Cédric Dumoulin (cedumoul@ulb.ac.be)
Arnaud Deraemaeker (aderaema@ulb.ac.be)

Session 2: MDOF systems

Exercise 1: Multiple DOFs System

Consider the following three degrees-of-freedom model:

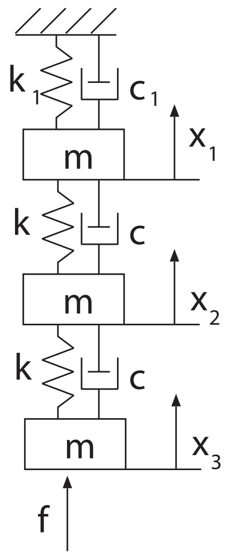

Figure 1 – 3-DOFs system

A) Write the equations of motion in the time domain in a matrix form.

B) For the numerical values $m = 1\,kg$, $k = k_1 = 16\,N/m$, $c = c_1 = 0.1\,Ns/m$, compute the eigenfrequencies and the mode shapes of the conservative system. Represent the modeshapes, and check the orthogonality conditions. .

C) Compute the mode shapes and the eigenfrequencies when $k_1 = 0$. Represent the mode shapes. Comment

D) For the value of $k_1 = k$, compute the impulse response for x_3 by projecting the equations of motion in the modal basis. Represent the Bode diagram for the same coordinate x_3, and for the acceleration $\ddot{x}_3$. Is the modal damping hypothesis valid . Multiply by a factor 5 the damping coefficient of mode 2 and plot the Bode diagram for x_3 on the same graph as with the initial value of the damping. Comment.

E) Consider the case when c_1=0. Is the modal damping hypothesis still verified ? Draw the Bode diagram for x_3 using the full system of equations (solve frequency by frequency). Compare with the modal approach in which the coupling is neglected. Comment

Note: Use the `eig` function to compute the eigenfrequencies and mode shapes.

Exercise 2: Tuned Mass Damper

Now we would like to design an optimal tuned mass damper which is aimed at damping the first vibration mode of the MDOF system. It is suggested to install the TMD on the 3-DOFs system shown in Exercise 1 (Figure 1) as illustrated in Figure 2.

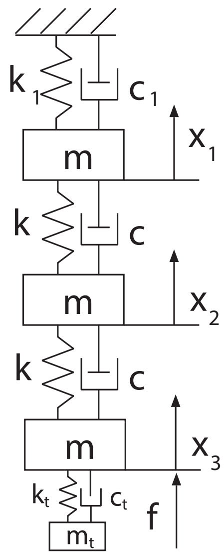

Figure 2 – 3-DOFs system equipped with a Tuned Mass Damper (TMD)

F) Write the equations of motion of the system (Figure 2) in the frequency domain.

G) For the numerical values given in B) and based on the results obtained in B), determine the optimal parameters of the TMD.
(Tips: Determine the equivalent mass and the equivalent stiffness of the multiple DOFs system corresponding to the first vibration mode).

H) Compute the direct frequency responses of the 3-DOFs system with and without the TMD. Represent and compare the Bode diagrams for both systems for the coordinate x_3.