Summary

Inventory

- Expensive, important!
- As much as 50% of invested capital!
- Balance between low (stockouts) and high (costs) inventory levels?
- Cost minimization?
- Inventory?
  - Stored resource used to satisfy a current or future need:
  - Raw materials, work-in-process, finished goods.

Importance of inventory management

1. Decoupling function
   - Buffer between manufacturing processes.
2. Storing resources
   - When demand varies over time (agriculture).
3. Irregular supply or demand
   - Seasonal demand.
4. Quantity discounts
   - Cost of products vs carrying costs.
5. Avoiding stockouts and shortages
   - Shortage costs.

Inventory decisions

- How much to order?
- When to order?

- Minimize total inventory costs:
  - Cost of the items,
  - Cost of the ordering,
  - Cost of carrying (holding) inventory,
  - Cost of safety stock,
  - Cost of stockouts.

Inventory cost factors

ORDERING COST FACTORS	CARRYING COST FACTORS
Developing and sending purchase orders	Cost of capital
Processing and inspecting incoming inventory	Taxes
Bill paying	Insurance
Inventory inquiries	Spoilage
Utilities, phone bills, and so on, for the purchasing department	Theft
Salaries and wages for purchasing department employees	Obsolescence
Supplies such as forms and paper for the purchasing department	Salaries and wages for warehouse employees Utilities and building costs for the warehouse Supplies such as forms and paper for the warehouse

First model

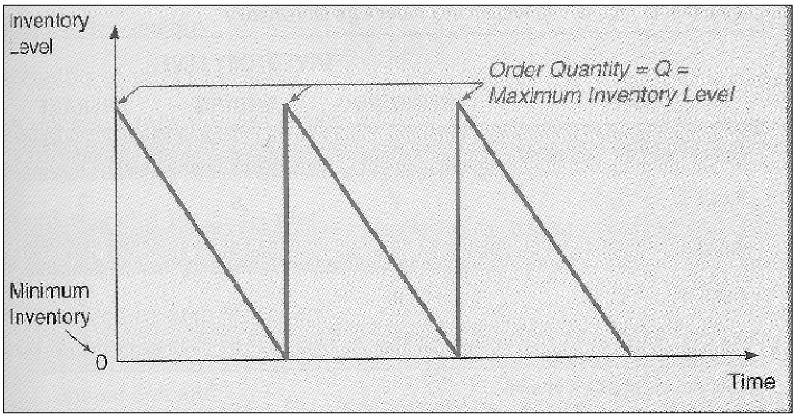
- Simple.
- For a single inventory item.
- Many assumptions:
  - Demand known and constant.
  - Lead time (time to receive an order) known and constant.
  - No quantity discounts.

EOQ model

- EOQ = economic order quantity
- Wilson model
- Assumptions:
  - Demand known and constant.
  - Lead time known and constant.
  - Receipt of inventory is instantaneous.
  - No quantity discounts.
  - Only ordering cost and carrying cost.
  - Stockouts can be avoided completely.

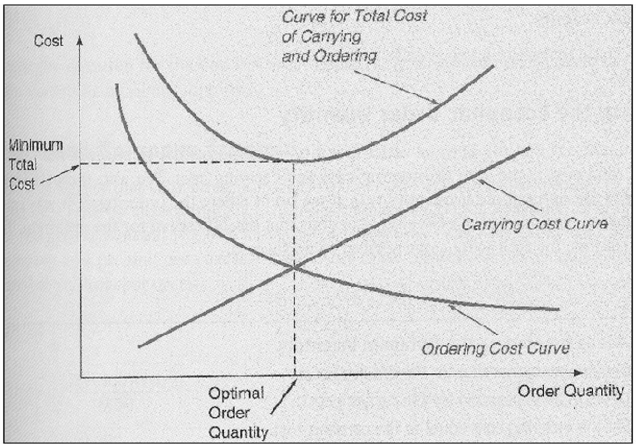
Inventory usage

- Sawtooth shape.



