# FINITE ELEMENT MODELS

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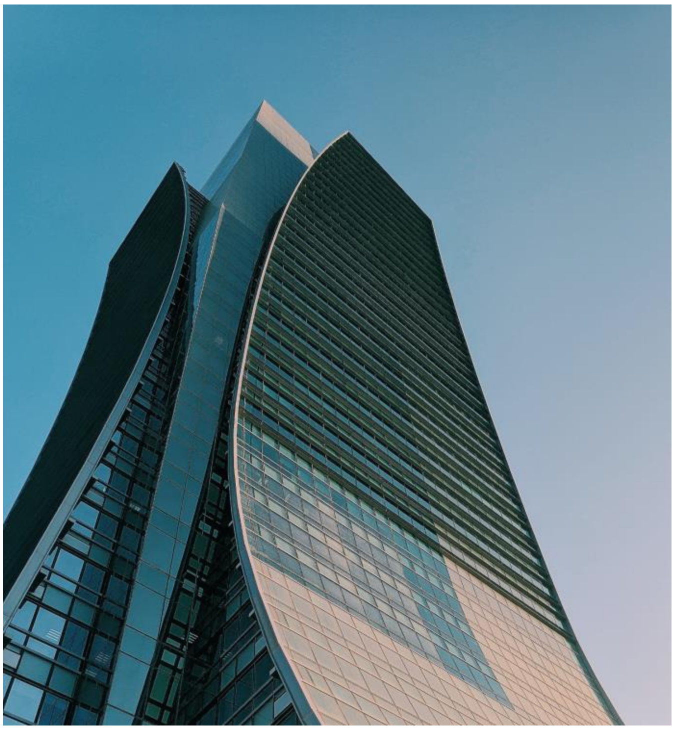
# COMPLEX STRUCTURES



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## Complex structures in mechanical engineering

3D kinematics

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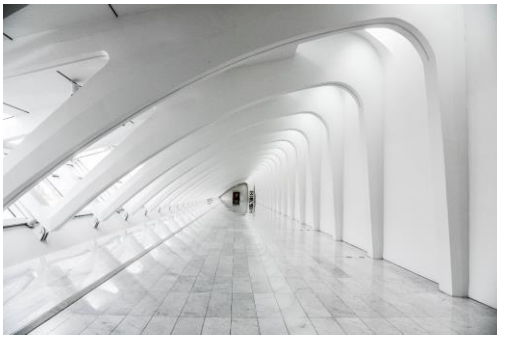
## Complex structures in civil engineering



3D kinematics

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## Finite element models of complex structures

30 storey building model

Beam and plate elements

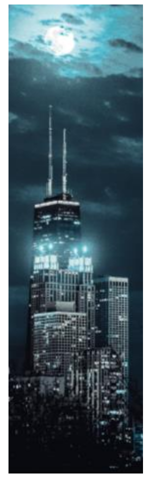
- 9222 nodes
- 11700 elements
- 55332 degrees of freedom

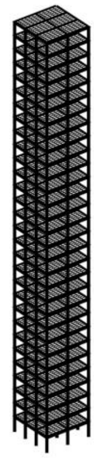
↓

$$M\ddot{x} + Kx = f$$

55332 equations with 55332 unknowns

SDTools
VIBRATION SOFTWARE & CONSULTING





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## MODE SHAPES

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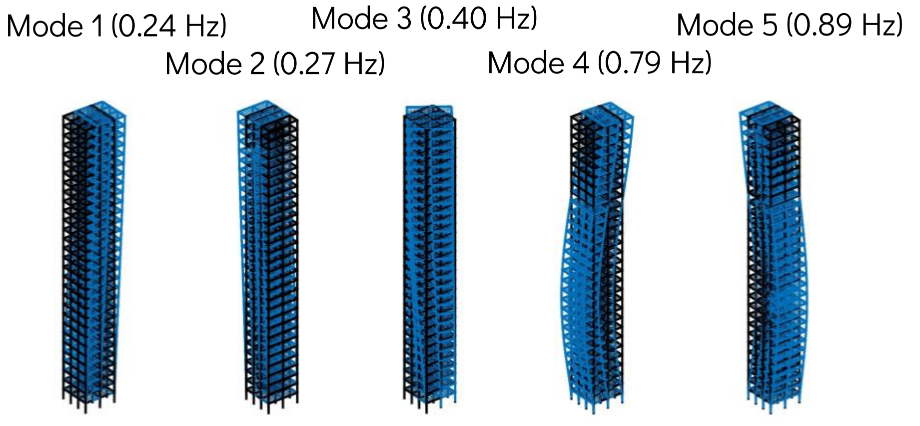
## Building mode shapes




