A Note on an Induced Subgraph Characterization of Domination Perfect Graphs.

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Abstract

Let $\gamma(G)$ and $\iota(G)$ be the domination and independent domination numbers of a graph G, respectively. Introduced by Sumner and Moorer [23], a graph G is domination perfect if $\gamma(H) = \iota(H)$ for every induced subgraph $H \subseteq G$. In 1991, Zverovich and Zverovich [26] proposed a characterization of domination perfect graphs in terms of forbidden induced subgraphs. Fulman [15] noticed that this characterization is not correct. Later, Zverovich and Zverovich [27] offered such a second characterization with 17 forbidden induced subgraphs. However, the latter still needs to be adjusted.

In this paper, we point out a counterexample. We then give a new characterization of domination perfect graphs in terms of only 8 forbidden induced subgraphs and a short proof thereof. Moreover, in the class of domination perfect graphs, we propose a polynomial-time algorithm computing, given a dominating set D, an independent dominating set Y such that $|Y| \leq |D|$.
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MSC: 05C69, 05C75.

1 Introduction

1.1 Basic definitions and notations

In this paper, graphs are undirected and simple. Standard notions are explained, for instance, by Diestel [11]. V and E denote the vertex and edge sets of a graph G, respectively. For a given vertex v, $N(v)$ denotes the set of all neighbors (i.e. adjacent vertices) while, for a given vertex set X, $G[X]$ denotes the subgraph of G induced by X. Moreover, if G and H are two graphs, we say that G is H-free if H does not appear as an induced subgraph of G. Furthermore, if G is H_1-free, H_2-free, ..., H_k-free for some graphs $H_1, H_2, \dots, H_k$, we say that G is $(H_i)_{i=1}^k$-free.

A *dominating set* of a graph $G = (V, E)$ is a set D of vertices such that every vertex $v \in V \setminus D$ has at least one neighbor in D. The *domination number* of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set. A dominating set with such cardinality is called *minimum* while a dominating set is *minimal* if no proper subset is a dominating set.

A graph is *complete* if it contains all possible edges. A set D of vertices is *independent* (also called *stable*) if the subgraph induced by D has no edge. An independent set X of a graph $G = (V, E)$ is *maximal* if for every vertex $v \in V \setminus X$, $X \cup \{v\}$ is not independent. A dominating set D of a graph G is called *independent* if D is independent. It is known [5, 6], that an independent dominating set is a maximal independent set, and conversely. The *independent domination number* of a graph G, denoted by $\iota(G)$, is the minimum cardinality of an independent dominating set in G. Thus, an independent dominating set is *minimum* if its cardinality is minimum.

Sumner and Moore [23] introduced the notion of *domination perfect graph*, as a graph G such that $\gamma(H) = \iota(H)$, for all induced subgraph H of G. A graph is said *minimal domination perfect* if the graph is not domination perfect but all proper induced subgraphs are.

1.2 Previous works

The class of domination perfect graphs has been studied. Looking for a characterization, many authors focused on special subclasses of graphs. We present here a brief survey on domination perfect graphs.

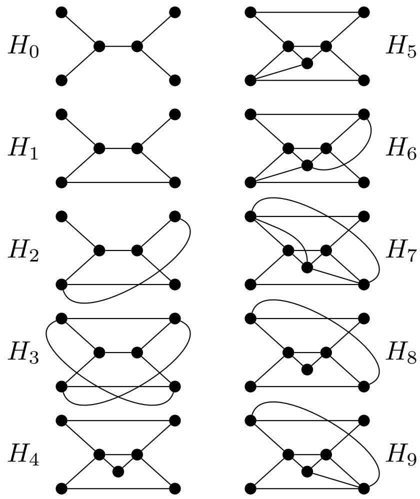

Figure 1: An illustration of graphs H_i, for $i = 0, \ldots 9$.

The line graph $L(T)$ of a tree T is always domination perfect [7, 20]. More generally, every line graph is domination perfect, proved by Allan and Laskar [2] and independently by Gupta (see Theorem 10.5 [17]). In fact, Allan and Laskar gave a sufficient condition in the following theorem.

Theorem 1 (Allan and Laskar [2]). *Every claw-free graph is domination perfect.*

Topp and Volkmann [24] generalized their results to new classes of graphs.

