An algorithm for cost optimization of PMU and communication infrastructure in WAMS

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Abstract

Power system state estimation relies increasingly on PMUs measurements to effectively control and monitor growing and stressed transmission networks and also affected by transient and dynamic events. High PMU cost has motivated optimal PMU placement solutions but recent works have shown the effect of communication infrastructure cost in PMU configuration. In this paper, we present a new method for the design of Wide Area Measurement Systems. A topological analysis algorithm based on the Variable Neighborhood Search heuristic is proposed and tested in several networks, including the common IEEE test networks and the 5804-bus Brazilian transmission system. Our results show the flexibility, effectiveness, and scalability of the proposed methodology when compared with recent research presented in the literature.

Keywords: Phasor Measurement Unit (PMU), optimal placement of PMUs, Variable Neighbourhood Search (VNS), communication infrastructure, power

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1. Introduction

The impact of increasing renewable power, free access to transmission networks and deregulation of the electricity sector is changing the nature of power systems. This fact brings more stress to electrical power system operation, which makes it increasingly important to improve the control and protection systems offered by Wide Area Measurement Systems (WAMS) based on Phasor Measurement Units (PMU) [1].

State Estimation (SE) processes a redundant set of measures in order to obtain in real time a reliable estimate of the operating state of the power system. Conventionally, network state monitoring is done with conventional meters (magnitude of bus, active/reactive power flows and injections along with some voltage magnitude) whose measurements are transmitted to the Supervisory Control and Data Acquisition Systems (SCADA). With the introduction of PMUs in the 1990s the conventional meters with RTUs are now being replaced by PMUs that transmit magnitude and phase measurements to the SCADA system. The PMUs are synchronized through the Global Positioning System (GPS), which allows WAMS to be in control and operation of power systems. Another advantage is the PMU high data sampling combined with SE process, ensures quick control voltage compared to conventional measurements. However, PMUs can be very costly in large power systems and has motivated the Optimal PMU Placement (OPP) problem on transmission networks.

Many algorithms have been proposed to solve the OPP problem, by taking into account criteria such as observability, contingencies (single PMU outage or single branch failure events), critical measures and critical assemblies. Most of these algorithms focuses on minimizing PMU costs, leading to objective functions which simply optimize the number of PMUs and their placements. A few recent studies have called attention to other aspects such as the Communication Infrastructure (CI), which has greater contribution to monitoring system costs
than PMUs. In such studies, the problem is formulated in a more comprehensive way, seeking to optimize the allocation of PMUs and the communication infrastructure for a minimum total cost.

1.1. Contribution

This paper proposes a new methodology for the OPP problem, based on a topological analysis of the observability. Furthermore, it is introduced a second optimization algorithm which aims to optimize simultaneously PMU allocation and communication network for a minimum total cost in WAMS. Other costs besides PMU and CI are outside the scope of this work. This latter algorithm is derived from the first one and embeds Dijkstra’s single-source shortest-path, and Kruskal minimum spanning tree algorithms to achieve a minimum total cost. To construct these algorithms, a Variable Neighbourhood Search (VNS) heuristic [2] is implemented. To the best of our knowledge, it is the first time that a VNS heuristic is applied to OPP problems.

These problems are related to Dominating Set and Connected Dominating Set of graph theory [3] [4]. These concepts have motivated the construction of a new metric, named here *Dominance*, which in combination with other known graph theory metrics plays a fundamental role to turn the heuristic and the algorithm more efficient. Our algorithm has proved to be flexible to incorporate various cost parameters and contingencies, confirmed its scalability feature when generating results from small networks to large and complex electrical transmission networks and finally, proved good efficiency with overall good results when compared with other works.

1.2. Related work

There are several works on Optimal PMU Placement (OPP) developed over approximately 25 years. These algorithms are classified according to the type of observability analysis.

They differ from those who make a numerical analysis [5], those who operate a topological analysis of observability [6] and those who conjugate both
approaches [7]. Numerical algorithms require a large amount of calculations and precision is related to cumulative errors influence, whereas topological algorithms, that make use of graph theory, are faster but may still present observability deficits for SE [8].