Inventory costs

- Objective: minimize total annual costs (ordering and carrying).



Parameters

- Q = number of pieces per order.
- Q* = EOQ = optimal number of pieces per order.
- D = annual demand for the item.
- C_o = ordering cost per order.
- C_h = carrying (holding) cost per unit per year.

Annual ordering cost

- Number of orders placed: $\frac{D}{Q}$
- Annual ordering cost: $\frac{D}{Q} \times C_o$

2008/2009 281

Annual holding cost

- Average inventory level: $\frac{Q}{2}$
- Annual holding cost: $\frac{Q}{2} \times C_h$

2008/2009 282

Total annual cost

- Ordering and holding cost:

$$TC = \frac{D}{Q} \times C_o + \frac{Q}{2} \times C_h$$

- Minimal value:

$$\frac{dTC}{dQ} = -\frac{D}{Q^2} \times C_o + \frac{1}{2} \times C_h = 0$$

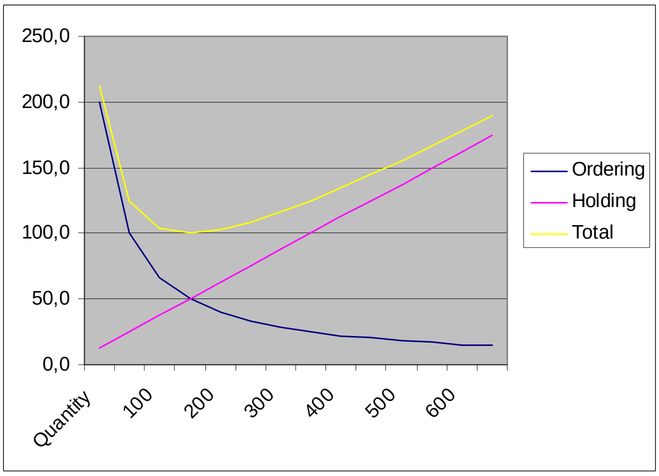
$$\frac{D}{Q^2} \times C_o = \frac{1}{2} \times C_h \quad \Rightarrow \quad Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

Example : Sunco Pump Cy

- Optimal number of pump housings to order?
- Annual demand = 1000 units.
- Ordering cost = $10 per order.
- Carrying cost = $0.50 per unit per year.
- EOQ :

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 1000 \times 10}{0.50}} = 200$$

Example : Sunco Pump Cy



Purchase cost of inventory items

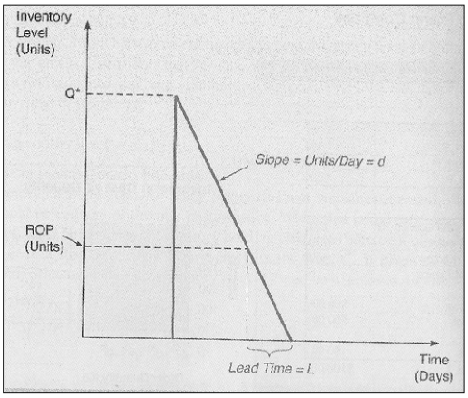
- Total inventory cost: could include the actual cost of the items purchased items:
  - Price per unit = P.
  - Total annual price is independent from Q : D×P.
- Average monetary value of the inventory: P×Q/2.
- Often, C_h = I×P where I is the annual inventory holding charge as a percent of P. Then:

$$Q^* = \sqrt{\frac{2DC_o}{IP}}$$

Reorder point (ROP)

