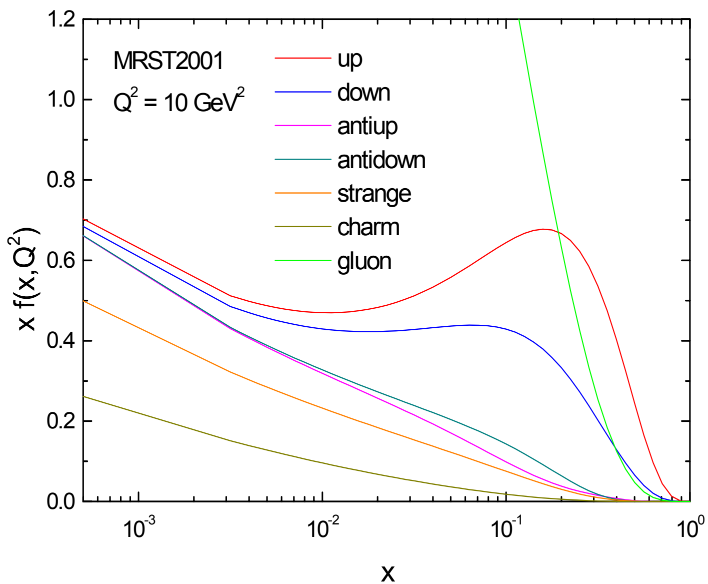


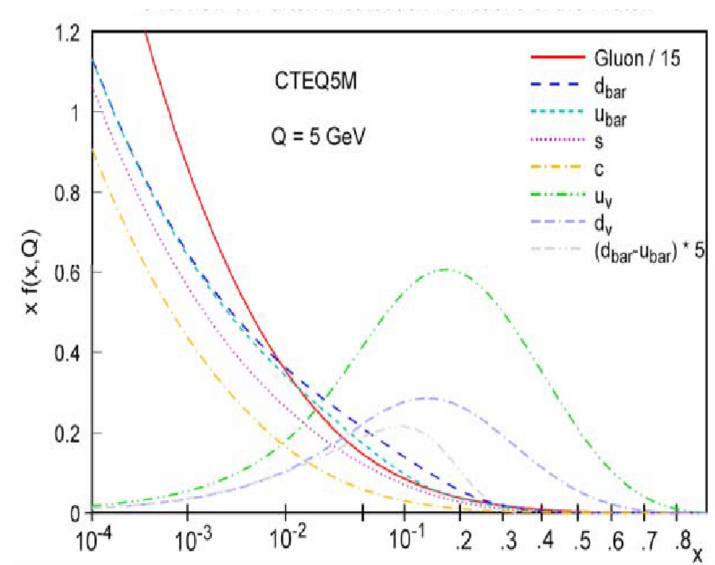
MRST do not use these data



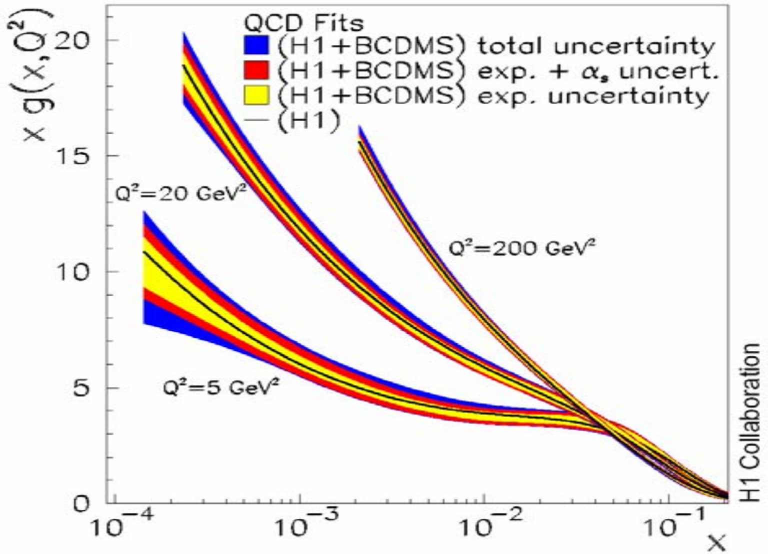
Results…


xP(x)
xU
xu_v
xU̅
xD
xd_v
xD̅
xg
H1 Collaboration
x
H1 PDF 2000: Q^2 = 4 GeV^2
Fit to H1 data
experimental errors
model uncertainties
Fit to H1 + BCDMS data
parton distribution