Mode shapes are generally 'mass normalized' : $\mu_i = 1$

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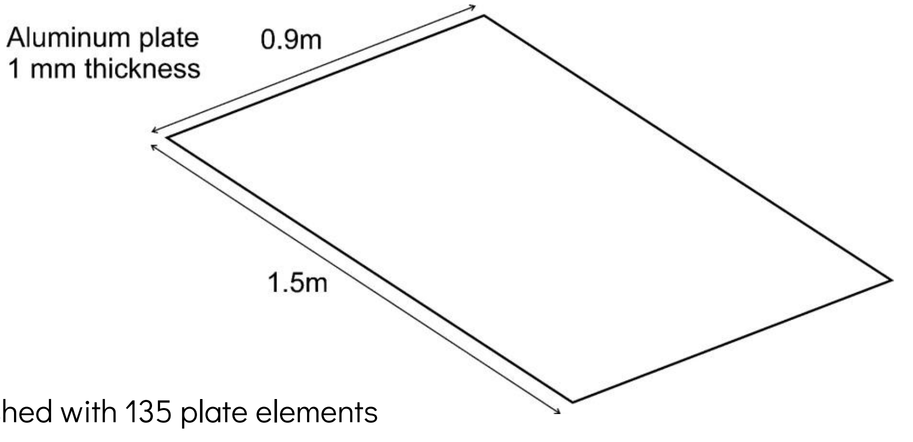
## Cantilever plate

Cantilever plate




Meshed with 135 plate elements

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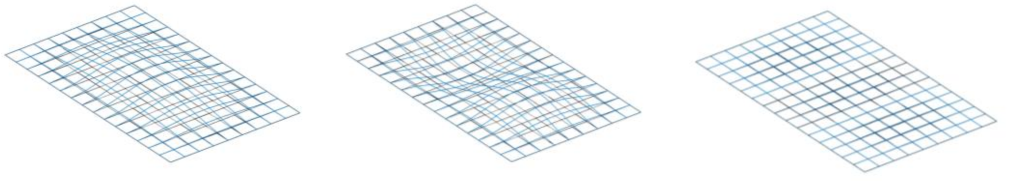
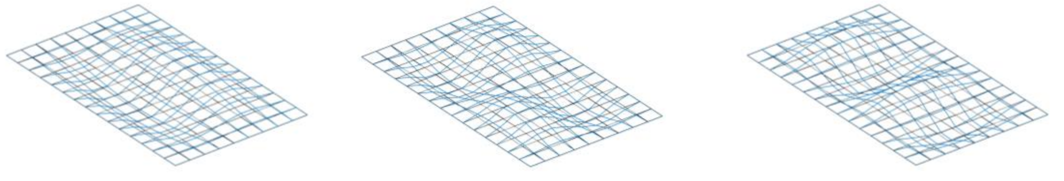
## Cantilever plate mode shapes

Mode 1 (8.2 Hz) Mode 2 (11.81 Hz) Mode 3 (18.49 Hz)



Mode 4 (22.11 Hz) Mode 5 (25.38 Hz) Mode 6 (28.68 Hz)