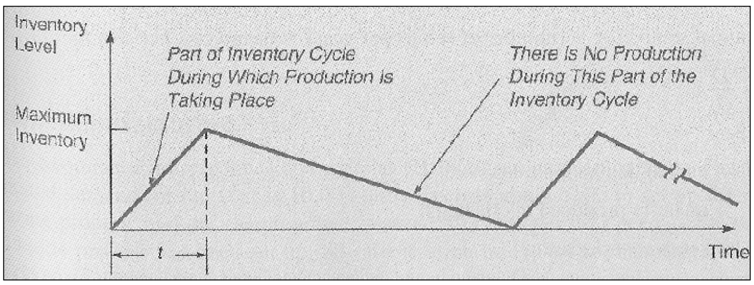
- When to reorder ?
  - Demand per day: d
  - Lead time in days: L
  - Reorder point:

$$ROP = d \times L$$



EOQ with continuous receipt

- Production run model.
- Setup cost instead of ordering cost.



Parameters

- Q = number of pieces per order (production run).
- D = annual demand for the item.
- C_s = setup cost per production run.
- C_h = carrying (holding) cost per unit per year.
- p = daily production rate.
- d = daily demand rate.
- t = length of the production run in days.

Annual setup cost

- Number of setups per year: $\frac{D}{Q}$
- Annual setup cost: $\frac{D}{Q} \times C_s$

Annual holding cost

- Maximum inventory level:

$$Q = pt \quad \Rightarrow \quad t = Q/p$$

$$\max = (p-d)\frac{Q}{p} = Q\left(1-\frac{d}{p}\right)$$

- Average inventory level: $\frac{Q}{2}\left(1-\frac{d}{p}\right)$

- Annual holding cost: $\frac{Q}{2}\left(1-\frac{d}{p}\right)\times C_h$

Total annual cost

- Setup and holding cost: $TC = \frac{D}{Q}\times C_s + \frac{Q}{2}\left(1-\frac{d}{p}\right)\times C_h$

- Minimal value:

$$\frac{dTC}{dQ} = -\frac{D}{Q^2}\times C_s + \frac{1}{2}\left(1-\frac{d}{p}\right)\times C_h = 0$$

$$\frac{D}{Q^2}\times C_s = \frac{1}{2}\left(1-\frac{d}{p}\right)\times C_h \quad \Rightarrow \quad Q^* = \sqrt{\frac{2DC_s}{C_h\left(1-\frac{d}{p}\right)}}$$

Example: Brown Mfg

- Production of refrigeration units in batches.
- Demand = 10,000 units per year.
- Setup cost = $100.
- Carrying cost = 50¢ per unit per year.
- Production rate = 80 units per day.
- Daily demand = 60 units per day (167 days per year).
- What is the optimal batch size?

Example: solution

- D = 10,000
- Cs = 100
- Ch = 0.50
- p = 80
- d = 60

$$Q^* = \sqrt{\frac{2\times 10{,}000\times 100}{0.50\left(1-\frac{60}{80}\right)}}$$

$$= \sqrt{\frac{2{,}000{,}000}{0.5\times 0.25}}$$

$$= 4{,}000$$

Quantity discount model

- Reduced cost C for items purchased in larger quantities.
- Discount schedule. Example:

Discount number	Discount quantity	Discount	Discount cost
1	0 to 999	0%	\$5.00
2	1,000 to 1,999	4%	\$4.80
3	More than 2,000	5%	\$4.75

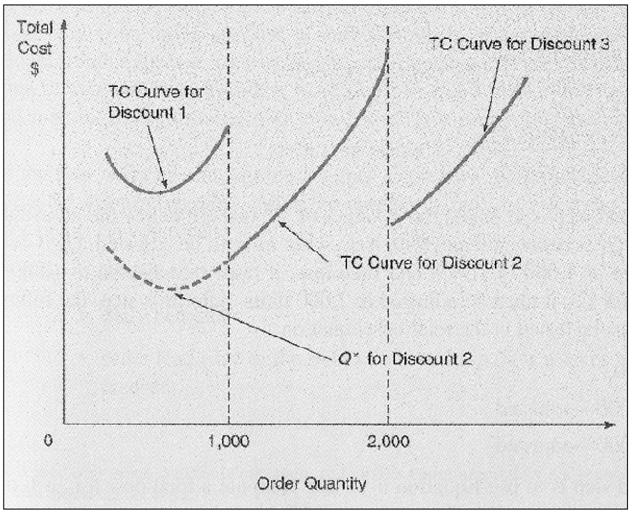
- Trade-off between reduced purchasing cost and increased carrying cost ?

