Acoustics exercise session of October 11, 2019 - Answers

Solution 1

$$\frac{\partial p(x,t)}{\partial x} = \frac{df\,(x-ct)}{d\,(x-ct)} \cdot \frac{\partial\,(x-ct)}{\partial x} = f'\,(x-ct) \tag{3}$$

$$\frac{\partial^2 p(x,t)}{\partial x^2} = \frac{df'\,(x-ct)}{d\,(x-ct)} \cdot \frac{\partial\,(x-ct)}{\partial x} = f"\,(x-ct) \tag{4}$$

$$\frac{\partial p(x,t)}{\partial t} = \frac{df\,(x-ct)}{d\,(x-ct)} \cdot \frac{\partial\,(x-ct)}{\partial t} = -cf'\,(x-ct) \tag{5}$$

$$\frac{\partial^2 p(x,t)}{\partial t^2} = \frac{df'\,(x-ct)}{d\,(x-ct)} \cdot \frac{\partial\,(x-ct)}{\partial t} = c^2 f"\,(x-ct) \tag{6}$$

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = f"\,(x-ct) - \frac{c^2}{c^2} f"\,(x-ct) = 0 \tag{7}$$

Solution 2

The delay between sound and image is :

$$\Delta t = \frac{15}{c_{sound}} - \frac{15}{c_{light}} \simeq \frac{15}{340} = 0.044s \tag{8}$$

This is roughly one twenty-fourth of a second ($\frac{1}{24} = 0.042$). The sound and image track should be shifted by one image. This was actually done on old film reels.

Solution 3

First fan : The rotation speed of 1200 rpm translates into a rotation frequency of 20Hz. We should therefore expect tones at 20Hz and all its multiple **but** the fan having 7 blades, one should expect that harmonics whose order is a multiple of 7 will be highly dominant (140, 280, 420). Finally there is always some non-periodic, wide-band noise, for instance associated with turbulence. The spectrum could therefore look this. Remember it is schematic and that the vertical scale is undefined.

Second fan : The fan has 8 blades but one of them is broken (this problem is called wind-milling). One of the blade is broken. The fundamental frequency is clearly 20Hz and we see all multiples of that. Harmonic 8, 16 and other multiple of 8 are nevertheless more important than their neighbours. Note that if a system is designed to be compatible with an eight blade fan i.e. to support an excitation at 160Hz, it may be severely affected if, by the loss of one blade, the excitation suddenly jumps down from 160 to 20 Hz.

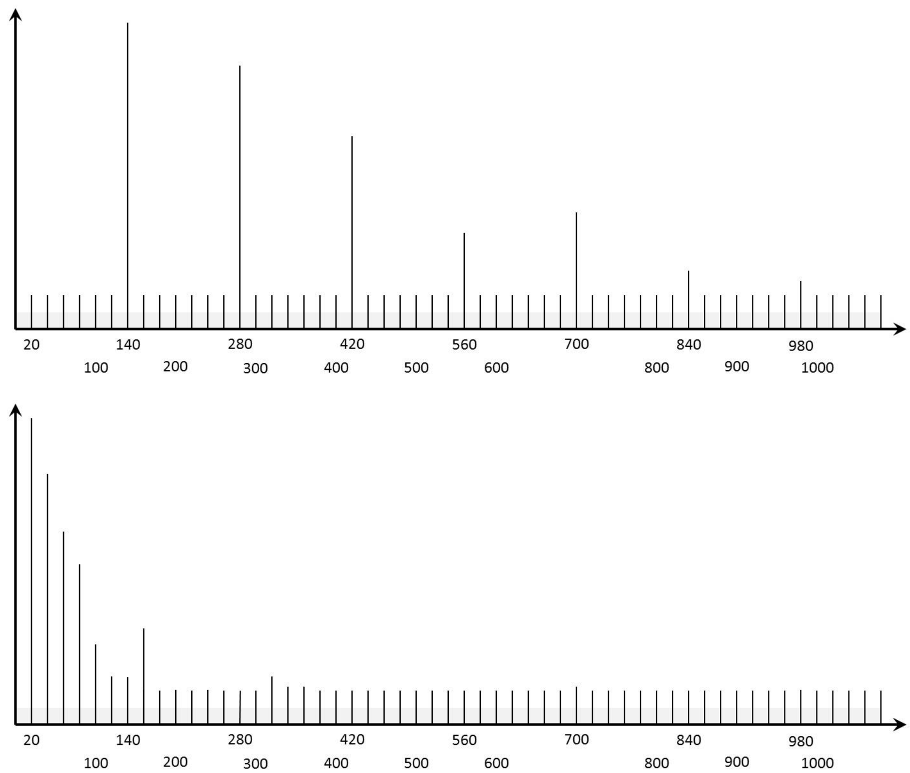


FIGURE 3 : Two fans. Typical expected noise spectrum. The grey background schematically represent the broad-band component.