MRST2001
Q^2 = 10 GeV^2
up
down
antiup
antidown
strange
charm
gluon
x f(x,Q^2)
x


CTEQ5M
Q = 5 GeV
Gluon / 15
d_bar
u_bar
s
c
u_v
d_v
(d_bar-u_bar) * 5
x f(x,Q)
x

The gluon at low x (HERA)



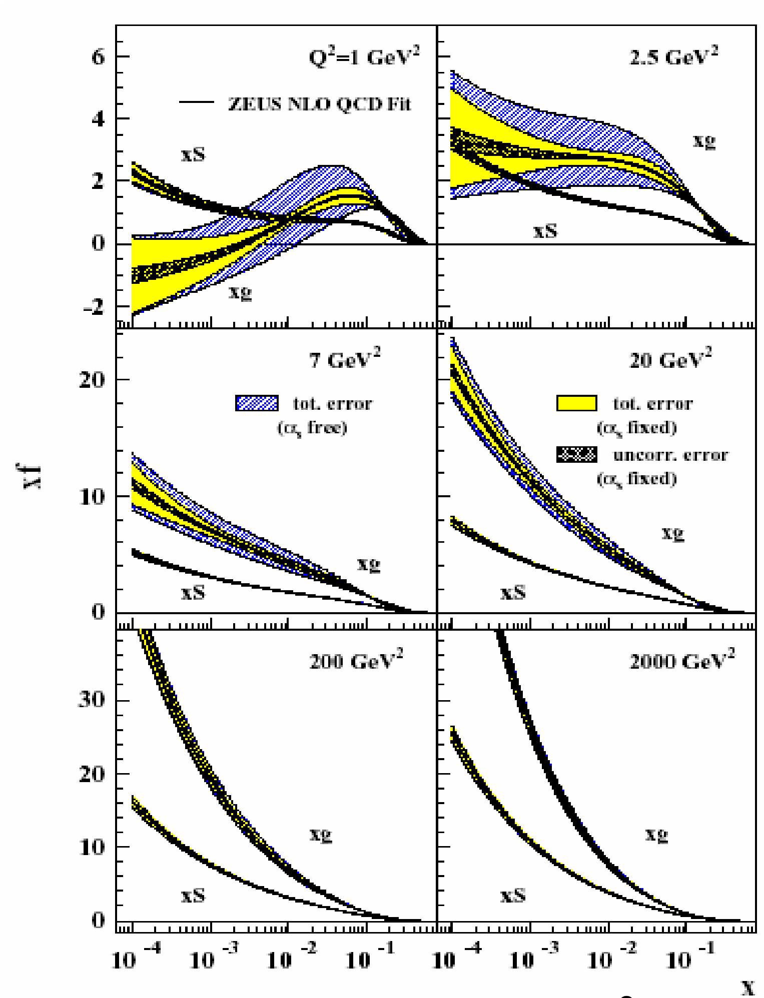
Fast increase of F_2 at small x → fast increase of number of gluons and sea quarks

→ « saturation », recombination effects ?

→ DGLAP not applicable

- BFKL evolution
- non-linear evolution

(remember small x is large $\gamma{*}p$ energy : $W^2 \simeq Q^2 / x$)



$Q^2 \simeq 1$ GeV^2
and maybe even < 0. Problem with observables ?
Indication of higher order effects ? (non-DGLAP)

6. Parton distribution uncertainties

Experimental uncertainties

- ❑ selection of data

  choice of accepted Q^2, W domain
- ❑ effect of experimental errors ?

  correlated / uncorrelated systematics
- ❑ how to combine « poorly compatible » experiments ?

➢ Hessian estimate of errors (correlation matrix)

deviation in χ^2 of the global fit from the minimum χ^2 value is assumed to be quadratic in the deviation of the fitted parameters errors from their best value → errors obtained from the covariance matrix, with $\Delta\chi^2 = 1$

BUT - hypothesis on the quadratic behaviour of uncertainties : (very) questionable

- (there may exist) strong correlations between parameters (if larger number than necessary)
- inconsistencies between experiments

→ which tolerance to define errors on pdf's ? $\Delta\chi^2$ = 100 (CTEQ), 50 (MRST), 1 (H1 – only DIS) ?

