BRUFACE **Vibrations and Acoustics**
MA1 **Academic year 2017-2018**

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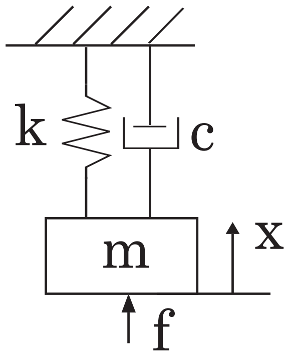
Session 1 : Fundamental concepts

Exercise 1

Consider the following one-degree-of-freedom (1 DOF) system



- Write the equation of motion in the time domain. Give the expression of ω_n and of ξ
- For this system

  a) Give the expression of the impulse response and represent it using the following numerical values: $m = 1\,kg$, $k = 16\,N/m$, $c = 0.1\,Ns/m$)

  b) Give the expression of the harmonic forced response and represent it using the Bode diagram

  c) Plot the Nyquist diagram and indicate the resonant frequency as well as some intermediary frequencies on the diagram. Comment.

  d) Repeats points a),b) and c) with the following successive values of damping: $c = 0.1\,Ns/m$, $c = 0.5\,Ns/m$, $c = 10\,Ns/m$. What are the corresponding values of ξ? Plot the respective responses on the same Bode diagram (both amplitude and phase)

  e) Write the relationship between the impulse response and the forced harmonic response. Verify numerically that it is valid.
  Note : the discrete Fourier transform (DFT) is defined as :

$$c_n = \frac{1}{T}\int_0^T u(t)\, e^{-in\omega_0 t} dt$$

  When the signal u(t) is sampled at m regular time intervals with time spacing Δt, for a small value of Δt we have

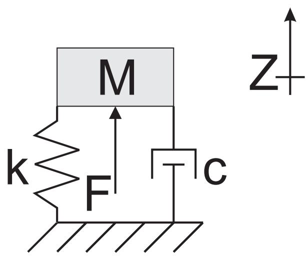
$$c_n T \simeq \sum_{j=0}^{m} \Delta t\, u(j\Delta t)\, e^{-in\omega_0 j\Delta t} = \Delta t \sum_{j=0}^{m} u(j\Delta t)\, e^{-in\omega_0 j\Delta t} = (\Delta t) fft(u)$$

where $fft(u)$ is the discrete Fourier transform of a sampled signal as computed in Matlab using the Fast Fourier Transform algorithm (FFT). For the limit of $T \to \infty$, $c_n T$ is the continuous Fourier transform. Therefore a good approximation of the continuous Fourier transform can be obtained by taking a long period T, a short time spacing Δt, and performing the discrete fourier transform $fft(u)$ in Matlab, then multiplying by Δt.

Exercise 2

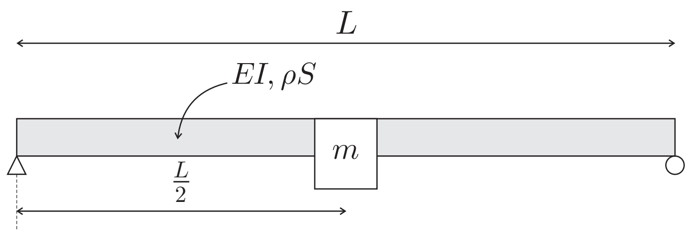
Consider a bridge crane consisting of a simply supported beam to which the lifting system is attached, as shown in Figure 1(a). The system can be modeled as a simply supported beam to which a mass m (equal to the total mass of the lifting system) is attached, as shown in Figure 2(a).

