Effect of gravity on oscillatory Marangoni convection in half-zone formed by high Pr-number liquids

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Abstract

A study of bifurcations and of onset of chaotic regime for the thermoconvective oscillatory flow in cylindrical liquid bridge is presented. Three-dimensional Navier–Stokes equations in Boussinesq approximation are solved numerically by finite volume method. Silicone oil 1 cSt at low temperature, $Pr = 18.8$, is chosen as test liquid. The simulations are done at both normal and zero gravity conditions. While onset of the oscillations in the system occurs earlier at 1-g case than in 0-g, the onset of chaos takes place earlier in 0-g case. Sequences of supercritical and subcritical bifurcations were observed on the way to chaos.

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1. Introduction

Oscillation phenomenon in floating zone melt was first reported by Chun and Wuest (1979) and by Schwabe and Scharmann (1979). A model used for studying the floating zone technology of crystal growth is called liquid bridge or half zone. Nowadays, the onset of the periodic oscillatory flow in liquid bridge is well studied and its mechanism is understood (see e.g., Wanschura et al., 1995), but the influence of gravity on the dynamics of the system, especially far above the onset of oscillations, is not examined. Performing direct numerical simulations of convection in $Pr = 35$ liquid bridge, Shevtsova et al. (2001) showed that compared to zero-gravity case, 1-g conditions change both the wave number $m$ and the critical temperature difference $\Delta T_{cr}$ at which the oscillatory regime appears.

The present study aims at analyzing the role of gravity in the development of the oscillatory flow in $Pr = 18.8$ liquid bridge from its origin and until the onset of non-periodicity and chaos. The problem in case of 1-g was thoroughly studied, see Melnikov et al. (2004), and the results successfully compared with experimental data of Ueno et al. (2003). It was obtained that on its way to chaos the system passes periodic and two-frequency quasi periodic regimes with a narrow periodic window before the dynamics become chaotic.

2. Mathematical formulation

A cylindrical fluid volume held between two differentially heated horizontal flat concentric disks of radius $R$ separated by a distance $d$ is under consideration (Fig. 1). The temperatures $T_{hot}$ and $T_{cold}$ ($T_{hot} > T_{cold}$) at solid–liquid interfaces are varied, yielding a temperature difference of $\Delta T = T_{hot} - T_{cold}$. The free surface is considered cylindrical and non-deformable. The fluid has
constant material properties except surface tension $\sigma$ and kinematic viscosity $\nu$ which are decreasing linear functions of temperature. It was shown in Shevtsova et al. (2001) that for large Prandtl number liquids (which is the case in the present study) the temperature dependence of the viscosity noticeably influences the dynamics of the system.

$$\sigma(T) = \sigma(T_{\text{cold}}) - \sigma_T(T - T_{\text{cold}}),$$

$$\nu(T) = \nu(T_{\text{cold}}) + \nu_T(T - T_{\text{cold}}),$$

where $\sigma_T = -\sigma_T = \text{const}$, $\nu_T = \sigma_T = \text{const}$. The governing equations in Boussinesq approximation for flow and heat transfer in non-dimensional primitive-variable formulation are

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla P + 2\nu \mathbf{S} \times \nabla(\Theta + z) + (1 + R_\gamma(\Theta + z))\nabla^2 \mathbf{V} + Gr(\Theta + z)e, \quad (1)$$

$$\nabla \cdot \mathbf{V} = 0, \quad (2)$$

$$\frac{\partial \Theta}{\partial t} + \mathbf{V} \cdot \nabla \Theta + V_z = \frac{1}{Pr} \cdot \nabla^2 \Theta, \quad (3)$$

where $t$ is time (scaled by $d^2/\nu$), velocity $\mathbf{V} = (V_r, V_\phi, V_z)$ is dimensionalyzed by $\nu d$, $\Theta_0 = (T - T_{\text{cold}})/\Delta T$ is the temperature and $\Theta$ is the deviation from the linear temperature profile $\Theta = \Theta_0 - z$, $\mathbf{S} = 1/2(\partial \mathbf{V}/\partial \xi_k + \partial \mathbf{V}_k/\partial \xi)$ is the strain rate tensor, $P$ is the pressure, $\nabla = (\frac{\partial}{\partial r}, \frac{1}{r} \frac{\partial}{\partial \phi}, \frac{\partial}{\partial z})$, $\Gamma = d/R$ is the aspect ratio. $e = \frac{\mathbf{g}}{g}$ is a unit vector parallel to the gravity acceleration $\mathbf{g}$.

At the rigid walls no-slip conditions are used $\mathbf{V}(z = 0) = \mathbf{V}(z = 1) = 0$ and a constant temperature is imposed $\Theta(z = 0) = \Theta(z = 1) = 0$.