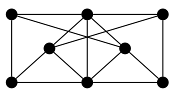

Figure 2: An illustration of the graph H_{10}.

Theorem 2 (Topp and Volkmann [24]). *If G is H_{10}-free and $(G_i)_{i=1}^{13}$-free (see Figure 2 and Figure 3) then G is domination perfect.*

As observed in [27], the original version of this theorem in [24] was stated with two additional graphs, which were shown to be redundant.

Harary and Livingston [18] studied the class of domination perfect trees and offered a complex characterization of this class. Other characterizations of these particular trees are mentioned in [9, 13, 19]. Actually, determining a minimum dominating set and a minimum independent dominating set in trees can be achieved in linear time [7, 10, 14].

Sumner [22] gave a characterization of domination perfect graphs in the classes of chordal and planar graphs while Zverovich and Zverovich [26] tackled the case of triangle-free graphs. Consider the class $\mathcal{S}$

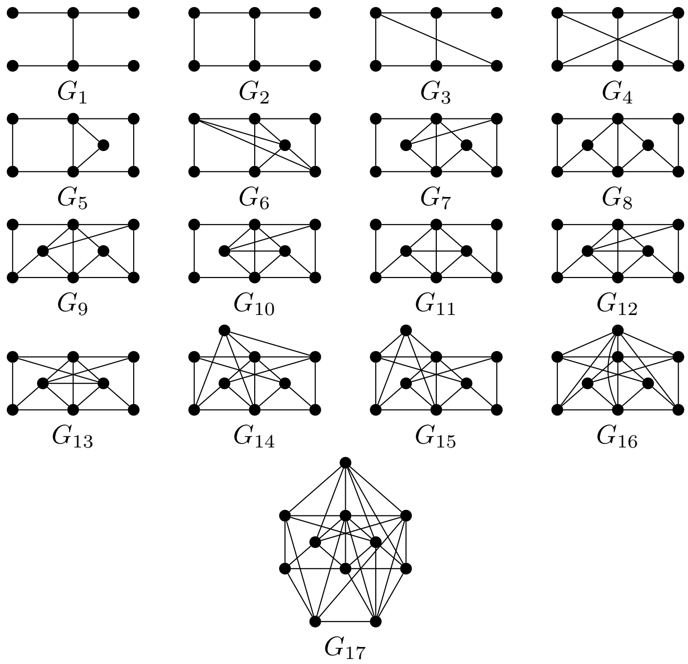

Figure 3: An illustration of graphs G_i, for $i = 1, \dots 17$.

of graphs defined by

$$\mathcal{S} = \{H \text{ graph on at most 8 vertices } | \gamma(H) = 2, \iota(H) > 2\}.$$

Theorem 3.

- *(Sumner [22]) Let G be a chordal graph. G is domination perfect if and only if G is H_0-free.*
- *(Sumner [22]) Let G be a planar graph. G is domination perfect if and only if G is $\mathcal{S}$-free.*
- *(Zverovich and Zverovich [26]) Let G be a triangle-free graph. G is domination perfect if and only if G is $(H_i)_{i=0}^3$-free.*

where graphs H_i are drawn in Figure 1.

Sumner and Moore [23] attempted to extend previous results to all graphs.

Theorem 4 (Sumner and Moore [23])**.** *If G is $\mathcal{S}$-free and G is H_{10}-free then G is domination perfect, where H_{10} is depicted in Figure 2.*

Other sufficient conditions were found [15, 27], stated in the following theorems, where graphs H_i, U_i and T_i are respectively represented in Figure 1, Figure 4 and Figure 5.

Theorem 5 (Fulman [15])**.** *If G is $(H_i)_{i=0}^4$-free, H_7-free and $(U_i)_{i=1}^2$-free, then G is domination perfect.*

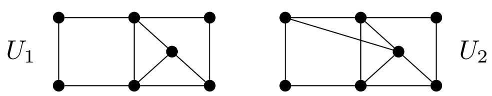

Figure 4: An illustration of graphs U_1 and U_2.