(a) Bridge crane

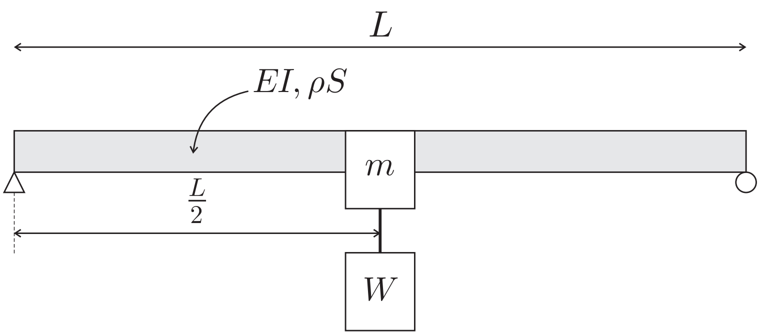


(b) 1DOF model.

Figure 1: Bridge crane and equivalent SDOF system



(a) Model of the bridge crane



(b) Model of the bridge crane with an additional mass W

Figure 2: Bridge crane model with and without additional mass

The beam is a steel IPE 750x196 (see tables) of length $L = 10m$ ($E = 210GPa$, $\rho = 7800kg/m^3$), and the mass m is 5 Tons. Assuming that the mass of the beam is small with respect to the mass m, which is located at position $x = L/2$:

1. Determine the stiffness k, and the mass M of the equivalent 1 DOF system represented in Figure 1(b) (use the tables provided). Use the energy method in order to determine the part of the equivalent mass due to the mass of the beam. Give the expression of the eigenfrequency of the equivalent 1 DOF system.

   Hint for the equivalent mass due to the mass of the beam:

   - Assume that the displacement profile of the beam is proportional to the static displacement of the beam due to a point force in the middle of the beam
   - compute the associated kinetic energy and identify the equivalent mass by comparing with the kinetic energy of the 1 DOF system

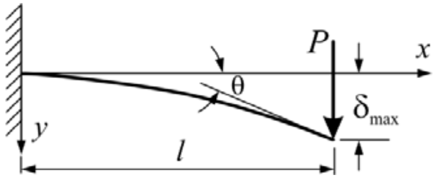
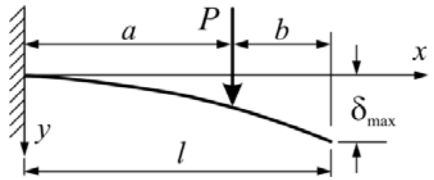
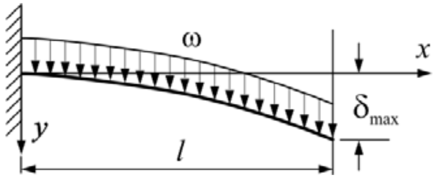
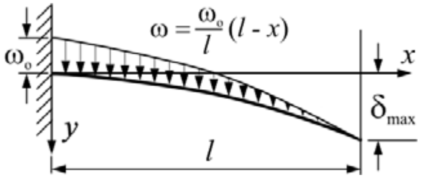
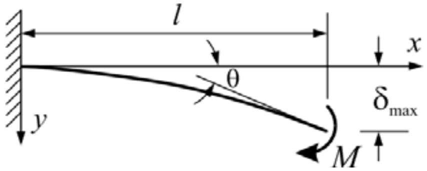
2. Plot the shape of the frequency response function of the equivalent 1 DOF system assuming a damping value of $\xi = 0.01$. What is the amplification factor with respect to the static response ? How many eigenfrequencies are present? How many eigenfrequencies would be present if you were to plot the frequency response function of the real system?

3. Assume that an additional weight of mass $W = 1000kg$ is attached to the lifting system (Figure 2(b)). At time $t = 0$, The rope attaching this additional mass is cut. Plot the shape of the evolution of the displacement in the vertical direction at position $x = L/2$ as a function of time (assume a damping value of $\xi = 0.01$ and use the equivalent 1 DOF representation). Indicate the value of the period of oscillation. How many oscillations are needed to reduce the amplitude of vibration by approximately one half ?