On the cylindrical free surface ($r = 1, \ 0 \leq \phi \leq 2\pi, \ 0 \leq z \leq 1$)

$$V_r = 0$$

and

$$2(1 + R_\gamma(\Theta + z))\mathbf{S} \cdot e_\gamma + Re \left( e_\gamma \frac{\partial}{\partial \phi} + e_\phi \frac{1}{r} \frac{\partial}{\partial r} \right) (\Theta + z) = 0.$$
three-dimensional and oscillatory. We will not discuss the symmetry breaking bifurcation, except stating that the presence of the gravity destabilizes the flow with respect to $\tilde{g} = 0$ conditions. The onset of the oscillations occurs as a supercritical Hopf bifurcation at $DT_{cr}^{1/2} = 5$ under normal gravity conditions vs. $DT_{cr}^{1/2} = 7.56$ K when $\tilde{g} = 0$.

In both cases the oscillatory instability begins in form of standing waves with different wave numbers $m$ (see Fig. 2): $m = 2$ in absence of gravity and $m = 1$ under terrestrial conditions. The wave number is the quantity of the hot (cold) patterns represented the temperature disturbances field. In Fig. 2(a), one can see two vertical “hot” twin patterns, corresponding to azimuthal wave number $m = 2$. These patterns are straight as they are pulsating; they shall disappear in a half of oscillatory period and reappear in a period at the same positions. Unlike Fig. 2(a) there is only one pattern in Fig. 2(b), which is twisted. It is a sign of traveling wave (see Shevtsova et al., 2001).

In 1-g case, standing wave becomes traveling already at $\Delta T \approx 6.28$ K (see Melnikov et al., 2004). The present calculations show that under 0-g conditions, the pulsating regime is stable up to $\Delta T \approx 12.20$ K.

### 3.2. Quasiperiodic and chaotic regimes

For far supercritical $\Delta T$ a hysteretic dynamic behavior of the system was observed when two or more attractors coexist for the same set of parameters: periodic and quasi periodic; periodic and chaotic; or periodic, quasi periodic and chaotic. Previously, the phenomena of the bi-stability has been found for the liquid bridge system for $Pr = 4$ at the absence of gravity (see Shevtsova et al., 2003). Here we report about similar phenomena for the case of $Pr = 18.8$ fluid also in the absence of gravity. Figs. 3(a) and (b) represent two different solutions for $\Delta T = 20$ K, $Gr = 0$ resulted from the numerical integration of the governing Eqs. (1)-(3) started with different initial guesses. The temperature time series and corresponding Fourier spectra are illustrated in Fig. 3. The first solution is obtained in the following way: increasing successively $\Delta T$, e.g., choosing each time the previous solution as initial guess with $m = 2$ spatial symmetry, we found the chaotic attractor as a solution for the 0-g case at $\Delta T = 20$ K, see Fig. 3(b). However, taking as an initial guess the $m = 1$ periodic solution of the normal gravity case, the solution for 0-g will converge to the $m = 2$ stable periodic attractor, see Fig. 3(a). Both attractors are stable and solution does not switch between them. For this $m = 2$ periodic branch one may see in Fig. 3(a) a sub harmonic, which
equals a half of the main frequency. The frequencies of this solution are \( \omega_1 = 13.53 \) and \( \omega_2 = 27.06 \) and the main frequency is the one with the largest pick. Due to this, the periodic thin limit cycles will split on two ones.

For the first time we report that dynamic behavior of the liquid bridge systems can not be described in frame of only supercritical bifurcations. Definitely, subcritical bifurcations appear with increasing the control parameter \( \Delta T \). Schematic bifurcation diagrams for 0-g and 1-g cases are shown in Fig. 4. It sketches amplitude of disturbances in arbitrary units vs. the temperature difference. The single mode periodic branches of the solution after first subcritical bifurcations are shown by horizontal dashed lines in Fig. 4. The single mode solution means that one of the modes is strongly dominant, e.g., they correspond to \( m = 2 \) for the 0-g, and \( m = 1 \) for the 1-g case. The diagram demonstrates that these periodic branches coexist together with the mixed mode quasi periodic and/or chaotic ones. The two solutions under 0-g, mentioned above, one may track by following the vertical line at \( \Delta T = 20 \) in Fig. 4(a).

There is kind of “connection” between the single mode periodic branches for the 0-g and 1-g cases: choosing one of them as initial guess and changing the gravity, we will arrive to the periodic solution for other case. Thus, “switching off (on)” the gravity one may jump from one periodic solution to another one.