Theorem 6 (Zverovich and Zverovich [27])**.** *If G is $(H_i)_{i=0}^3$-free and $(T_i)_{i=1}^2$-free, then G is domination perfect.*

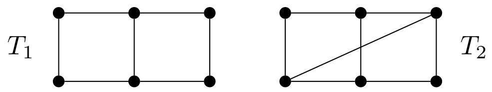

Figure 5: An illustration of graphs T_1 and T_2.

Sumner [22] established that a graph is domination perfect if and only if $\gamma(H) = \iota(H)$ only for all induced subgraph H with $\gamma(H) = 2$, and supposed impossible to provide a finite list of forbidden induced subgraphs characterizing domination perfect graphs. However, Zverovich and Zverovich [26] gave a first characterization with a list of 4 forbidden induced subgraphs. Nevertheless, Fulman [15] brought out a counterexample. Later, Zverovich and Zverovich [27] proposed another characterization with a list of 17 forbidden induced subgraphs.

Theorem 7 (Zverovich and Zverovich [27])**.** *Let G be a graph. Then G is domination perfect if and only if G is $(G_i)_{i=1}^{17}$-free, where graphs G_i are depicted in Figure 3.*

In this paper, we point out a counterexample to Theorem 7 and we state a new characterization of domination perfect graphs. Moreover, we check the validity of results deduced from [27] and adapt some of them.

Zverovich [25] generalized the notion of domination perfect graphs by considering, instead of an equality of invariants, the difference between two invariants bounded by a constant. Explicitly, he was interesting by the following classes of graphs, generically called *k-bounded classes of dominant-independent perfect graphs* : for any $k \geqslant 0$, $\iota\gamma(k) = \{G \mid \iota(H) - \gamma(H) \leqslant k \text{ for any } H \text{ subgraph of } G\}$. He also investigated another invariant : the *independence number* $\alpha(G)$, which is the minimum size of an independent set in G. Thenceforth, he worked on the similar classes of graphs : for any $k \geqslant 0$, $\alpha\gamma(k) = \{G \mid \alpha(H) - \gamma(H) \leqslant k \text{ for any } H \text{ subgraph of } G\}$ and $\alpha\iota(k) = \{G \mid \alpha(H) - \iota(H) \leqslant k \text{ for any } H \text{ subgraph of } G\}$. He found a characterization in terms of finite list of forbidden induced subgraphs for the two previous classes of graphs. He conjectured that the first classes of graphs can be characterized by a finite list of forbidden induced subgraphs.

Besides, other graph invariants were investigated for comparison. The famous example is naturally perfect graphs [3, 4] whose chromatic number and clique number are equal for all induced subgraph. Furthermore, Gutin and Zverovich [16] studied the class of Γ-perfect graphs, graphs whose independence number and upper domination number are equal for all induced subgraph, and the similar class of IR-perfect graphs with the independence number and the upper irredundance number. Later, Dohmen, Rautenbach and Volkmann [12] obtained different results on the corresponding k-bounded classes of graphs. Alvarado, Dantas and Rautenbach [1] investigated and characterized perfect graphs, according to all pairs of invariants among the total domination number, the paired domination number and the domination number while Rautenbach and Zverovich [21] characterized certain subclasses of perfect graphs when the pair of graph invariants is either the strong domination number and the domination number, or, the domination number and the independent strong domination number, or, the independent domination number and the independent strong domination number.

2 Main results

2.1 Counterexample to characterization in [27]

Under Theorem 7, a graph G is domination perfect if and only if G does not contain any of $G_1, \ldots, G_{17}$ in Figure 3 as an induced subgraph. Nonetheless, graphs H_5 and H_6 in Figure 1 are counterexamples. Indeed, none contains any $(G_i)_{i=1}^{17}$ as an induced subgraph, as the system GraphsInGraphs [8] confirmed. And none is domination perfect, since $\gamma(H_5) = \gamma(H_6) = 2 \neq 3 = \iota(H_5) = \iota(H_6)$.

Observe that every graph H_i is isomorphic to certain G_i or is an induced subgraph of certain G_i : $H_0 \cong G_1, H_1 \cong G_2, H_2 \cong G_3, H_3 \cong G_4, H_4 \cong G_5, H_7 \cong G_6$ and H_5, respectively H_6, is an induced subgraph of G_7, G_8, G_{10}, G_{11}, G_{12}, G_{14} and G_{15}, respectively G_9, G_{13}, G_{16} and G_{17}. Since every graph G_i contains at least one subgraph isomorphic to a certain H_i as an induced subgraph, the class of $(H_i)_{i=0}^{9}$-free graphs is included in the class of $(G_i)_{i=1}^{17}$-free graphs.