➢ Lagrange multipliers : a series of global fits using Lagrange parameters attached to each given measurement, constraining the measured cross sections by the quoted errors → how does the global description deteriorates as one moves away from the unconstrianed best fit – while spanning a range of Lagrange multipliers

But very heavy procedure

Theoretical uncertainties

- higher QCD orders – in DIS : NNLO
- *log (1/x)* and *log (1-x)* effects
- absorptive corrections – parton recombinations
- other higher twist contributions
- form of the parameterisation at starting scale
-   number of parameters ?
- … and relevance of the chosen factorisation scheme for the chosen parameterisation form
- choice of starting scale of evolution
- choice of α_S
- simplification assumptions

  isospin violation

  $s \neq \bar{s}$
- treatment of heavy flavours
- nuclear effects
- inclusion of e-w corrections (significant at NNLO)
- …

Remark : pdf's in Monte Carlos

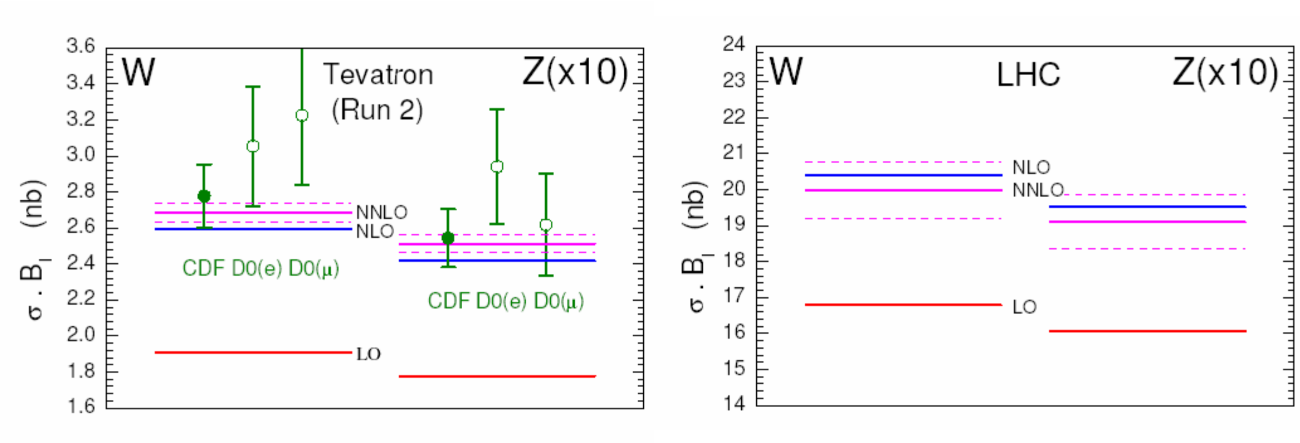
Present Monte Carlos are generally LO + simulation of higher orders through parton shower (JETSET)

JETSET follows DGLAP evolution – HERWIG is believed to be closer to BFKL evolution

Higher orders

All order summation is finite (factorisation theorem) *but* how fast is the convergence ?

- trust convergence if corrections decrease when computing next order



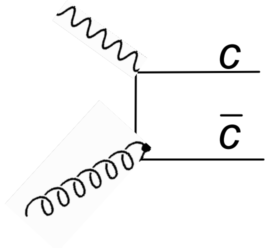
- sensitivity to scale = indication of size of next order contribution

$$\mu \frac{d}{d\mu} C^{(n)}(x, Q^2, \mu) \sim O(\alpha_s^{n+1})$$

small scale sensitivity at NL for DIS and D-Y

large for heavy quarks and prompt photon

Heavy quarks



No HQ in the nucleon at small scale

- dynamically generated (photon gluon fusion)

Works at not too large Q^2 but logarithmic divergence at large Q^2 $\approx \log \frac{Q}{m_q}$

- at large Q^2, treated as massless quarks

→ Fixed / variable flavour number scheme

Jets

full NNLO calculations not available yet

→ estimated through scale dependence :

μ often varied from 0.5 E_T to 2 E_T

Resummations

- Fixed order calculations ←→ resummation of all order contributions : *leading logarithms*

  Necessary when 2 scales, e.g. Q^2 and jet E_T

  ! double counting !

