# THE SUBGROUP LATTICE OF $L_2(64) = PGL(2, 64)$

THOMAS CONNOR AND DIMITRI LEEMANS

| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
|---|---|---|---|---|---|
| 1 | $L_2(64)$ | 262080 | 1 | 2 (65), 5 (520), 8 (2016), 9 (2080), 13 (4368) | |
| 2 | $2^6 : 7 : 3 \cdot 3$ | 4032 | 65 | 3, 4, 12 (64) | 1 |
| 3 | $2^6 : 7 : 3$ | 1344 | 65 | 6, 7, 19 (64) | 2 |
| 4 | $2^6 : 3 \cdot 3$ | 576 | 65 | 7, 36 (64) | 2 |
| 5 | $L_2(8)$ | 504 | 520 | 14 (9), 20 (28), 32 (36) | 1 |
| 6 | $2^6 : 7$ | 448 | 65 | 11, 14 (72) | 3 |
| 7 | $2^6 : 3$ | 192 | 65 | 11, 15 (84) | 3, 4 |
| 8 | $D_{130}$ | 130 | 2016 | 10, 18 (5), 35 (13) | 1 |
| 9 | $D_{126}$ | 126 | 2080 | 12, 16 (3), 20 (7) | 1 |
| 10 | 65 | 65 | 2016 | 33, 62 | 8 |
| 11 | $2^6$ | 64 | 65 | 17 (63) | 6, 7 |
| 12 | 63 | 63 | 2080 | 19, 36 | 2 (2), 9 |
| 13 | $A_5$ | 60 | 4368 | 34 (5), 35 (6), 61 (10) | 1 |
| 14 | $2^3 : 7$ | 56 | 4680 | 37, 60 (8) | 5, 6 |
| 15 | $2^4 : 3$ | 48 | 5460 | 21, 34 (20) | 7 |
| 16 | $D_{42}$ | 42 | 6240 | 19, 32 (3), 61 (7) | 9 |
| 17 | $2^5$ | 32 | 4095 | 22 (3), 23 (3), 24 (3), 25 (3), 26 (3), 27 (3), 28 (3), 21, 29 (3), 30 (3), 31 (3) | 11 |
| 18 | $D_{26}$ | 26 | 10080 | 33, 75 (13) | 8 |
| 19 | 21 | 21 | 2080 | 60, 74 | 3 (2), 12, 16 (3) |
| 20 | $D_{18}$ | 18 | 14560 | 36, 61 (3) | 5, 9 |
| 21 | $2^4$ | 16 | 1365 | 38 (3), 39 (3), 40 (3), 41 (3), 42 (3) | 15 (4), 17 (3) |
| 22 | $2^4$ | 16 | 4095 | 41, 43, 44, 45 (3), 46, 39, 47, 48, 49, 50, 51, 38, 52 | 17 (3) |
| 23 | $2^4$ | 16 | 4095 | 53, 43, 54, 55 (3), 39, 40, 56, 57, 50, 38, 46, 45, 51 | 17 (3) |

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*Date*: June 7, 2013.