Notice also that H_1, respectively H_2, is an induced subgraph of H_8, respectively H_9. Accordingly, the class of $(H_i)_{i=0}^{9}$-free graphs is exactly the same as the class of $(H_i)_{i=0}^{7}$-free graphs. From now, we consider both classes equivalently.

2.2 A new characterization of domination perfect graphs

Lemma 1. *Let G be a $(H_i)_{i=0}^{7}$-free graph on n vertices, where graphs H_i are drawn in Figure 1, and D a dominating set of G. Then Algorithm 1 gives in $\mathcal{O}(n^4)$ an independent dominating set Y such that $|Y| \leq |D|$.*

Algorithm 1 Polynomial-time algorithm to find an independent dominating set with at most the cardinality of a given dominating set.

Require: $G = (V, E)$ a $(H_i)_{i=0}^{7}$-free graph
Require: D a dominating set of G
Ensure: Y an independent dominating set of G such that $|Y| \leq |D|$
1: $Y \leftarrow$ a minimal dominating set included in D
2: **while** $G[Y]$ contains an edge **do**
3: Let d_0d_1 be an edge of $G[Y]$
4: Let N_i be the set of private neighbors of d_i, for $i = 0, 1$
5: **if** $G[N_0]$ is complete **then**
6: Let t be an arbitrary vertex in N_0
7: $Y \leftarrow (Y \setminus \{d_0\}) \cup \{t\}$
8: **else if** $G[N_1]$ is complete **then**
9: Let t be an arbitrary vertex in N_1
10: $Y \leftarrow (Y \setminus \{d_1\}) \cup \{t\}$
11: **else**
12: Let u_0v_0 be a missing edge in N_0
13: Let u_1v_1 be a missing edge in N_1
14: **if** $G[\{u_0, v_0, u_1, v_1\}]$ is a perfect matching **then**
15: Let u_0u_1, v_0v_1 be edges of the perfect matching
16: $Y \leftarrow (Y \setminus \{d_0, d_1\}) \cup \{u_0, v_1\}$
17: **else**
18: Let $u_1u_0v_1v_0$ be the path on 4 vertices
19: $Y \leftarrow (Y \setminus \{d_0, d_1\}) \cup \{u_1, v_0\}$
20: **end if**
21: **end if**
22: $Y \leftarrow$ a minimal dominating set included in Y
23: **end while**
24: **return** Y

Proof of Lemma 1. Clearly, Algorithm 1 performs in $\mathcal{O}(n^4)$ since the loop 'while' is applied at most $\mathcal{O}(n^2)$ times, a minimal dominating set can be computed in $\mathcal{O}(n^2)$, the same worst time for computing private neighbors (at worst for instance with a breadth-first search), checking the completeness of a subgraph or finding a missing edge, and other operations take constant time.

Let D be a dominating set of G. Consider initially $Y = D$. We show that each iteration of the loop 'while' decreases strictly the number of edges in $G[Y]$ and the resulting set Y is still a dominating set. Notice that in each case, the cardinality of Y is not greater than that of D (see lines $6, 8, 12, 15, 21$ and

24 in Algorithm 1) and lines 1 and 28 in Algorithm 1 assure the minimality of Y at the beginning of the loop 'while'. Let d_0d_1 be an arbitrary edge in $G[Y]$. We consider $N_i = \{x \in V(G)\setminus Y | N(x) \cap Y = \{d_i\}\}$, the set of private neighbors of d_i according to Y, for $i = 0, 1$. Because Y is minimal and $d_0d_1 \in E(G)$, no N_i is empty, otherwise $D \setminus \{d_i\}$ would be a smaller dominating set. We distinguish different cases, depending on the cardinality of N_0 and N_1.

- **Case 1.** If there exists i such that $G[N_i]$ is complete, say N_0, then $(Y \setminus \{d_0\}) \cup \{t\}$, where t is an arbitrary vertex in N_0, is a dominating set with fewer edges than Y. Note that $|N_0| = 1$ or $|N_1| = 1$ are special cases.