- DGLAP evolution : hard scale given by Q^2

  resums $\alpha_s^n \log^n Q^2$ terms (+ NLO etc.),

  corresponds to strong ordering in k_T of (virtual) partons

- BFKL evolution : in DIS domain (sufficiently large Q^2), very high energy

  resums $\alpha_s^n \log^n \frac{1}{x}$ terms

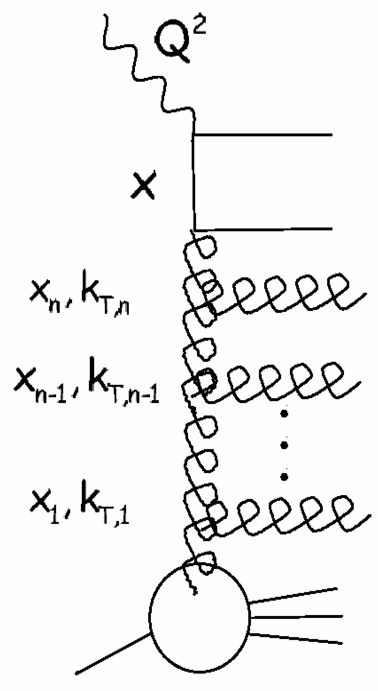
  corresponds to strong parton ordering in x (long. momentum)