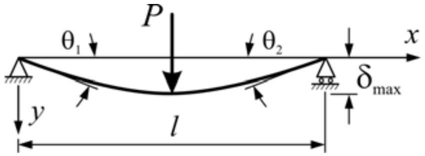
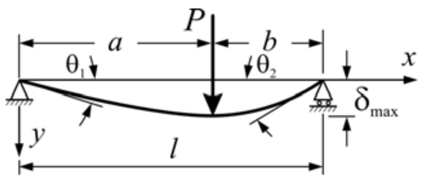
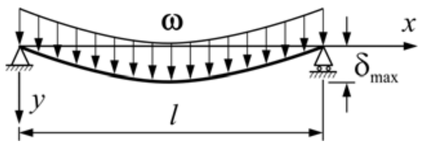
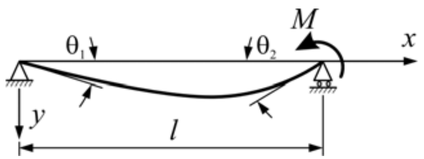
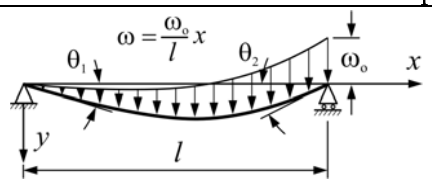
EUROPEAN I BEAMS: Dimensions and Properties (IPE)

Designation	G	h	b	t_w	t_f	r	A	h_i	d	I_y	I_z	i_y	i_z	W_y	W_z	W_{ply}	W_{plz}	I_w	I_T
	kg/m	mm	mm	mm	mm	mm	cm^2	mm	mm	cm^4	cm^4	cm	cm	cm^3	cm^3	cm^3	cm^3	dm^6	cm^4
IPE 500 A	79.4	497	200	8.4	14.5	21	101	468.0	426.0	42930	1939	20.6	4.38	1728	194	1946	302	1.13	64.3
IPE 500	90.7	500	200	10.2	16.0	21	116	468.0	426.0	48200	2142	20.4	4.31	1928	214	2194	336	1.25	89.1
IPE 500 O	107	506	202	12.0	19.0	21	137	468.0	426.0	57780	2622	20.6	4.38	2284	260	2613	409	1.55	143
IPE 500 R	111	508	198	12.6	20.0	21	142	468.0	426.0	59930	2600	20.5	4.28	2360	263	2709	415	1.55	162
IPE 500 V	129	514	204	14.2	23.0	21	164	468.0	426.0	70720	3271	20.8	4.47	2752	321	3168	507	1.97	242
IPE 550 A	92.1	547	210	9.0	15.7	24	117	515.6	467.6	59980	2432	22.6	4.55	2193	232	2475	362	1.72	89.3
IPE 550	105	550	210	11.1	17.2	24	134	515.6	467.6	67120	2668	22.3	4.45	2441	254	2787	401	1.89	123
IPE 550 O	123	556	212	12.7	20.2	24	156	515.6	467.6	79160	3224	22.5	4.55	2847	304	3263	481	2.31	187
IPE 550 R	134	560	210	14.0	22.2	24	170	515.6	467.6	86600	3447	22.5	4.50	3093	328	3562	521	2.49	242
IPE 550 V	159	566	216	17.1	25.2	24	202	515.6	467.6	102300	4265	22.5	4.60	3616	395	4205	632	3.12	372
IPE 600 A	108	597	220	9.8	17.5	24	137	562.0	514.0	82920	3116	24.6	4.77	2778	283	3141	442	2.62	122
IPE 600	122	600	220	12.0	19.0	24	156	562.0	514.0	92080	3387	24.3	4.66	3069	308	3512	486	2.86	165
IPE 600 O	154	610	224	15.0	24.0	24	197	562.0	514.0	118300	4521	24.5	4.79	3879	404	4471	640	3.88	316
IPE 600 R	144	608	218	14.0	23.0	24	184	562.0	514.0	110300	3993	24.5	4.66	3629	366	4175	580	3.42	271
IPE 600 V	184	618	228	18.0	28.0	24	234	562.0	514.0	141600	5570	24.6	4.88	4582	489	5324	780	4.85	506
IPE 750 X 137	137	753	263	11.5	17.0	17	175	719.0	685.0	159900	5166	30.3	5.44	4246	393	4865	614	7.00	135
IPE 750 X 147	147	753	265	13.2	17.0	17	187	719.0	685.0	166100	5289	29.8	5.31	4411	399	5110	631	7.16	157
IPE 750 X 161	160	758	266	13.8	19.3	17	204	719.4	685.4	186100	6073	30.2	5.45	4909	457	5666	720	8.28	208
IPE 750 X 173	173	762	267	14.4	21.6	17	221	718.8	684.8	205800	6873	30.5	5.57	5402	515	6218	810	9.42	270
IPE 750 X 185	185	766	267	14.9	23.6	17	236	718.8	684.8	223000	7510	30.8	5.65	5821	563	6691	884	10.3	334
IPE 750 X 196	196	770	268	15.6	25.4	17	251	719.2	685.2	240300	8175	31.0	5.71	6241	610	7174	959	11.3	406
IPE 750 X 210	210	775	268	16.0	28.0	17	268	719.0	685.0	262200	9011	31.3	5.80	6765	672	7762	1054	12.6	512
IPE 750 X 222	222	778	269	17.0	29.5	17	283	719.0	685.0	278200	9604	31.3	5.82	7152	714	8225	1122	13.5	601