- **Case 2.** We may suppose that $|N_0| \geq 2$ and $|N_1| \geq 2$. Let u_0v_0, respectively u_1v_1, be a missing edge in N_0, respectively in N_1 (see Figure 6). Look at the adjacency between the 4 vertices

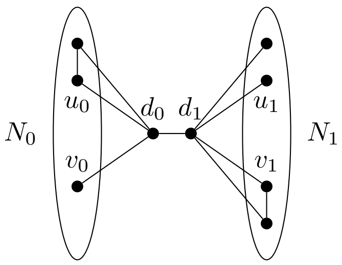

Figure 6: The edge d_0d_1 with two non-complete private neighborhoods.

  u_0, u_1, v_0, v_1. Because G is (H_0, H_1, H_2, H_3)-free, without loss of generality, we may assume that either only u_0u_1 and v_0v_1 are edges in G, or only u_0u_1, u_0v_1 and v_0v_1 are edges in G. We discern each case.

  - **Case 2.1.** We have a perfect matching between the vertices u_0, v_0, u_1, v_1, i.e. u_0u_1 and $v_0v_1 \in E(G)$. Clearly, $(Y \setminus \{d_0, d_1\}) \cup \{u_0, v_1\}$ has fewer edges than Y. Suppose that $Z = (Y \setminus \{d_0, d_1\}) \cup \{u_0, v_1\}$ is not a dominating set. Hence, there exists a vertex $t \in V(G) \setminus Y$ which is not adjacent to Z, especially to u_0 and v_1. Since Y is a dominating set, t must be adjacent to d_0 or d_1 or both. Because G is (H_4, H_5, H_6)-free, t can not be adjacent to d_0 and d_1. Therefore t is only adjacent to d_0 or d_1, say d_0 (see Figure 7). Thus $G[\{d_0, d_1, t, u_0, v_1, u_1\}]$

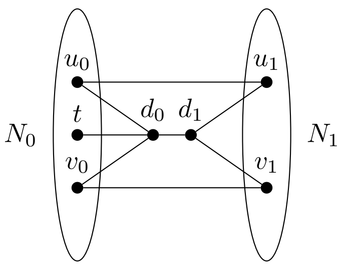

Figure 7: An illustration of Case 2.1.

    is an induced H_1 or H_2, depending on the adjacency between t and u_1, a contradiction.
  - **Case 2.2.** In the last case, only u_0u_1, u_0v_1 and v_0v_1 are edges in G, i.e. they induce a path on 4 vertices. Obviously $(Y \setminus \{d_0, d_1\}) \cup \{u_1, v_0\}$ has fewer edges than Y. Now, assume that $Z = (Y \setminus \{d_0, d_1\}) \cup \{u_1, v_0\}$ is not a dominating set. Thus, there exists a vertex $t \in V(G) \setminus Y$ which is not adjacent to Z, especially to u_1 and v_0. Since Y is a dominating set, t must be adjacent to d_0 or d_1 or both. Because G is (H_7, H_8, H_9)-free, t is only adjacent to one of d_i,

say d_0 (see Figure 8). Therefore $G[\{d_0, d_1, t, v_0, v_1, u_1\}]$ is an induced H_1 or H_2, depending on the adjacency between t and v_1, a contradiction.

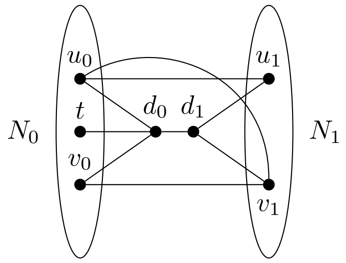

Figure 8: An illustration of Case 2.2.

□

Theorem 8. *Let G be a $(H_i)_{i=0}^{7}$-free graph, where graphs H_i are drawn in Figure 1. Then*

$$\iota(G) = \gamma(G).$$

Proof. Let D be a minimum dominating set of G. By Lemma 1, we obtain an independent dominating set Y such that $|Y| \leq |D|$, which implies that $\iota(G) \leq \gamma(G)$. But the cardinality of every minimum dominating set is a lower bound on the cardinality of any independent dominating set, i.e. $\gamma(G) \leq \iota(G)$. Thus,

$$\gamma(G) = \iota(G).$$

□

Because $\iota(H_i) = 3 \neq 2 = \gamma(H_i)$, for $i = 0, \ldots, 9$, we deduce from the last theorem the following one which gives a new characterization of domination perfect graphs.