Total annual inventory cost

- Assumption: $C_h = IC$
- Purchasing cost + ordering cost + carrying cost:

$$TC = DC + \frac{D}{Q} \times C_o + \frac{Q}{2} \times I \times C$$

Total cost curve



4-step procedure

1. For each discount, calculate Q*.
2. For any discount, if Q* is too low to qualify for the discount, adjust Q* upward to the lowest qualifying quantity.
3. Compute the total cost for each Q* resulting from steps 1 and 2.
4. Select the Q* that has the lowest total cost.

Example: Brass Dpt Store

- Purchase of toy race cars.
- Quantity discount schedule:

Discount number	Discount quantity	Discount	Discount cost
1	0 to 999	0%	\$5.00
2	1,000 to 1,999	4%	\$4.80
3	More than 2,000	5%	\$4.75

- Ordering cost = \$49 per order.
- Annual demand = 5000 race cars.
- Carrying charge = 20% of cost.

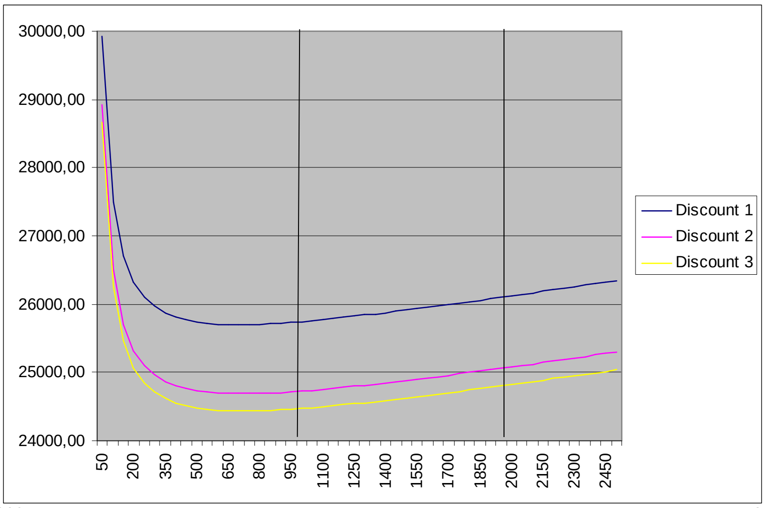
Example - steps 1 and 2

$$Q_1^* = \sqrt{\frac{2 \times 5000 \times 49}{0.2 \times 5.00}} = 700$$

$$Q_2^* = \sqrt{\frac{2 \times 5000 \times 49}{0.2 \times 4.80}} = 714 \quad \rightarrow \quad Q_2^* = 1000$$

$$Q_3^* = \sqrt{\frac{2 \times 5000 \times 49}{0.2 \times 4.75}} = 718 \quad \rightarrow \quad Q_3^* = 2000$$

Example - cost curve



Example - steps 3 and 4

- Total cost computations:

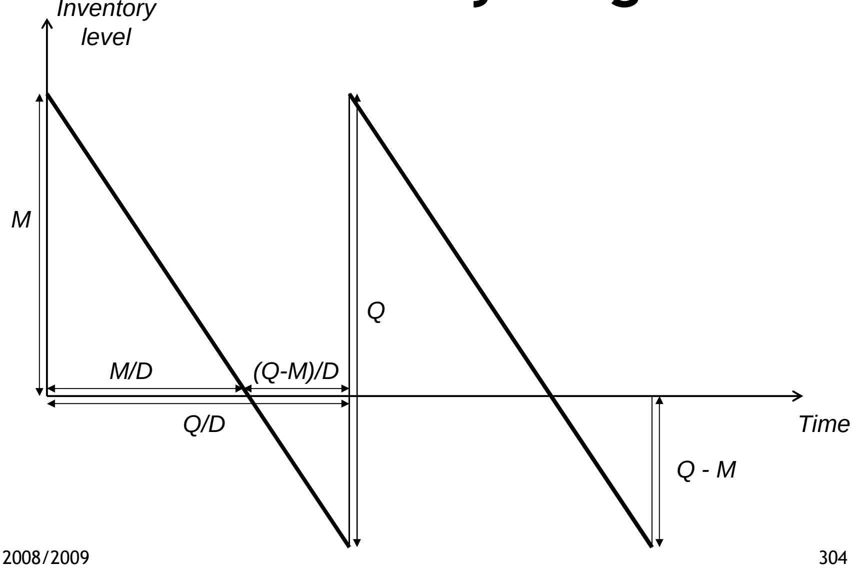
DISCOUNT NUMBER	UNIT PRICE	ORDER QUANTITY	ANNUAL MATERIAL COST ($)	ANNUAL ORDERING COST ($)	ANNUAL CARRYING COST ($)	TOTAL ($)
1	$5.00	700	25,000	350.00	350.00	25,700.00
2	4.80	1,000	24,000	245.00	480.00	24,725.00
3	4.75	2,000	23,750	122.50	950.00	24,822.50

- Optimal solution: $Q_1^* = 1000$

EOQ model with stockouts allowed

- Assumption: all demand is backlogged (no lost sales).
- Additional parameter: stockout (shortage) cost – C_s
- Order quantity: Q
- Maximum shortage: Q – M
- Q ? M ?

Inventory usage



Annual ordering cost

- Number of setups per year: $\frac{D}{Q}$
- Annual setup cost: $\frac{D}{Q} \times C_s$

2008/2009 305

Annual holding cost

- Holding cost per cycle: $\frac{M}{2} \times \frac{M}{D} \times C_h$
- Annual holding cost: $\frac{M^2 C_h}{2D} \times \frac{D}{Q} = \frac{M^2 C_h}{2Q}$

2008/2009 306

Annual shortage cost

- Shortage cost per cycle:

$$\frac{(Q-M)}{2}\times\frac{(Q-M)}{D}\times C_s$$

- Annual shortage cost:

$$\frac{(Q-M)^2 C_s}{2D}\times\frac{D}{Q}=\frac{(Q-M)^2 C_s}{2Q}$$

Total annual cost

- Ordering, holding and shortage costs:

$$TC(Q,M)=\frac{D}{Q}\times C_o+\frac{M^2}{2Q}\times C_h+\frac{(Q-M)^2}{2Q}\times C_s$$

- Minimal value: $$\frac{\partial TC}{\partial Q}=\frac{\partial TC}{\partial M}=0$$

$$Q^*=\sqrt{\frac{2C_oD(C_h+C_s)}{C_hC_s}}=EOQ\times\sqrt{\frac{C_h+C_s}{C_s}}$$

$$M^*=\sqrt{\frac{2C_oDC_s}{C_h(C_h+C_s)}}=EOQ\times\sqrt{\frac{C_s}{C_h+C_s}}$$

Safety stock

- When demand is uncertain.
- To avoid stockouts when demand is higher than expected.
- Increase ROP to reduce the probability of a stockout, according to the probability distribution of the demand during the lead time.

ABC analysis

- A company can have many inventory items.
- Items are divided into three groups:
  - Group A : items that are critical to the operation of the company (typically more than 70% of the inventory value, but 10% of all inventory items).
  - Group B : important items, but not critical (typically 20% of inventory value and 20% of inventory items).
  - Group C : not important (10% of inventory value, but 70% of inventory items).
- Monitor closely items of group A, and part of items in group B.