

RESPONSE SPECTRA

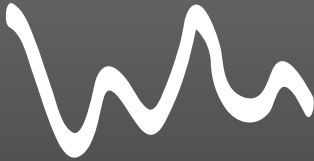


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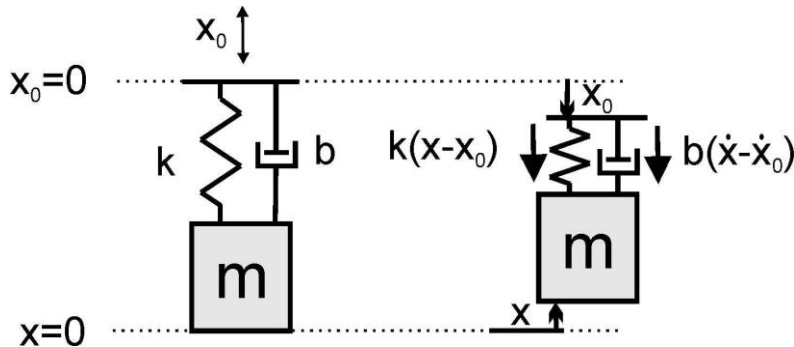
1

TYPES OF RESPONSE SPECTRA



2

Equation of motion



$$m\ddot{x} = -k(x - x_0) - b(\dot{x} - \dot{x}_0)$$

$$x_r = x - x_0$$

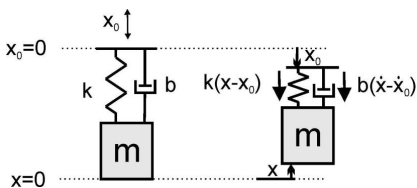
$$m\ddot{x}_r + b\dot{x}_r + kx_r = -m\ddot{x}_0$$

Excitation

3

3

Response to ground acceleration in the time domain



$$m\ddot{x} = -k(x - x_0) - b(\dot{x} - \dot{x}_0)$$

$$x_r = x - x_0$$

$$m\ddot{x}_r + b\dot{x}_r + kx_r = -m\ddot{x}_0$$

Earthquake base acceleration

Duhamel's integral

$$x(t) = \int_{-\infty}^{\infty} f(\tau)h(t - \tau)d\tau = f(t) * h(t)$$

Impulse response

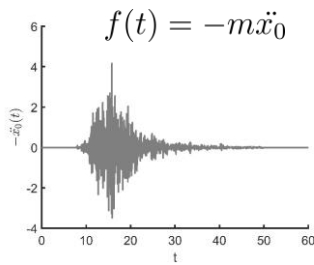
$$h(t) = \frac{e^{-\xi\omega_n t}}{m\omega_d} \sin(\omega_d t)$$

$$\longrightarrow x_r(t) = \int_{-\infty}^{\infty} -\ddot{x}_0(\tau) \frac{e^{-\xi\omega_n(t-\tau)}}{\omega_d} \sin(\omega_d(t-\tau))d\tau$$

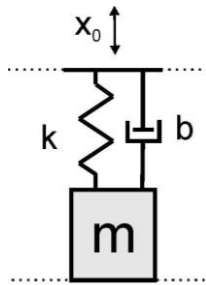
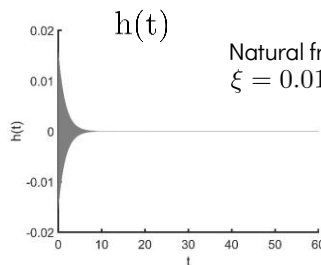
4

4

Santa Cruz earthquake (1990)

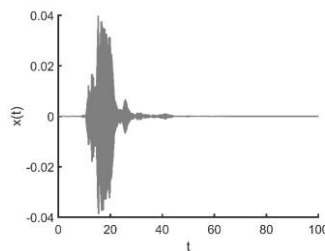


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Response to earthquake excitation

