Experimental Verification of Rail Corrugation Theories on a Scaled Roller Rig

C. Collette\textsuperscript{1}, R. Bastaits\textsuperscript{2}, M. Horodinca\textsuperscript{1}

\textsuperscript{1}University of Brussels, Department of Mechanical Engineering and Robotics, Active Structures Laboratory
F.D. Roosevelt avenue 50, 1050 Brussels

email: christophe.collette@ulb.ac.be, renaud.bastaits@ulb.ac.be, mihaita.horodinca@ulb.ac.be

Abstract— In this work, the classical rail corrugation theory \cite{2} based on measurements is applied to a scaled roller rig from the INRETS. Possible wavelength fixing mechanisms are detected and discussed. Results are compared and validated with experiments on a scaled roller rig. Through this comparison, the authors focus their study on one specific type of rail corrugation, related to torsional vibrations of the wheel set axle (i.e. rutting corrugation). The specific combination of factors under which the phenomenon appears is established.

Keywords— rail corrugation, roller rig, friction

I. INTRODUCTION

Due to track shapes and hard schedule constraints, some light rails have always been subjected to a wavy wear of the railhead, called corrugation. The phenomenon is commonly regarded as a combination of two mechanisms \cite{1}: the first a wavelength fixing mechanism (i.e. for each vehicle, the same mode is excited) and secondly a damage mechanism (i.e. damage caused by one vehicle tends to exacerbate vibrations of subsequent vehicles).

During last twenty years, many authors have developed rail corrugations theories. All of them are based on the assumption of a linear dependency between creep force and creepage. Using this assumption, these theories were well suited for a computation in the frequency domain. Recent increases of computation capabilities allowed authors to account for non-linearities in time domain simulations. These models will not be discussed in the present paper. A brief survey of classical rail corrugation theories \cite{2} is presented in the next section, including a new vectorial tool for the representation of the creep force variations in the wheel-rail contact patch. Then, the theory will be applied to a scaled roller rig from the New Technologies Laboratory (INRETS-France). Specific conditions under which wheel-rail vibrations can be excited are established, and the corrugation mechanism is reproduced experimentally by simulating the wheel-rail contact conditions under which rail corrugation is known to be prone to develop.

II. RAIL CORRUGATION THEORIES

During last twenty years, various models have been developed to define the interaction between wheel and rail, with various objectives. More specifically, in the field of rail corrugation, the first work have been carried out by Frederick \cite{1}. These works have been generalized for longitudinal creepages by Tassily and Vincent in \cite{2} and \cite{3}. The feedback loop description of rail corrugation \cite{4} is well suited for a description in the frequency domain. To this aim, the basic assumption of the theory is the linearity. The theory is then only valid for small variations around a steady state behavior between the wheel and the rail, i.e. for small variations of the velocity between the wheel and the rail and for small lateral relative displacements between the wheel and the rail.

The linearity means that at a given frequency \( f = V/\lambda \), the output (force, rate of corrugation) is proportional to the input (rail roughness or wheel roughness). The theories described in this section establish an analytical formulation for the rate at which rail corrugation is prone to develop using a more or less complex combination of track and wheelset mechanic admittances. More recently, the shapes of the wheel and rail have been taken into account by Hempelmann \cite{5}, and a constant wavelength fixing mechanism have been proposed by Muller \cite{6} for very short pitch corrugation, that will not be considered in this work.

A. Presentation of the model

After each passage of a wheel on the rail, the profile of the rail has been slightly modified due to the wear engendered by the contact between these two bodies. Mathematically, the height of the rail profile, \( z \), can be described by a first order ordinary differential equation of the form

\[
\frac{dz}{dn} = G(f)z
\]  

(1)

where \( n \) a discrete time representing the number of passing wheel sets on the rail. In equation (1) \( G(f) \) designate the wear rate. The following developments are dedicated to introduce vehicle, track and contact mechanics to derive an analytical formulation for the wear rate.

From Archard’s law, we know that the amount of mass removed by the friction between two bodies \( \Delta m \) is proportional to the energy by unit area dissipated in the contact patch \( w \) due to the fictional work:

\[
\Delta m = C_w w
\]  

(2)

where \( C_w \) is the proportionality coefficient.