  but not necessarily in k_T

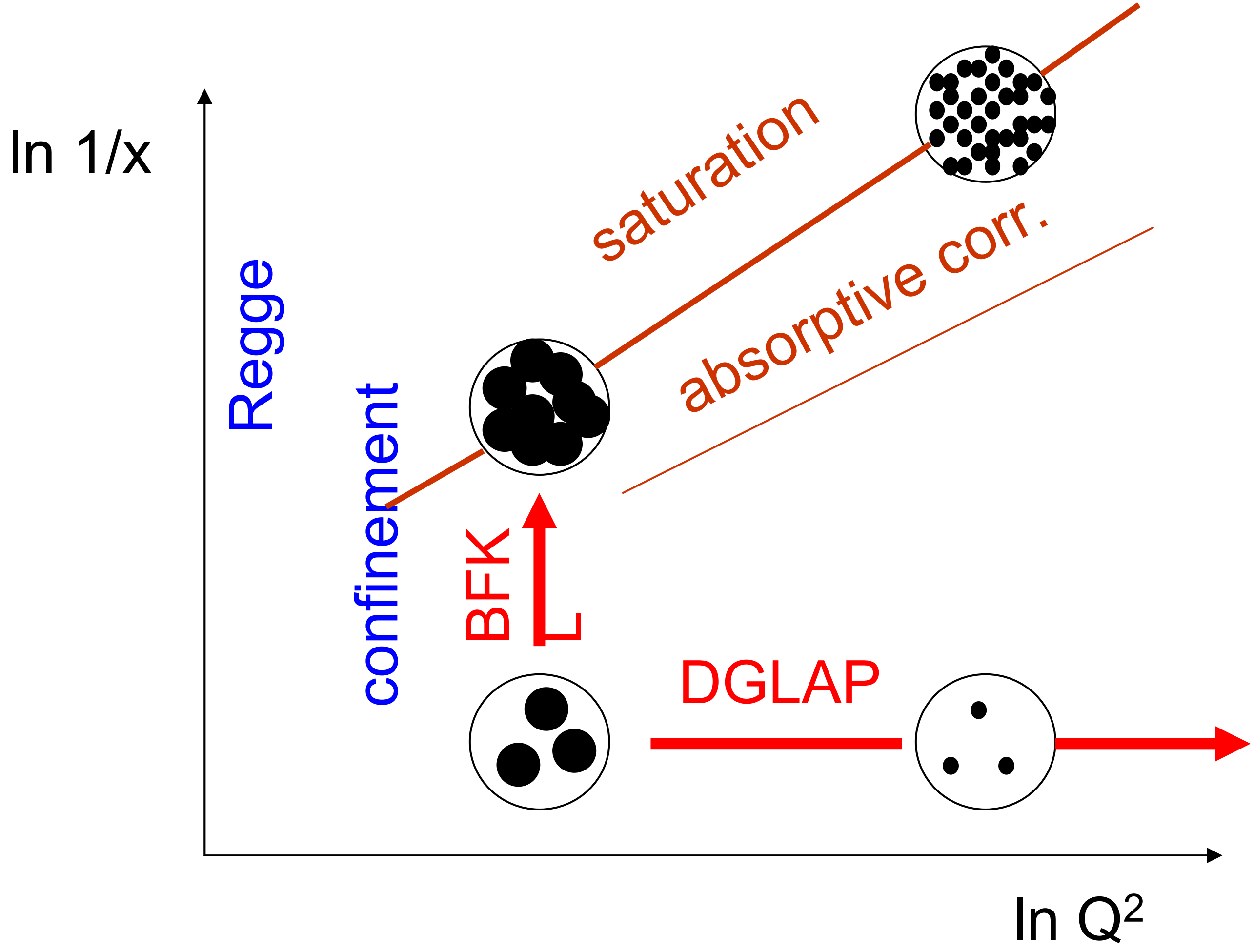
  Predicts fast increase

- CCFM evolution : connexion between DGLAP and BFKL

  angular ordering : $\theta = \frac{k_T}{xp}$



At very high parton density : saturation – parton recombination - non linear evolutions

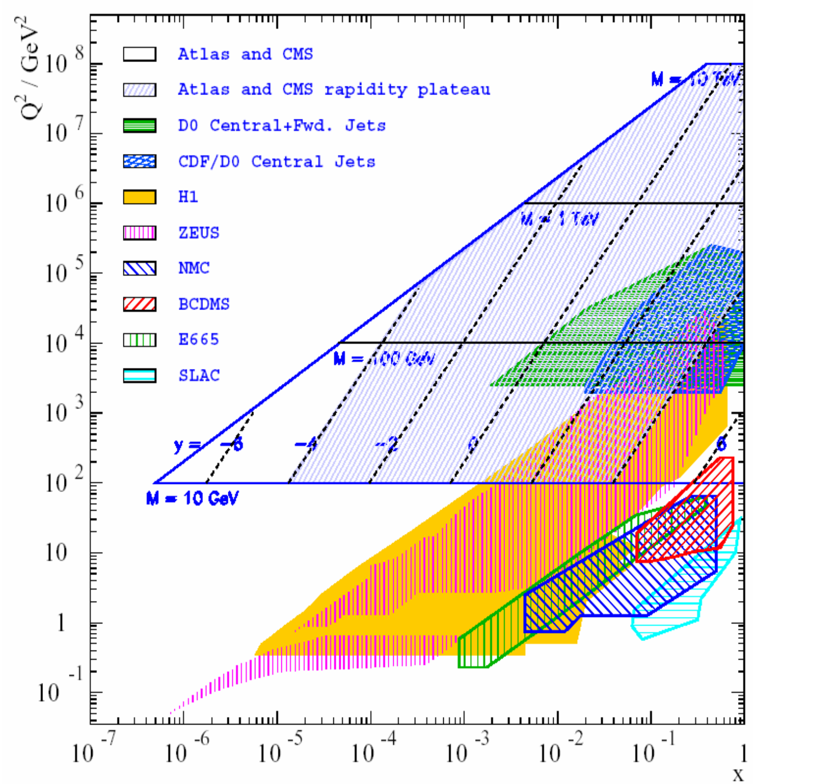


ln 1/x
saturation
absorptive corr.
Regge
confinement
BFK
L
DGLAP
ln Q^2

At the LHC…

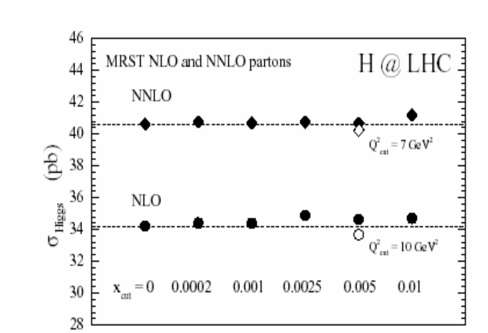
Precision predictions for SM processes are essential for discoveries :