BEAM DEFLECTION FORMULAE

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}(10l^3 - 10l^2 x + 5lx^2 - x^3)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$

BEAM DEFLECTION FORMULAS

BEAM TYPE	SLOPE AT ENDS	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM AND CENTER DEFLECTION
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1 = \theta_2 = \dfrac{Pl^2}{16EI}$	$y = \dfrac{Px}{12EI}\left(\dfrac{3l^2}{4} - x^2\right)$ for $0 < x < \dfrac{l}{2}$	$\delta_{max} = \dfrac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1 = \dfrac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \dfrac{Pab(2l - b)}{6lEI}$	$y = \dfrac{Pbx}{6lEI}\left(l^2 - x^2 - b^2\right)$ for $0 < x < a$ $y = \dfrac{Pb}{6lEI}\left[\dfrac{l}{b}(x-a)^3 + \left(l^2 - b^2\right)x - x^3\right]$ for $a < x < l$	$\delta_{max} = \dfrac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{\left(l^2 - b^2\right)/3}$ $\delta = \dfrac{Pb}{48EI}\left(3l^2 - 4b^2\right)$ at the center, if $a > b$
8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)			
	$\theta_1 = \theta_2 = \dfrac{\omega l^3}{24EI}$	$y = \dfrac{\omega x}{24EI}\left(l^3 - 2lx^2 + x^3\right)$	$\delta_{max} = \dfrac{5\omega l^4}{384EI}$
9. Beam Simply Supported at Ends – Couple moment M at the right end			
	$\theta_1 = \dfrac{Ml}{6EI}$ $\theta_2 = \dfrac{Ml}{3EI}$	$y = \dfrac{Mlx}{6EI}\left(1 - \dfrac{x^2}{l^2}\right)$	$\delta_{max} = \dfrac{Ml^2}{9\sqrt{3}\,EI}$ at $x = \dfrac{l}{\sqrt{3}}$ $\delta = \dfrac{Ml^2}{16EI}$ at the center
10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta_1 = \dfrac{7\omega_o l^3}{360EI}$ $\theta_2 = \dfrac{\omega_o l^3}{45EI}$	$y = \dfrac{\omega_o x}{360lEI}\left(7l^4 - 10l^2x^2 + 3x^4\right)$	$\delta_{max} = 0.00652\dfrac{\omega_o l^4}{EI}$ at $x = 0.519\,l$ $\delta = 0.00651\dfrac{\omega_o l^4}{EI}$ at the center