If we assume constant the time \( \Delta t \) each point of the rail stays in the contact patch, we get

\[
w = kP\Delta t = k\gamma F\Delta t
\]  

(3)

Putting together equation (2) and (3), and expressing the mass loss as the product of the profile height \( \Delta z \) and the material density \( \rho \) we get

\[
\Delta z = \frac{C_w k}{\rho} P \Delta t
\]  

(4)

Identifying (4) and (1), we get

\[
G(f) = -\text{real}(P)\frac{C_w k}{\rho}
\]  

(5)
The analytical formulation for the frictional power dissipated in the contact patch \( P \) is obtained by including the structural dynamics of the system.

**B. Structural dynamics**

**B.1 Dynamics of the rail**

The frequency responses of the rail can be expressed using the general formulation

\[
\Delta u_x^r = R_{xx}^r \Delta N + R_{xy}^r \Delta F_y
\]

(6)

\[
\Delta u_y^r = +R_{yx}^r \Delta N + R_{yy}^r \Delta F_y
\]

(7)

\[
\Delta u_z^r = R_{oz}^r \Delta F_y
\]

(8)

\[
\Delta u_y^r = +R_{xy}^r \Delta M
\]

(9)

with \( x \) \( y \) \( z \) \( \phi \) being respectively the longitudinal, lateral, vertical and spin (rotation about \( z \) axis) directions.

**B.2 Dynamics of the wheel**

The same symbolic formulations can be used to describe the dynamic behavior of the wheel:

\[
\Delta u_x^w = +R_{xx}^w \Delta N + R_{xy}^w \Delta F_y
\]

(10)

\[
\Delta u_y^w = +R_{yx}^w \Delta N + R_{yy}^w \Delta F_y
\]

(11)

\[
\Delta u_z^w = +R_{oz}^w \Delta F_y
\]

(12)

\[
\Delta u_y^w = +R_{xy}^w \Delta M
\]

(13)

For convenience, we write the equations from the two last sections in a matrix form

\[
\Delta \mathbf{u} = \mathbf{R} \Delta \mathbf{F}
\]

(14)

where \( \mathbf{u} \) is

\[
\mathbf{u} = (u_x, u_y, u_\phi) = (u_x^w + u_x^r, u_y^w + u_y^r, u_\phi^w + u_\phi^r)
\]

(15)

**C. Kinematics**

**C.1 Wheel-rail creepages**

The longitudinal, lateral and spin creepages are given by

\[
\Delta \gamma_x = \frac{\Delta \dot{x}_w - \Delta \dot{x}_r}{V}
\]

(16)

\[
\Delta \gamma_y = \frac{\Delta \dot{y}_w - \Delta \dot{y}_r}{V}
\]

(17)

\[
\Delta \gamma_\phi = \frac{\Delta \dot{\phi}_w - \Delta \dot{\phi}_r}{V}
\]

(18)

where \( V \) is the speed of the wheel forward motion.

**C.2 Dynamics of the wheel**

\[u\]

where \( u \) is

\[
\mathbf{u} = (u_x, u_y, u_\phi) = (u_x^w + u_x^r, u_y^w + u_y^r, u_\phi^w + u_\phi^r)
\]

(15)

**D. Contact forces**

**D.1 Normal force**

The increment of the elastic compression between the wheel and the rail \( \Delta d \) is related to the increments of the vertical displacements of the wheel and the rail \( \Delta u_z \) and the change in rail profile \( \Delta z \) by:

\[
\Delta d + \Delta u_z = \Delta z
\]

(19)

Equation (19) can be rewritten

\[
\frac{\Delta N}{k_c} + \Delta u_z = \Delta z
\]

(20)

with \( k_c \) denoting the Hertzian contact stiffness.

**D.2 Lateral forces**

For small creepages between the wheel and the rail, the relations between creepages and creep forces are given by the linear theory of Kalker [7]:

\[
\begin{bmatrix}
F_x \\
F_y \\
M
\end{bmatrix} = Gc^2 \begin{bmatrix}
C_{11} & 0 & 0 \\
0 & C_{22} & cC_{23} \\
0 & cC_{23} & c^2C_{33}
\end{bmatrix} \begin{bmatrix}
\gamma_x \\
\gamma_y \\
\gamma_\phi
\end{bmatrix}
\]

(21)

where \( C_{11}, C_{22}, C_{23}, C_{33} \) are the creep coefficients, \( G \) the shear modulus and \( c^2 \) the product of semi-axes of the elliptical contact patch.