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
|---|---|---|---|---|---|
| 24 | $2^4$ | 16 | 4095 | 56, 46, 44, 42, 37, 55, 39, 49, 53, 41, 58, 52, 48, 43, 51 | 17 (3) |
| 25 | $2^4$ | 16 | 4095 | 59, 41, 42, 58, 46, 56, 53, 57 (3), 44, 43, 48, 45, 38 | 17 (3) |
| 26 | $2^4$ | 16 | 4095 | 38, 48, 53, 49, 55, 39, 47 (3), 44, 56, 57, 52, 54, 42 | 17 (3) |
| 27 | $2^4$ | 16 | 4095 | 58, 44, 50 (3), 40, 56, 46, 42, 48, 41, 43, 54, 47, 53 | 17 (3) |
| 28 | $2^4$ | 16 | 4095 | 53, 59 (3), 50, 52, 55, 56, 44, 49, 40, 45, 42, 48, 39 | 17 (3) |
| 29 | $2^4$ | 16 | 4095 | 58 (3), 46, 54, 49, 55, 47, 43, 38, 59, 42, 45, 40, 52 | 17 (3) |
| 30 | $2^4$ | 16 | 4095 | 41, 53, 56, 51 (3), 59, 38, 49, 58, 57, 50, 47, 52, 40 | 17 (3) |
| 31 | $2^4$ | 16 | 4095 | 43, 59, 44, 54 (3), 48, 41, 39, 46, 51, 57, 49, 52, 40 | 17 (3) |
| 32 | $D_{14}$ | 14 | 18720 | 60, 75 (7) | 5, 16 |
| 33 | 13 | 13 | 2016 | 76 | 10, 18 (5) |
| 34 | $A_4$ | 12 | 21840 | 63, 74 (4) | 13, 15 (5) |
| 35 | $D_{10}$ | 10 | 26208 | 62, 75 (5) | 8, 13 |
| 36 | 9 | 9 | 2080 | 74 | 4 (2), 12, 20 (7) |
| 37 | $2^3$ | 8 | 585 | 64 (7) | 14 (8), 24 (7) |
| 38 | $2^3$ | 8 | 4095 | 65, 66, 67, 63, 68, 69, 70 | 21, 22, 23, 25, 26, 29, 30 |
| 39 | $2^3$ | 8 | 4095 | 71, 66, 63, 67, 70, 64, 72 | 21, 22, 23, 24, 26, 28, 31 |
| 40 | $2^3$ | 8 | 4095 | 63, 71, 67, 65, 73, 68, 72 | 21, 23, 27, 28, 29, 30, 31 |
| 41 | $2^3$ | 8 | 4095 | 65, 63, 71, 66, 73, 64, 69 | 21, 22, 24, 25, 27, 30, 31 |
| 42 | $2^3$ | 8 | 4095 | 68, 70, 73, 64, 72, 69, 63 | 21, 24, 25, 26, 27, 28, 29 |
| 43 | $2^3$ | 8 | 4095 | 64, 69, 73, 68, 67, 66, 71 | 22, 23, 24, 25, 27, 29, 31 |
| 44 | $2^3$ | 8 | 4095 | 70, 66, 73, 72, 64, 69, 71 | 22, 24, 25, 26, 27, 28, 31 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
|---|---|---|---|---|---|
| 45 | $2^3$ | 8 | 4095 | 72, 66 (3), 67, 69, 68 | 22 (3), 23, 25, 28, 29 |
| 46 | $2^3$ | 8 | 4095 | 71, 67, 73, 69, 64, 66, 68 | 22, 23, 24, 25, 27, 29, 31 |
| 47 | $2^3$ | 8 | 4095 | 70 (3), 66, 73, 68, 65 | 22, 26 (3), 27, 29, 30 |
| 48 | $2^3$ | 8 | 4095 | 72, 66, 73, 69, 71, 70, 64 | 22, 24, 25, 26, 27, 28, 31 |
| 49 | $2^3$ | 8 | 4095 | 71, 64, 70, 72, 65, 68, 66 | 22, 24, 26, 28, 29, 30, 31 |
| 50 | $2^3$ | 8 | 4095 | 73 (3), 66, 65, 67, 72 | 22, 23, 27 (3), 28, 30 |
| 51 | $2^3$ | 8 | 4095 | 65 (3), 67, 64, 71, 66 | 22, 23, 24, 30 (3), 31 |
| 52 | $2^3$ | 8 | 4095 | 64, 71, 68, 70, 66, 72, 65 | 22, 24, 26, 28, 29, 30, 31 |
| 53 | $2^3$ | 8 | 4095 | 67, 69, 73, 70, 64, 72, 65 | 23, 24, 25, 26, 27, 28, 30 |
| 54 | $2^3$ | 8 | 4095 | 70, 71 (3), 73, 67, 68 | 23, 26, 27, 29, 31 (3) |
| 55 | $2^3$ | 8 | 4095 | 72, 67 (3), 70, 68, 64 | 23 (3), 24, 26, 28, 29 |
| 56 | $2^3$ | 8 | 4095 | 67, 64, 70, 72, 73, 69, 65 | 23, 24, 25, 26, 27, 28, 30 |
| 57 | $2^3$ | 8 | 4095 | 69 (3), 70, 67, 71, 65 | 23, 25 (3), 26, 30, 31 |
| 58 | $2^3$ | 8 | 4095 | 73, 68 (3), 64, 69, 65 | 24, 25, 27, 29 (3), 30 |
| 59 | $2^3$ | 8 | 4095 | 71, 69, 72 (3), 68, 65 | 25, 28 (3), 29, 30, 31 |
| 60 | 7 | 7 | 2080 | 76 | 14 (18), 19, 32 (9) |
| 61 | $S_3$ | 6 | 43680 | 74, 75 (3) | 13, 16, 20 |
| 62 | 5 | 5 | 2016 | 76 | 10, 35 (13) |
| 63 | $2^2$ | 4 | 1365 | 75 (3) | 34 (16), 38 (3), 39 (3), 40 (3), 41 (3), 42 (3) |
| 64 | $2^2$ | 4 | 4095 | 75 (3) | 37, 39, 41, 42, 43, 44, 46, 48, 49, 51, 52, 53, 55, 56, 58 |
| 65 | $2^2$ | 4 | 4095 | 75 (3) | 38, 40, 41, 47, 49, 50, 51 (3), 52, 53, 56, 57, 58, 59 |
| 66 | $2^2$ | 4 | 4095 | 75 (3) | 38, 39, 41, 43, 44, 45 (3), 46, 47, 48, 49, 50, 51, 52 |
| 67 | $2^2$ | 4 | 4095 | 75 (3) | 38, 39, 40, 43, 45, 46, 50, 51, 53, 54, 55 (3), 56, 57 |
| 68 | $2^2$ | 4 | 4095 | 75 (3) | 38, 40, 42, 43, 45, 46, 47, 49, 52, 54, 55, 58 (3), 59 |