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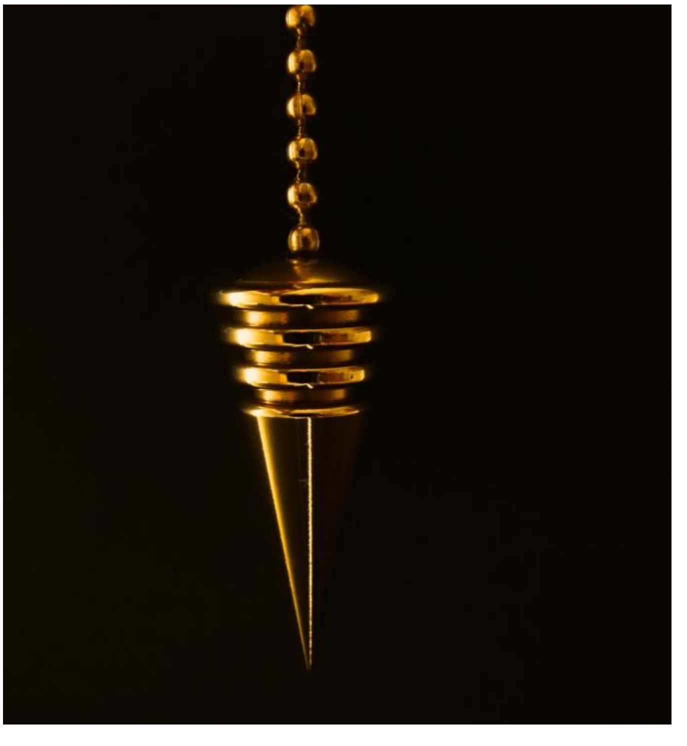
## HARMONIC RESPONSE



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## Computation of the dynamic response

$$M\ddot{x} + Kx = f$$

Orthogonality conditions    Projection in the modal basis

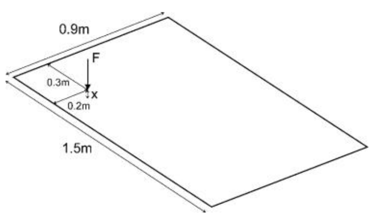
$$\begin{aligned}\psi_i^T M \psi_j &= \delta_{ij}\mu_i \\ \psi_i^T K \psi_j &= \delta_{ij}\mu_i\omega_i^2\end{aligned} \quad \longrightarrow \quad \boxed{\mu_i\ddot{z}_i + \mu_i\omega_i^2 z_i = F_i}$$

•In practice, the number of dofs in the finite element model is dictated by the details of the geometry and for large models, it is not possible to compute all the modeshapes.
•In addition, the number of modes in the frequency band of interest is usually quite low (i.e 10 to 50 modes)

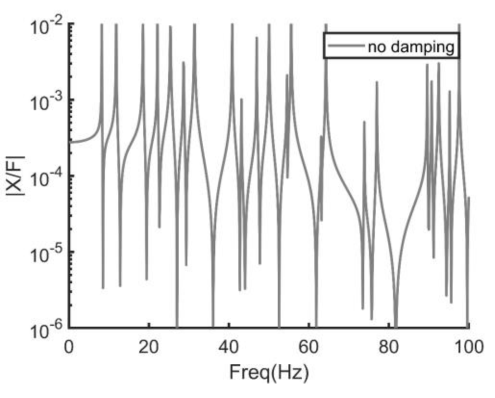
⟶ Important reduction when projecting on the modal basis



## Harmonic response of the cantilever plate




$$(K - \omega^2 M)X = F$$




$$X(\omega) = \sum_{i=1}^{n} \frac{\psi_i^T F \psi_i}{\mu_i(\omega_i^2 - \omega^2)}$$