For larger creepages, the equation (21) becomes non-linear. The creepage-creep-force law reaches a maximum when wheel is sliding on the rail. This non-linearity becomes important in case of high creepages [8]. Many formulations exist in the literature to express this non-linearity [9]. For instance, Tassily and Vincent [2] adopted a continuous exponential law of the form:

\[
F_x = \mu N [1 - \exp \left( -\frac{C_0 \gamma_x}{\mu N^2} \right)]
\]

(22)

Variations of \( F_x \) about a stationary component can then be expressed as follow:

\[
\Delta F_x = \frac{\partial F_x}{\partial N} \Delta N + \frac{\partial F_x}{\partial \gamma_x} \Delta \gamma_x
\]

(23)

\[
\begin{bmatrix}
F_x \\
F_y \\
M
\end{bmatrix} = \begin{bmatrix}
\frac{F_x}{N} + \left( \frac{F_x - \mu N}{N^2} \right) \frac{C_0 \gamma_x}{3\mu} \\
\frac{F_y}{N} + \left( \frac{F_y - \mu N}{N^2} \right) \frac{C_0 \gamma_y}{3\mu} \\
\frac{M}{N} + \left( \frac{M - \mu F_y}{\mu N^2} \right) C_0
\end{bmatrix} \begin{bmatrix}
\Delta \gamma_x \\
\Delta \gamma_y \\
\Delta \gamma_\phi
\end{bmatrix}
\]

(24)

Similarly for the lateral component of the friction force:

\[
F_y = \mu N [1 - \exp \left( -\frac{C_0 \gamma_y}{\mu N^2} \right)]
\]

(25)

\[
\Delta F_y = \frac{\partial F_y}{\partial N} \Delta N + \frac{\partial F_y}{\partial \gamma_y} \Delta \gamma_y
\]

(26)

\[
\begin{bmatrix}
F_x \\
F_y \\
M
\end{bmatrix} = \begin{bmatrix}
\frac{F_x}{N} + \left( \frac{F_x - \mu N}{N^2} \right) \frac{C_0 \gamma_y}{3\mu} \\
\frac{F_y}{N} + \left( \frac{F_y - \mu N}{N^2} \right) \frac{C_0 \gamma_y}{3\mu} \\
\frac{M}{N} + \left( \frac{M - \mu F_y}{\mu N^2} \right) C_0
\end{bmatrix} \begin{bmatrix}
\Delta \gamma_x \\
\Delta \gamma_y \\
\Delta \gamma_\phi
\end{bmatrix}
\]

(27)

\footnote{As explained in the introduction of this section, the additional dependency of \( N \) with the contact geometry is neglected in this work.}
E. Wear rates calculation

Combining equations from sections B, C and D, we can derive analytical formulations of the transfer functions between vertical roughness and normal, lateral and longitudinal forces. When multiplied by a factor taking the material properties into account, these expressions are then identified to the corrugation growth rates using formula (5). These rates can then be formally written as follow:

\[
G_x = \frac{\Delta F_x}{\Delta z}; \quad G_y = \frac{\Delta F_y}{\Delta z}; \quad G_\phi = \frac{\Delta M}{\Delta z} \quad (28)
\]

The extended expressions are detailed in [3] and given in appendix of this paper for simple cases. Using this theory, it is possible to evaluate the frequencies at which rail corrugation is prone to develop due to longitudinal or lateral frictional power dissipation. However, in the calculation of the time evolution of the profile height in equation (1), all these contributions are the terms of a scalar addition in the calculation of the total frictional power dissipated in the contact. Using this formulation, only wavelength fixing mechanisms can be considered in the theory, because the same multiplicative coefficient is used for the longitudinal and lateral components of the frictional force [10], that is not physical from a tribology point of view.

For this reason, an other formulation, based on a vectorial approach, is proposed to maintain the angular information of the frictional power at each step. The basic assumption of this approach is to define the frictional power as the matrix product of the force and the creepage, in stead of the conventional scalar product. Here below are detailed the components of the creep force, creepage and frictional power:

\[
F = (F_x, F_y, M)^T
\]

(29)

\[
\Gamma = (\gamma_x, \gamma_y, \gamma_\phi)^T
\]

(30)

\[
P = \Gamma F = V(\gamma_x F_x, \gamma_y F_y, \gamma_\phi F_\phi)^T
\]

(31)

In order to express the profile evolutions using a compact matrix formulation, we introduce a vector \( z \)

\[
z = (z_x, z_y, z_\phi)
\]

(32)

However, in this case, all components are in the same direction \( z \) and their scalar sum gives the height of the rail profile. Consequently, the right hand part of the equation (20) is expanded to

\[
\Delta z = \Delta z_x + \Delta z_y + \Delta z_\phi
\]

(33)

and \( \Delta z_i \) is defined as the change in profile height due to the friction between the wheel on the rail in the \( i^{th} \) direction and \( i = x, y, \phi \).