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| Nr. | Structure | Order | Length | Maximal Subgroups | Minimal Overgroups |
|---|---|---|---|---|---|
| 69 | $2^2$ | 4 | 4095 | 75 (3) | 38, 41, 42, 43, 44, 45, 46, 48, 53, 56, 57 (3), 58, 59 |
| 70 | $2^2$ | 4 | 4095 | 75 (3) | 38, 39, 42, 44, 47 (3), 48, 49, 52, 53, 54, 55, 56, 57 |
| 71 | $2^2$ | 4 | 4095 | 75 (3) | 39, 40, 41, 43, 44, 46, 48, 49, 51, 52, 54 (3), 57, 59 |
| 72 | $2^2$ | 4 | 4095 | 75 (3) | 39, 40, 42, 44, 45, 48, 49, 50, 52, 53, 55, 56, 59 (3) |
| 73 | $2^2$ | 4 | 4095 | 75 (3) | 40, 41, 42, 43, 44, 46, 47, 48, 50 (3), 53, 54, 56, 58 |
| 74 | 3 | 3 | 2080 | 76 | 19, 34 (42), 36, 61 (21) |
| 75 | 2 | 2 | 4095 | 76 | 18 (32), 32 (32), 35 (32), 61 (32), 63, 64 (3), 65 (3), 66 (3), 67 (3), 68 (3), 69 (3), 70 (3), 71 (3), 72 (3), 73 (3) |
| 76 | 1 | 1 | 1 | | 33 (2016), 60 (2080), 62 (2016), 74 (2080), 75 (4095) |

TABLE 1. Subgroup lattice of $L_2(64) = PGL(2, 64)$

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