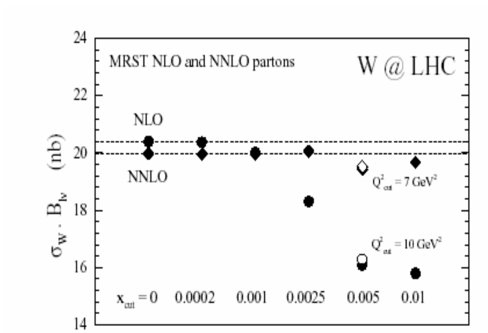
e.g. $pp \to t\bar{t}b\bar{b} \quad \leftrightarrow \quad pp \to t\bar{t}H \quad H \to b\bar{b}$



Experimentally : 2 orders of magnitude larger kinematic domain

Importance of settling theoretical uncertainties !

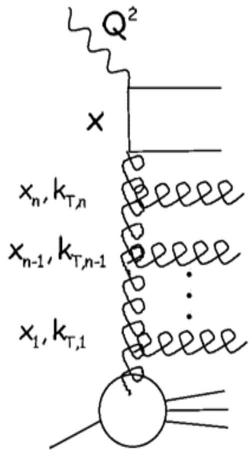




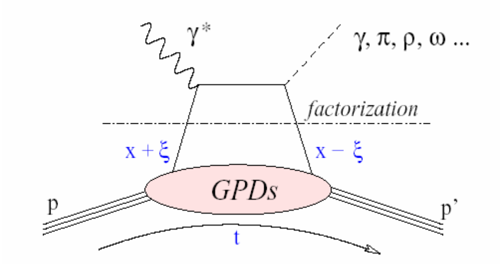
7. (Some of many) uncovered topics

Other parton distributions

- unintegrated k_T distributions
  relevant at very high energy, and when no strong k_T ordering
  (BFKL domain)
  e.g. large k_T jet or particle at large x



- generalised parton distributions
  correlations between partons



vector meson and real photon production (DVCS)
most relevant for large mass difference between initial and final state

- spin parton distributions
  dedicated experiments (HERMES, COMPAS, etc.)

Other hadrons or hadronic objects

- photon

  $\gamma\,\gamma$ scattering at LEP, hard photoproduction at HERA

  i.e. measurement of the hadronic structure of the photon

  (« resolved » photon ←→ « direct » photon = pointlike)

  $\gamma \to q\bar{q}$ + evolution, including gluon content of the photon

  NB in DGLAP evolution, inhomogeneous component (cf. NS SF)

- pion

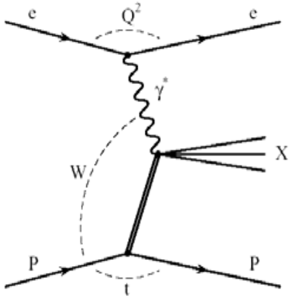
  Drell-Yan, leading neutron final states at HERA (interactions on the pion virtual cloud around the proton)

- pomeron : hadronic structure of diffractive exchange

  HERA (total diffractive production, vector mesons, charm, jets, etc.

  Tevatron (diffractive jet and W production)

  LHC : diffractive Higgs production



  Factorisation theorem proved

  but strong higher twist contributions

  + effects on evolution equations

  + underlying interaction → breaks simple application of of pdf transportation from HERA to Tevatron

  (« survival probability »)

Some references

- Introduction on DIS, SF, etc.
  F. Halzen, A.D. Martin, *Quarks and Leptons*, Wiley

- Introduction to pdf's and QCD
  CTEQ site http://www.phys.psu.edu/~cteq/ in particular
  QCD Handbook http://www.phys.psu.edu/~cteq/#Handbook
  W.K. Tung, Perturbative QCD and the parton structure of the nucleon
  see also : J.C. Collins, What exactly is a parton density? arXiv:hep-ph/0304122

- Present status of pdf's - draw your favourite pdf's
  MRST site http://durpdg.dur.ac.uk/hepdata/

- Pdf uncertainties : see e.g. (+ ref. therein)
  A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne
  Uncertainties of predictions from parton distributions
  I. Experimental errors arXiv:hep-ph/0211080
  II. Theoretical errors arXiv:hep-ph/0308087

- CERN PDFLIB manual http://consult.cern.ch/writeup/pdflib/