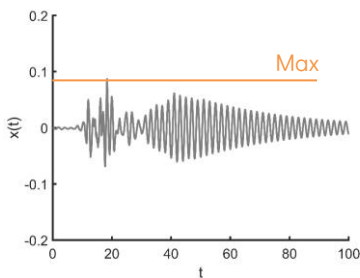


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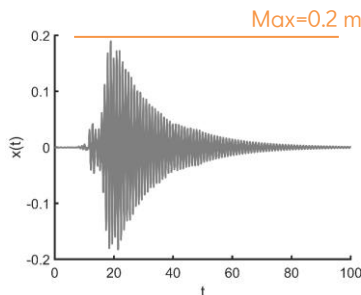
5

Response spectra

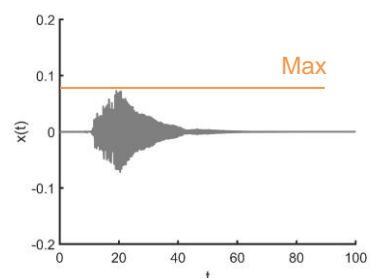
$$x_r(t) = \int_{-\infty}^{\infty} -\ddot{x}_0(\tau) \frac{e^{-\xi\omega_n(t-\tau)}}{\omega_d} \sin(\omega_d(t-\tau)) d\tau$$



Natural frequency = 0.5 Hz
 $\xi = 0.01$



Natural frequency = 1 Hz
 $\xi = 0.01$



Natural frequency = 2 Hz
 $\xi = 0.01$

- One value in a response spectrum corresponds to the **maximum of the absolute value of the amplitude** of a one dof system with given natural frequency and damping coefficient.
- It is built by varying the **natural frequency of the one dof system**, and taking the associated period $T=1/f$ for the x-axis

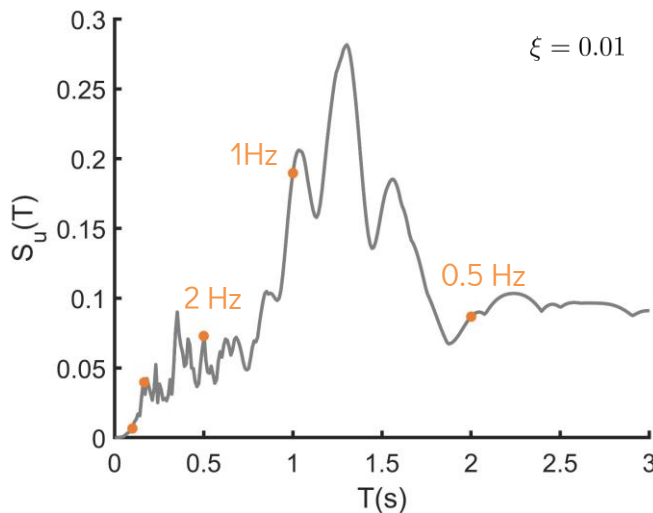
6

6

Displacement response spectrum

$$S_u(\xi, \omega_n) = |x_r(t)|_{max}$$

Santa Cruz Earthquake



- The x-axis does not correspond to the frequency of excitation
- It corresponds to the **eigenperiod of the one dof system considered**
- The natural frequency of the corresponding system is $f=1/T$
- The response spectrum is **specific to an earthquake**

7

Response spectra

$$x_r(t) = \int_{-\infty}^{\infty} -\ddot{x}_0(\tau) \frac{e^{-\xi\omega_n(t-\tau)}}{\omega_d} \sin(\omega_d(t-\tau)) d\tau$$