Theorem 9. *The following assertions are equivalent for every graph G :*

- *G is $(H_i)_{i=0}^{7}$-free.*
- *G is $(H_i)_{i=0}^{9}$-free.*
- *For every induced subgraph H of G, it holds that $i(H) = \gamma(H)$.*

By Theorem 9, we observe that Algorithm 1 actually computes, given a dominating set D of any domination perfect graph G, an independent dominating set Y such that $|Y| \leq |D|$, that implies if D is minimum then Y is also minimum.

2.3 Validity or adaptation of consequences of [27]

Based on their characterization, Zverovich and Zverovich [27] proposed two corollaries that we improve. To be self-contained, we include a slightly adapted proof of the first corollary. Consider $\overline{G}$ the *complement* of the graph $G = (V, E)$, defined by $\overline{G} = (V, (V \times V) \setminus E)$.

Corollary 1. *If a graph G is $(\overline{H_i})_{i=0}^{7}$-free, where H_i is depicted in Figure 1, with diameter at least 3 and minimum degree at least 1, then G has an edge that does not belong to any triangle.*

Proof. Let $\overline{G}$ be the complement of G (on n vertices). Clearly $\overline{G}$ is $(H_i)_{i=0}^{7}$-free. If $\gamma(\overline{G}) = 1$, then $\overline{G}$ has a dominating vertex, i.e. a vertex of degree $n-1$. Then, G has an isolated vertex, i.e. a vertex of degree 0, a contradiction. Assume that $\gamma(\overline{G}) \neq 2$. For every pair of vertices x and y, there exists a vertex z which is not adjacent in $\overline{G}$ to x and y. This implies in G that for every pair of vertices x and y, there is a vertex z adjacent to x and y, i.e. the diameter of G is at most 2, a contradiction. Thus, $\gamma(\overline{G}) = 2$ and by Theorem 8, we deduce that $\iota(\overline{G}) = 2$, i.e. the graph G has an edge that does not belong to any triangle. □

Consider $\mathcal{L}$ the class of graphs G satisfying all following conditions :

1. G is planar,
2. G is bipartite,
3. G has maximum degree 3,
4. G has girth $g(G) \geq k$, where k is fixed.

Corollary 2. *Decision problems associated to the domination problem and the independent domination problem are both NP-complete in the class $\mathcal{L}$.*

The proof in [27] of the previous corollary is still valid if we consider the list of graphs $(H_i)_{i=0}^{7}$ as forbidden induced subgraphs.

Moreover, Fischermann, Volkmann and Zverovich [14] characterized the class $\mathcal{I}$ of graphs G such that for every induced subgraph H of G, H has a unique minimum irredundant set if and only if it has a unique minimum dominating set.

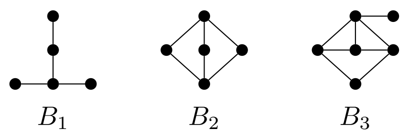

Figure 9: An illustration of graphs B_1, B_2 and B_3.

Theorem 10 (Fischermann, Volkmann, Zverovich [14])**.** *A graph G belongs to $\mathcal{I}$ if and only if G is $(B_i)_{i=1}^{3}$-free (see Figure 9).*

Their proof is based on the characterization of domination perfect graphs from Zverovich and Zverovich [27]. In order for the proof to remain correct, we need to check in the sufficient condition that every $(B_i)_{i=1}^{3}$-free graph G is domination perfect by using our characterization. Indeed, each of the graphs H_0, H_1, H_2, H_4 and H_5 has an induced subgraph isomorphic to B_1. The graph H_3 contains B_2 as an induced subgraph. Finally, B_3 is an induced subgraph of H_6 and H_7. Because G is $(B_i)_{i=1}^{3}$-free, G is also $(H_i)_{i=0}^{7}$-free. By Theorem 9, G is thus domination perfect. The end of the proof in [14] remains.

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