n=25

## Global damping models

$$M\ddot{x} + C\dot{x} + Kx = f$$

Global damping models

Rayleigh damping

$$C = \alpha K + \beta M$$

Global viscous model

$$C = \alpha K$$

Loss factor – Hysteretic damping

$$(K + i\omega C - \omega^2 M)X = F$$

$$(K(1 + i\eta) - \omega^2 M)X = F \qquad C = \frac{\eta}{\omega}K$$

→ Used most of the time for frequency domain computations





## Global damping models

For proportional (global) damping models

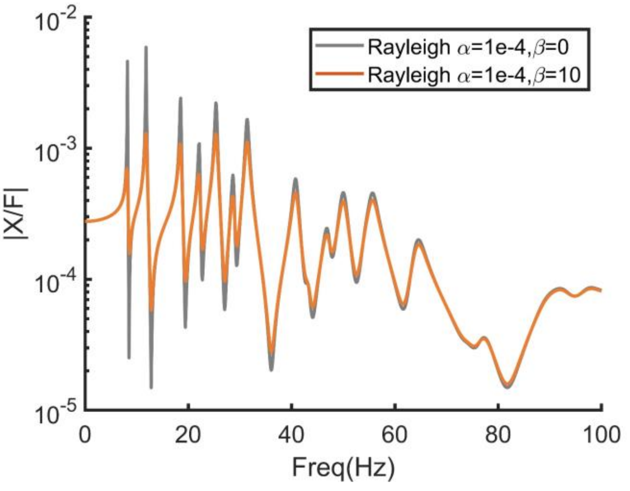
$$\mu_i \ddot{z}_i + 2\xi_i \mu_i \omega_i \dot{z}_i + \mu_i \omega_i^2 z_i = F_i$$

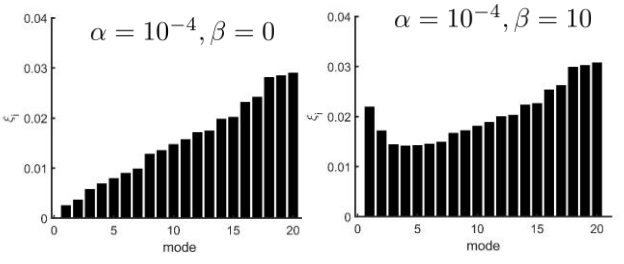
Rayleigh damping $\Rightarrow \xi_i = \frac{1}{2}\left(\alpha\omega_i + \frac{\beta}{\omega_i}\right)$

Loss factor $\Rightarrow \xi_i = \frac{\eta}{2}$ Constant modal damping

Modal damping $\Rightarrow \xi_i$ For each mode

## Harmonic response : comparison of global damping models




$$X(\omega) = \sum_{j=1}^{n} \frac{\psi_j^T F \psi_j}{\mu_j(\omega_j^2 - \omega^2 + 2i\xi_j \omega \omega_j)}$$

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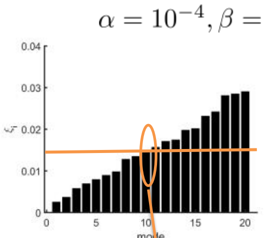
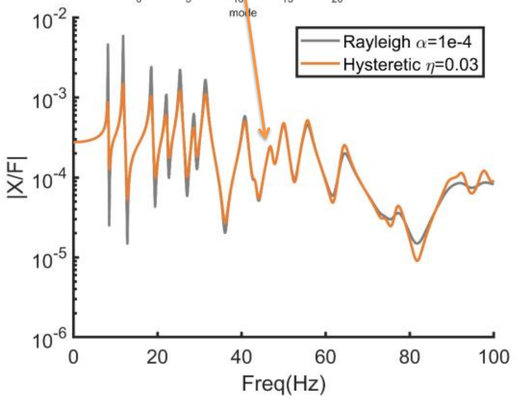
## Harmonic response : comparison of global damping models

$\alpha = 10^{-4}, \beta = 0$




$\eta = 0.03 \Rightarrow \xi_i = 0.015$

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## Local damping models

$$M\ddot{x} + C\dot{x} + Kx = f$$

Viscous damping

$C_i = \alpha_i K_i$ In each substructure

Loss factor – Hysteretic damping

$E(1 + i\eta)$ Loss factor can be different for each material

→ Non proportional damping

$\Psi^T C \Psi$ is not diagonal

If damping is small $\Rightarrow \xi_i = f(\alpha_i, \eta_i, ...)$

Usually identified experimentally, or from similar structures

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## TIME DOMAIN RESPONSE

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## Base excitation : time domain response

$$M\ddot{x}_r + C\dot{x}_r + Kx_r = -M\ddot{x}_b \qquad \ddot{x}_b = T\,\ddot{x}_0$$

$$\longrightarrow \quad M\ddot{x}_r + C\dot{x}_r + Kx_r = -M\,T\ddot{x}_0$$

Projection in the modal basis using mode shapes with fixed base ( $x_b = 0$)

$$x_r = \sum_{i=1}^{n} z_i(t)\psi_i \qquad x_r = \Psi Z$$

$$\Psi^T M \Psi \ddot{z}_r + \Psi^T C \Psi \dot{z}_r + \Psi^T K \Psi z_r = -\Psi^T M T \ddot{x}_0$$

$$\mu_i \ddot{z_{ri}} + 2\mu_i \xi_i \omega_i \dot{z_{ri}} + \mu_i \omega_i^2 z_{ri} = -\Gamma_i \ddot{x}_0$$