Relative displacement spectrum

$$S_u(\xi, \omega_n) = |x_r(t)|_{max}$$

Relative velocity spectrum

$$S_v(\xi, \omega_n) = |\dot{x}_r(t)|_{max}$$

Absolute acceleration spectrum

$$S_a(\xi, \omega_n) = |\ddot{x}_0(t) + \ddot{x}_r(t)|_{max}$$

Pseudo velocity response spectrum

$$S_{pv}(\xi, \omega_n) = \omega_n S_u(\xi, \omega_n)$$

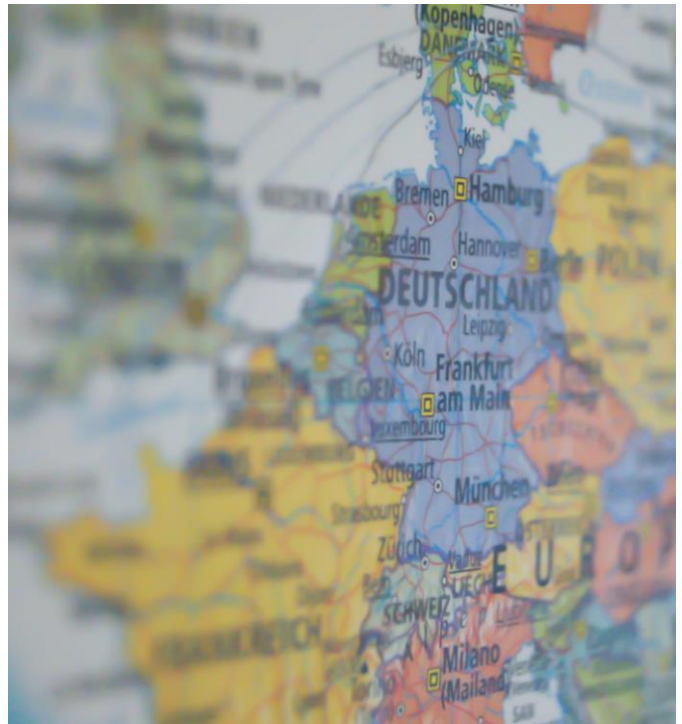
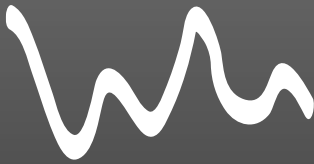
Pseudo acceleration response spectrum

$$S_{pa}(\xi, \omega_n) = \omega_n^2 S_u(\xi, \omega_n)$$

8

8

RESPONSE SPECTRA IN THE EUROCODE 8



9

Response spectra in the Eurocode

For an undamped one dof system :

$$m\ddot{x}_r + kx_r = -m\ddot{x}_0$$

$$\ddot{x}_r + \omega_n^2 x_r = -\ddot{x}_0$$

$$\ddot{x}_r + \ddot{x}_0 = -\omega_n^2 x_r$$

$$S_{pa} = S_e = \omega_n^2 S_u \simeq S_a$$

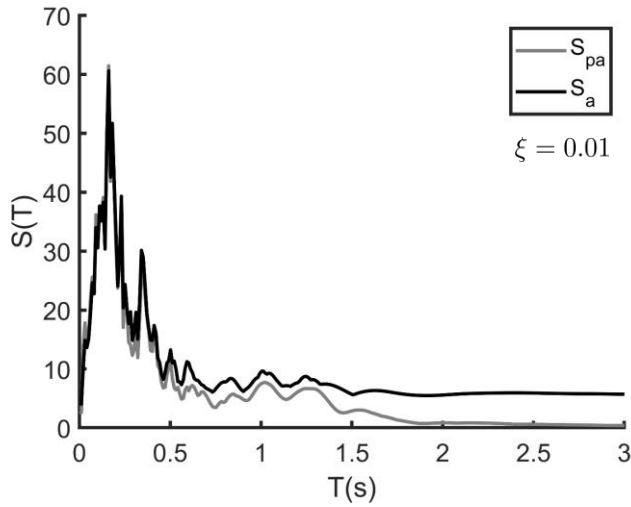


The response spectrum specified in the Eurocode (S_e) is an approximation of the total acceleration, valid when damping is small