Near the branching point at a subcritical bifurcation, two stable and one unstable state may coexist for some range of parameters. The coexistence of all the three branches for the same parameters’ set was discovered in the 1-g case. It is not easy to study such situations by direct numerical simulations since it takes numerous computations for understanding each state’s dynamics. They can be observed when non-linear interactions become strong. If control parameter is increased, the system will make a jump from stable state and will end up on some other branch. Decreasing it, the system will remain on this branch and then will jump back to the previous stable state. This hysteresis and discontinuous change of the dynamics is similar to a first order phase transition.

Starting from the points \( T_0^* \) and \( T_1^* \) the system exhibits quasi periodic solutions along with the periodic one. In both cases, 0-g and 1-g, the initial stable periodic branches are eventually destabilized by quasiperiodic branches forming 2-torii in phase plane, which itself are destabilized and lead to a chaotic regime. The transition to chaos under absence of gravity is due to the torus breakdown. Under normal gravity, between the quasi periodic and chaotic regimes there is a bifurcation of a limit cycle, so-called periodic window (see Melnikov et al., 2004). These sequences are obtained numerically and mapped in Figs. 4 and 5. Note, that quasi periodic and chaotic regimes are results of subcritical Hopf bifurcations.

Stability of the torii and strange attractors have also been studied while \( \Delta T \) is decreased. Starting from quasi periodic or chaotic branches we observed survival of them for less values of \( \Delta T \) than in the case when \( \Delta T \) is increased \((T_0^*, T_1^*)\). Thus, decreasing the temperature difference moves the secondary subcritical Hopf bifurcations downward. Also, the system reveals flow pattern with mixed mode \((m = 1 \text{ and } 2)\) while one of them slightly dominates.

A qualitative comparison of 0-g and 1-g cases is given in Fig. 5. One can see sequences of regimes obtained numerically and passed by the system when \( \Delta T \) is increased. The onset of the oscillations occurs earlier in the 1-g case: at \( \Delta T_1^* \approx 6.05 \text{ K} \) vs. \( \Delta T_0^* \approx 7.56 \text{ K} \) when \( g = 0 \). But the onset of chaos takes place earlier in the 0-g case. Actually it occurs even earlier than appearance of 2-torus under 1-g. This is a result demanding a more detailed investigation. Thus at relatively large values of \( \Delta T \) gravity stabilizes the flow and keeps the periodic branch stable up to much larger \( \Delta T \) than under the absence of gravity.

Summarizing the results of the simulations, one may say that for the gravity level of 0-g and 1-g, the route to chaos is two frequency quasi periodic and the branch of chaotic solution is characterized by random spikes in

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Fig. 4. Schematic bifurcation diagrams for 0-g (a) and 1-g (b) cases. Diagram sketches amplitude of disturbances in arbitrary units vs. the temperature difference. In both cases the initial 2D → 3D and SW → TW bifurcations are supercritical. Increasing \( \Delta T \), the system passes cascades of secondary subcritical bifurcations and regions of hysteretic behavior.
temperature time series. On its way to chaos, under absence of gravity the system passes four bifurcations: two supercritical and two subcritical, see Fig. 4(a). While the presence of gravity increases the quantity of bifurcations up to seven (see Fig. 4(b)). The latter case was thoroughly studied in Melnikov et al. (2004), but at the time of publishing the paper we were not sure in the existence of the subcritical bifurcations. Only analysis of hysteretic behavior proved it.

4. Conclusions

The influence of gravity on the dynamics of the high Prandtl number liquid bridge, $Pr = 18.8$, has been analyzed by direct numerical simulations. The presence of the gravity destabilizes the flow with respect to $\bar{g} = 0$ conditions. Transitions from 2D stationary flow to 3D pulsatory both under 0- and 1-g occurs as supercritical Hopf bifurcation. This standing wave solution is switched to the traveling wave as a result of second Hopf bifurcation with further increasing the temperature difference. In the absence of gravity, the first and the second supercritical bifurcations take place at larger values of $\Delta T$ compared to the terrestrial conditions.

With further increase of $\Delta T$ the system passes a few subcritical bifurcations in both cases 1- and 0-g. The periodic solutions lose stability in the first subcritical bifurcation which gives rise to quasi periodic branch characterized by 2-torus orbit in phase plane. It was found that in far supercritical region the periodic limit cycles coexist together with 2-tori and strange attractors for the same set of parameters. In the 0-g case further increasing the temperature difference breaks down the quasi periodic orbits and the system moves to chaotic solution forming a strange attractor.

Comparative analysis of the results shows that the 0-g case looks more complicated than the 1-g case. More detailed study is required for better understanding the system dynamics.

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References