Using the vectorial formulation described here above, variations of the frictional power are

\[
\Delta P = \frac{\partial P}{\partial \Gamma} \Delta \Gamma + \frac{\partial P}{\partial F} \Delta F = \begin{bmatrix}
- \frac{\rho}{R_{xx} + \gamma_x} F_x \\
- \frac{\rho}{R_{yy} + \gamma_y} F_y \\
- \frac{\rho}{R_{\phi\phi} + \gamma_\phi} F_\phi
\end{bmatrix}
\]

(34)

Again, using equations from section B, C and D, the general equation for the characterization of the evolution of the profile height (1) splits into three equations:

\[
\begin{bmatrix}
\frac{\partial z_x}{\partial n} \\
\frac{\partial z_y}{\partial n} \\
\frac{\partial z_\phi}{\partial n}
\end{bmatrix} = C \frac{k}{\rho} \begin{bmatrix}
\alpha_1 & 0 & 0 \\
0 & \alpha_2 & 0 \\
0 & 0 & \alpha_3
\end{bmatrix} \Delta P
\]

(35)

Or, using compact matrix notations:

\[
\frac{\partial z}{\partial n} = \mathbf{G} \Delta z
\]

(36)

These rates can be calculated either in the frequency domain, or in the time domain. In the latter case, the evaluation is no more restricted to small variations of the creep forces and the creepages.

In the frequency domain, in the neighborhood of natural frequencies of the wheel set, even a very small roughness amplitude can generate high creep and force variations. In this case, the hypothesis of infinitesimal variations around a mean value is no longer valid. Then, rather than giving precise values of the wear rate frequencies, the wear transfer functions should be regarded as indicating the critical frequency bands where corrugation is prone to develop.

III. EXPERIMENT

A. Description of the test rig

The experimental test bench is constituted by a single 1/4 scaled wheelset, rolling on a roller, having 1/4 scaled wheel and rail profiles respectively (see Fig. 1 a picture of the bench). The wheelset is supported by a mechanical frame allowing the following parameters to be controlled: vertical force applied on the wheelset (load), angle of attack between the axle and the rails (lateral creepage), relative longitudinal speed between wheels and rails, longitudinal and lateral creep forces in the contact. Wheelset lateral position with respect to the track, yaw angle and rolling speed are the controllable parameters of the system.

![Fig. 1. Picture of the 1/4 scaled test bench.](image)

B. Similarity laws

The similarity theory used for the construction of the bench is based on a conservation of stress field in the contact patch between the wheel and the rail, to maintain the
validity of contact laws established for the calculation of creep force of full scale system. This implies to keep the same material characteristics, and the application of an additional vertical force on the wheelset to compensate the unscaleable gravitational force. All geometrical quantities are scaled. As a consequence, ratios of the main quantities involved in the test rig are listed below:

- Length \([L]\): ratio 1/4;
- Area \([L^2]\): ratio 1/16;
- Volume \([L^3]\): ratio 1/64;
- Mass \([M]\): ratio 1/64;
- Time \([T]\): ratio 1/4;
- Speed \([L T^{-1}]\): ratio 1 (unchanged);
- Frequency \([T^{-1}]\): ratio 4;
- Force \([M L T^{-2}]\): ratio 1/16;
- Acoustic pressure \([M L^{-1} T^{-2}]\): ratio 1 (unchanged).

The main practical consequences of these similarity ratios are the following:

- The modal frequencies of the wheels are multiplied by 4: as pure tones up to 10 kHz can be found at full scale, measurements should be done up to 40 kHz on the test rig.
- The mass of the wheelset under test is divided by 64 compared with the full scale.
- Pressure and speed are not modified by similarity, whether it is the rolling speed or the vibration velocity of the wheelset.

On the other hand, the scale ratio is not perfectly ensured in the local conditions of the contact, which affect the size of the contact patch and the friction law. The size of the contact patch (surface of contact between the rail and the wheel) can be estimated from Hertz theory. Calculation of the contact ellipse dimensions on the test rig gives ratios between 1/3.7 and 1/3.8 compared with full scale instead 1/4 (Hertz theory is nonlinear).