10

10

Response spectra

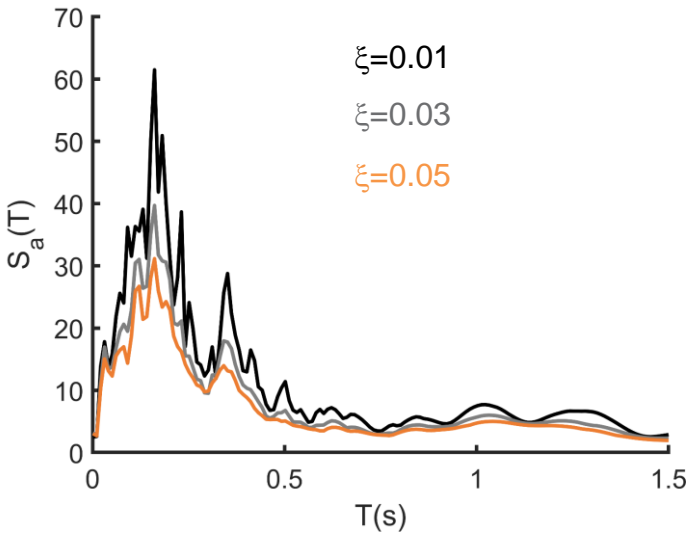


Santa Cruz Earthquake

$$S_a(\xi, \omega_n) = |\ddot{x}_0(t) + \ddot{x}_r(t)|_{max}$$

$$S_{pa}(\xi, \omega_n) = \omega_n^2 S_u(\xi, \omega_n)$$

Response spectra : influence of damping



Santa Cruz Earthquake

Response spectra in the Eurocode

Horizontal elastic spectrum

$$0 \leq T \leq T_B : S_e(T) = a_g S \left(1 + \frac{T}{T_B} (2.5\eta - 1) \right)$$

$$T_B \leq T \leq T_C : S_e(T) = a_g S 2.5\eta$$

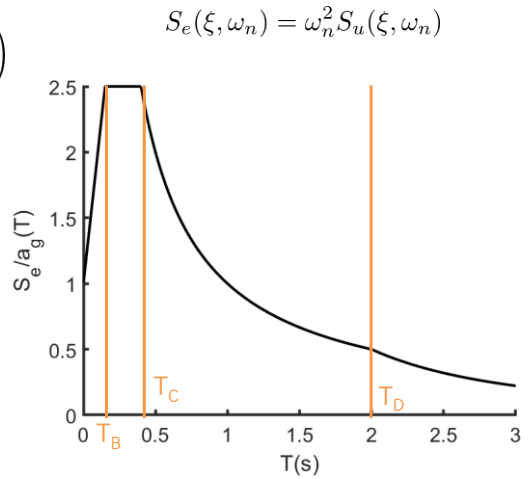
$$T_C \leq T \leq T_D : S_e(T) = a_g S 2.5\eta \frac{T_C}{T}$$

$$T_D \leq T \leq 4s : S_e(T) = a_g S 2.5\eta \frac{T_C T_D}{T^2}$$

a_g = ground acceleration

S = soil factor

η = damping correction factor (=1 for 5% damping)



13

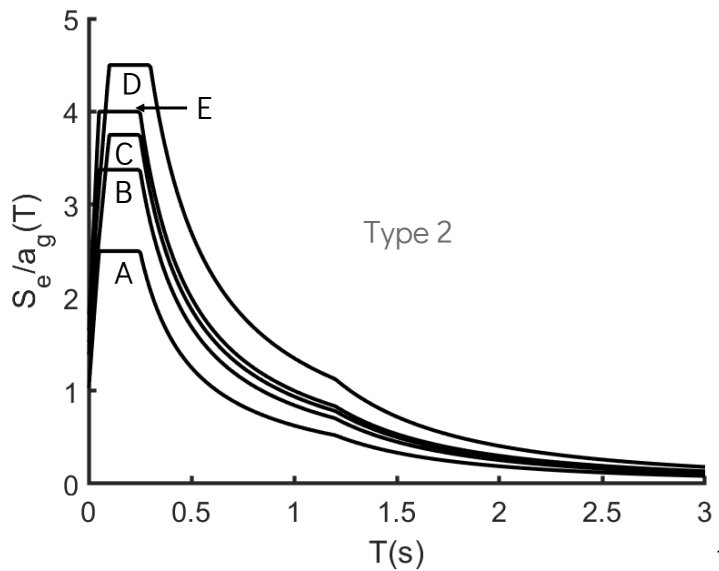
Response spectra in the Eurocode

Ground type	S	T_B	T_C
A	1.0	0.15	0.4
B	1.2	0.15	0.5
C	1.15	0.2	0.6
D	1.35	0.2	0.8
E	1.4	0.15	0.5