$\Gamma_i = \psi_i^T M T$ Modal acceleration factor

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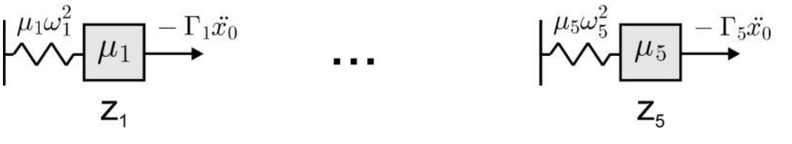
## Base excitation : time domain response

$$\mu_i \ddot{z_{ri}} + 2\mu_i \xi_i \omega_i \dot{z_{ri}} + \mu_i \omega_i^2 z_{ri} = -\Gamma_i \ddot{x}_0 \qquad \Gamma_i = \psi_i^T M T$$




Impulse response

$$h_i(t) = \frac{e^{-\xi_i \omega_i t}}{\mu_i \omega_{di}} \sin(\omega_{di} t) \qquad \omega_{di} = \omega_i \sqrt{1-\xi_i^2}$$

Convolution

$$z_i(t) = h_i(t) * -\Gamma_i \ddot{x}_0$$

Expansion

$$x_r(t) = \sum_{i=1}^{n} z_i(t)\psi_i$$

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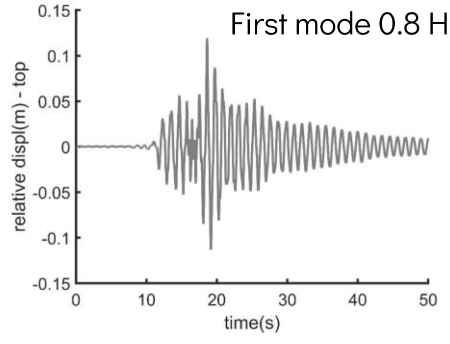
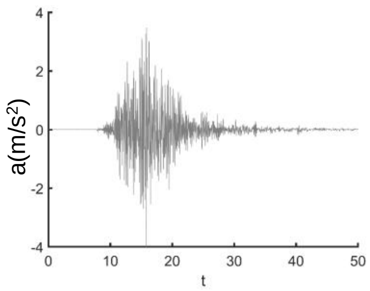
## Earthquake response of a 10 storey building

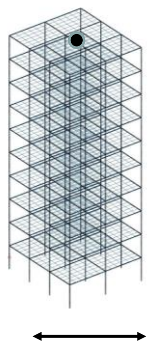
Modal damping / 20 modes (0-15 Hz)

$\xi = 0.01$

First mode 0.8 Hz



Ground motion (acceleration)

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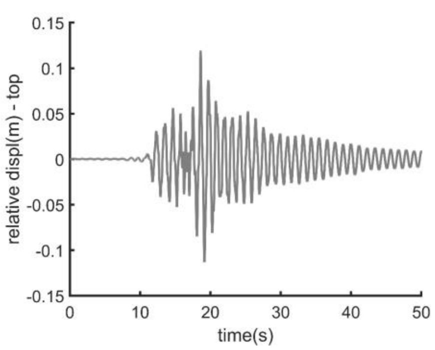
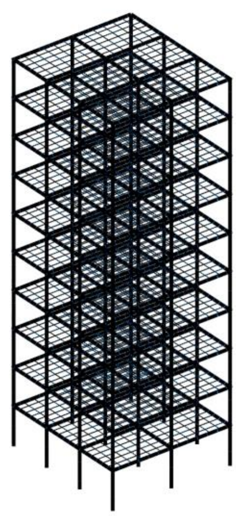
## Earthquake response of a 10 storey building

t=0



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