### C. Application of the theory

In this application, we will restrict ourself to an analysis in the longitudinal direction. The growth rates are given explicitly by formulas (37) and (38). The input for the calculation are the mean longitudinal creepage, the linear speed of the wheelset, the vertical hertzian stiffness of the contact between the wheel and the rail, and the vertical and longitudinal wheel and rail dynamic receptances, shown on Fig. 3 and Fig. 2.

The longitudinal creepage given by equation (16) is a non-dimensional quantity that characterizes the sliding velocity between the wheel and the rail, taking into account the material deformations of the bodies in contact. In other words, for small creepage values, the wheel-rail contact behaves like a damper in horizontal directions; for larger values, the interaction is a classical dry friction coulomb force. The larger the friction force and creepage variations, the bigger is the the amount of energy dissipated in the contact patch and corrugation. In the longitudinal direction, large values of the creepage are obtained by the application of differential torques on the wheelset and the roller to impose sliding conditions. Parameters are listed here below:

- \(V\): 5 m/s
- \(T_w\): 3.45 Nm

![Fig. 2. Wheelset receptances in lateral, longitudinal and vertical directions.](image)

![Fig. 3. Rail receptances in lateral, longitudinal and vertical directions.](image)

![Fig. 4. Growth rate obtained in the longitudinal direction.](image)

- \(T_r\): 5.6 Nm
- \(\gamma_z\): 0.08

Such conditions represent a vehicle that brakes and runs on a curved track section.

Using the data set described above and mechanical receptances of the bench, the theory predicts the growth with equation (38) and is depicted on Fig. 4.
As we can see from Fig. 4, the friction between the wheel and the rail will tend to give rise to corrugation at a frequency corresponding to the first torsional mode of the wheelset. The phenomenon is known as rail rutting corrugation. This prediction is verified in the next section by performing an experiment in the time domain.

**D. Test case**

The key condition for the application of the theory described in section II is a friction curve with an increasing slope for small creepages (viscous damping), and a saturation for larger values of the creepage (dry friction). This behavior has been confirmed by a first preliminary experiment. The experiment consisted in applying a torque on the wheelset when it is rolling on the roller, in order to force the slip. The experiment has been performed in various conditions, and the typical resulting friction curve is shown on Fig. 5.

The time evolution of the frictional power dissipated in the contact patch is represented on Fig. 8 in the creep-force plane. This filtered signal shows elliptic loops, each of them representing a variation of the frictional power. The area of the ellipse represents the amplitude of the variations and the periodicity corresponds to the frequency of the first torsional mode of the wheelset.

**IV. Conclusion**

In the first part of the paper, the authors have shown that the classical theory of rail corrugation can be advantageously rewritten using a vectorial approach for the representation of the frictional power dissipated in the contact. That way, it enables to introduce different wear coefficients in the longitudinal and lateral directions, and consequently offers the possibility to consider other damage mechanisms than just wear.

In the second part of the paper, a validation of the rail corrugation theory on a scaled test bench has been presented by comparing output corrugation growth rates from the theory and time history of the frictional power measured from an experimental test case.
Acknowledgments

The authors would like to acknowledge the European Community for supporting and funding this work under the fifth framework program (project Wheel-rail corrugation in urban transport - GRD2-2001-5006)

Appendix

I. Corrugation growth rates

If we neglect the coupling between vertical and lateral dynamics, the growth rates are approximated in linear and saturated regimes by the following expressions.

In linear regime:

$$G_\alpha(\omega) = \frac{2}{3} \left( \frac{1}{k_c} \right) \left[ f^2 (a^2 + b^2) - V^2 \right] + 2V fbd $$

(37)

In saturated regime:

$$G_\alpha(\omega) = \frac{c + \frac{1}{k_c} (a - \frac{V \gamma_\alpha}{\mu N})}{V (c + \frac{1}{k_c})^2 + d^2} + bd $$

(38)

where we call

$$a + ib = i \omega (R_{yy}^r + R_{yy}^w) $$

(39)

$$c + id = R_{zz}^r + R_{zz}^w $$

(40)

and $V$ the mean vehicle speed, $k_c$ the vertical stiffness of the spring, $f = \frac{F_\alpha}{\gamma_\alpha}$, and $\alpha = x, y$.

References