Type 1

Ground type	S	T_B	T_C
A	1.0	0.05	0.25
B	1.35	0.05	0.25
C	1.5	0.1	0.25
D	1.8	0.1	0.3
E	1.6	0.05	0.25

Type 2 (Belgium)



14

14

Design spectrum vs elastic spectrum

The design spectrum takes into account the inelastic behavior using the behavior factor q , and adds a lower bound factor β

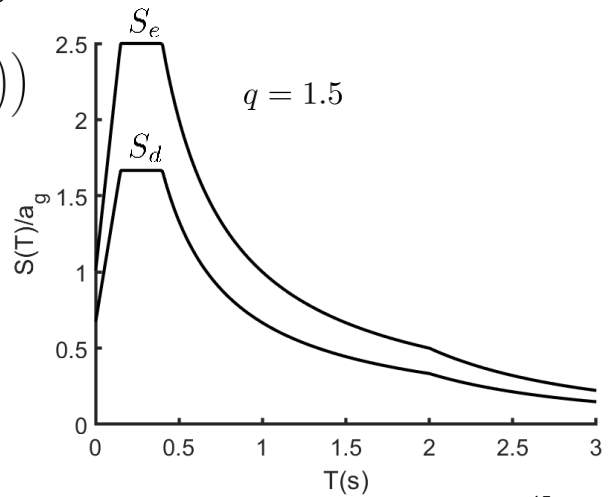
$$0 \leq T \leq T_B : S_d(T) = a_g S \left(\frac{2}{3} + \frac{T}{T_B} \left(\frac{2.5}{q} - \frac{2}{3} \right) \right)$$

$$T_B \leq T \leq T_C : S_d(T) = a_g S \frac{2.5}{q}$$

$$T_C \leq T \leq T_D : S_d(T) \begin{cases} = a_g S \frac{2.5}{q} \frac{T_C}{T} \\ \geq \beta a_g \end{cases}$$

$$T_D \leq T : S_d(T) \begin{cases} = a_g S \frac{2.5}{q} \frac{T_C T_D}{T^2} \\ \geq \beta a_g \end{cases}$$

S_d is the **design spectrum**



15

RESPONSE SPECTRA FOR MDOF SYSTEMS



16

Response spectra for MDOF systems

$$\ddot{z}_{ri} + 2\xi_i\omega_i\dot{z}_{ri} + \omega_i^2 z_{ri} = -\frac{\Gamma_i}{\mu_i}\ddot{x}_0 \quad \Gamma_i = \psi_i^T MT$$



$$z_{i,max} = |z_i(t)|_{max} = \frac{\Gamma_i}{\mu_i} S_u(\xi_i, \omega_i)$$

$$x_{i,max} = z_{i,max} \psi_i \quad \text{Contribution of mode } i \text{ to the displacement vector}$$

$$\ddot{x}_{a,i,max} = \frac{\Gamma_i}{\mu_i} S_{pa}(\xi_i, \omega_i) \psi_i$$

17

17

Response spectra for MDOF systems

Modal combinations

Absolute sum method (ABSSUM)

$$r_{max} = \sum_{i=1}^n |r_i|$$

—————> Leads to over estimation

Square root of sum of squares (SRSS)