Solution 4

$$R = c_p - c_v = 287.0\ J/(kg.K)) \tag{9}$$

$$\gamma = \frac{c_p}{c_v} = 1.4 \tag{10}$$

$$c = \sqrt{\gamma RT} = \sqrt{1.4 \cdot 287 \cdot 293.15} = 343.2\ m/s \tag{11}$$

$$\rho = \frac{p}{RT} = \frac{220000}{287 \cdot 293.15} = 2.615\ kg/m^3 \tag{12}$$

Solution 5

Use the relationships :

$$A = \Re(D),\ \ B = \Im(D),\ \ C = \sqrt{A^2 + B^2},\ \ \tan\phi = \frac{B}{A} \tag{13}$$

$$T = \frac{1}{f}, \ \ \omega = 2\pi f, \ \ c = \sqrt{\gamma R T}, \ \ k = \frac{\omega}{c}, \ \ \lambda = \frac{c}{f} \tag{14}$$

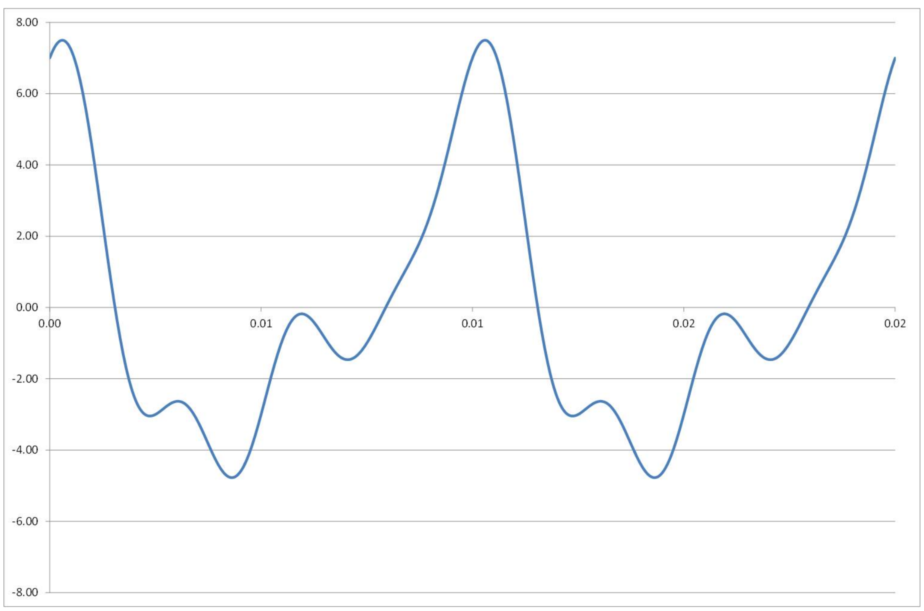
Temperature		60
Speed of sound		365.868378

Frequency	Real part	Imaginary part	Amplitude	Phase°	Period	Pulsation	Wave-number	Wavelength
100	4	1	4.12	14.04	0.0100	628.32	1.72	3.66
200	2	-1	2.24	-26.57	0.0050	1256.64	3.43	1.83
300	1	0	1.00	0.00	0.0033	1884.96	5.15	1.22
400	0	1	1.00	90.00	0.0025	2513.27	6.87	0.91

The signal is ($\omega_1 = 200\pi$, $\omega_2 = 400\pi$, $\omega_3 = 600\pi$, $\omega_4 = 800\pi$) :

$$\begin{aligned} p(t) &= \Re\big[(4+i)\, e^{i\omega_1 t} + (2-i)\, e^{i\omega_2 t} + e^{i\omega_3 t} + i e^{i\omega_4 t}\big] \\ &= 4\cos\omega_1 t - \sin\omega_1 t + 2\cos\omega_2 t + \sin\omega_2 t + \cos\omega_3 t - \sin\omega_4 t \end{aligned} \tag{15}$$

The period of the signal is $T = \frac{1}{100}$ i.e. 10ms.



Solution 6

The formula for active intensity is :

$$I_a = \frac{1}{2}\Re(p \cdot v^*) \tag{16}$$

We get :

$$\begin{aligned} I_a &= \frac{1}{2}\Re[(4+i)\cdot(-i)+(2-i)\cdot(1-i)+1\cdot(1+i)] \\ &= \frac{1}{2}(1+1+1) = 1.5\ W/m^2 \end{aligned} \tag{17}$$

Dimensional analysis (F=force ([N]), d=distance ([m]), S=surface ([m^2]), t=time ([s]), E=energy or work ([J]), P=power ([W])) :

$$I = p \cdot v = \frac{F}{S}\cdot\frac{d}{t} = \frac{F\cdot d}{A\cdot t} = \frac{E}{A\cdot t} = \frac{P}{A} \rightarrow W/m^2 \tag{18}$$

Solution 7

$$L = 10\log\left(10^{\frac{60}{10}} + 10^{\frac{55}{10}} + 10^{\frac{50}{10}} + 10^{\frac{48}{10}} + 10^{\frac{47}{10}}\right) = 61.85\ dB \tag{19}$$