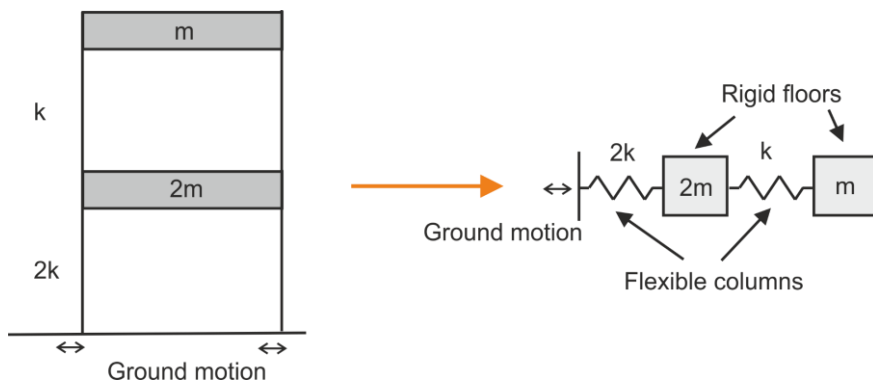
$$r_{max} = \sqrt{\sum_{i=1}^n r_i^2}$$

Most often used in practice

18

18

Example of a two-floor building



$$k = 197\,392 \text{ N/m}, m = 2500 \text{ kg}, \xi = 0.05$$

19

19

Example of a two-floor building : application of Eurocodes

$$K = \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix} \quad M = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}$$

Mode shapes and natural frequencies

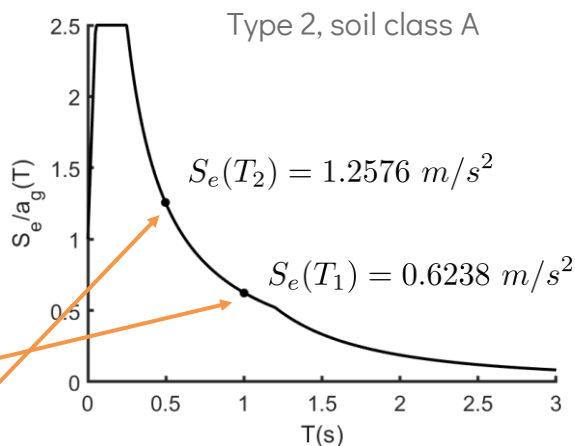
$$\Psi = \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\omega_1 = 2\pi \text{ rad/s}$$

$$\omega_2 = 4\pi \text{ rad/s}$$

$$T_1 = 1 \text{ s}$$

$$T_2 = 0.5 \text{ s}$$



20

20

Example of a two-floor building

$S_e(T_1) = 0.6238 \text{ m/s}^2$	$\xrightarrow{q = 1.5}$	Design spectrum $S_d(T_1) = 0.4158 \text{ m/s}^2$
$S_e(T_2) = 1.2576 \text{ m/s}^2$		$S_d(T_2) = 0.8384 \text{ m/s}^2$

Modal participation factors

$$\frac{\Gamma_1}{\mu_1} = \frac{\psi_1^T M \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\psi_1^T M \psi_1} = \frac{4}{3} \qquad \frac{\Gamma_2}{\mu_2} = \frac{\psi_2^T M \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\psi_2^T M \psi_2} = -\frac{1}{3}$$

Max modal displacements $a_g = 1 \text{ m/s}^2$

$$Z_{1,max} = \frac{\Gamma_1}{\mu_1} a_g S_d(T_1) / \omega_1^2 = 0.0140 \text{ m}$$

$$Z_{2,max} = \frac{\Gamma_2}{\mu_2} a_g S_d(T_2) / \omega_2^2 = -0.0018 \text{ m}$$

21

21

Example of a two-floor building

Max displacement of top of building :

$$X_{1,max} = Z_{1,max} \psi_1 = 0.014 \text{ m} \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

$$X_{2,max} = Z_{2,max} \psi_2 = -0.0018 \text{ m} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Top floor displacement,
 contribution of each
 mode

$$X_{SRSS, TOP} = \sqrt{X_{1,max}(2)^2 + X_{2,max}(2)^2} = 0.0142 \text{ m}$$

22

22

Example of a two-floor building

Shear force at the bottom of ground level columns

$$F_{1,max} = 2kZ_{1,max}\psi_1(1) = 2.77 \text{ kN} \leftarrow$$
$$F_{2,max} = 2kZ_{2,max}\psi_2(1) = 682 \text{ N} \leftarrow$$

Shear force ground level columns, contribution of each mode

$$F_{SRSS,BOT_SHEAR} = \sqrt{F_{1,max}(2)^2 + F_{2,max}(2)^2} = 2.859 \text{ kN}$$

23

23