All these algorithms can still choose to use a deterministic or metaheuristic methods. Linear programming, binary programming and quadratic programming are among the deterministic techniques have been used in OPP problem [9]. The deterministic techniques rely heavily on linear programming and numerical methods [10]. Also, several algorithms based on metaheuristics have been developed to date such as Simulated Annealing, Genetic Algorithms, Imperialistic Competition Algorithm (ICA), Tabu Search, Binary Particle Swarm Optimization (BPSO), Ant Colony and hybrid methods [11]. Due to the fact that OPP problems are NP-complete [3], many works have been implementing metaheuristic algorithms for the advantages of computational time (especially in large systems) and the ability to deal with conditions such as single PMU outage, topological variation, critical measures, areas of interest, among others.

Regarding topological analysis of observability, [12] was one of the first works to use graph theory for PMU allocation in a network. The study used a dual algorithm, modified bisector search and simulated-annealing, resuming that one-third to one-fourth of the network buses would need PMUs for the power system to become observable.

In [13] it is proposed a non-dominated sorting genetic algorithm (NSGA), which uses graph-theoretical procedure for PMU placement problem to deal with simultaneous optimization of two conflicting objectives, such as minimization of number of PMUs and maximization of measurement redundancy. Pareto-optimal solutions are presented for IEEE 38 and 118 test networks.

The concept of depth of unobservability is introduced in [14] who used the spanning tree of a power system graph and a tree search technique to find optimal location of PMUs. A based Simulated Annealing algorithm is used to solve the PMU placement in a pragmatic approach, thereby cushioning cost impact for utilities.
In [15] a binary search algorithm is used to set benchmarks to the optimal PMU placement solutions for IEEE 14-bus, IEEE 24-bus, IEEE 30-bus and New England 39-bus test systems. The method is also applied on a 298-bus system to determine the optimal placement of PMUs when conventional measurements are available.

A broad comparison between metaheuristic and deterministic methods applied to OPP problems is done in [10] which concludes that integer linear programming is the most adaptable mathematical form to model a network.

A topological and three stage iterative method is proposed in [6] to place a minimum number of PMUs in an electrical network for complete observability; PMU outage condition is also considered and results are presented for IEEE test networks to confirm algorithm efficiency.

An extensive review on heuristics for OPP problem is presented in [11], introducing a general comparison between heuristic methods in terms of their specific features and results on test networks.

The OPP problem has always been formulated through a PMU cost minimization paradigm. Recently, some authors have drawn attention to other factors that could have a major influence for a total cost in WAMS deployment, such as Communication Infrastructure (CI). Therefore, in [16] the problem is modeled as a total cost minimization problem, taking into account both PMU and CI costs and using genetic algorithm to confirm that the lowest total cost is not synonymous of a smallest number of PMUs.

In [9], this previous approach is deepened and other aspects and devices involved in communication infrastructure are detailed. Additionally, it is considered contingency conditions and pre-existence of some PMUs and communication cables in certain parts of the network in the methodology.

Finally in [17] is highlighted the same approach of total cost optimization and is presented a realistic cost-effective model for optimal PMU placement considering practical and unaccounted cost implications, based on a real-life project. The results confirmed that a minimum number of PMUs does not necessarily indicate minimum financial implications of the OPP project.
1.3. WAMS

The Wide Area Measurement Systems (WAMS) represent an important and promising technological advance in monitoring and control of power transmission systems. Synchrophasor technology (PMU) is at the heart of WAMS due to precision and celerity of its measures allowing a more dynamic analysis and intervention of the electrical network operators.

Basically a WAMS consists of four main elements: 1) The PMUs, 2) the Phasor Data Concentrator (PDC), 3) communication network and 4) a Power System Communication Center as illustrated in Figure 1. The measured data by the PMUs are sent via a communication network to a PDC which has the function of organizing them, based on GPS time record, and subsequent transmission to power system control center for state estimation [18]. This last stage of the applications processing is not considered in this paper.

Viable architectures for deploying WAMS can be classified into centralized, decentralized, and distributed [19]. The decision to choose one or another architecture depends on monitoring, protection and control schemes desired. Centralized and distributed architectures are most commonly used in the control and monitoring of large areas. The centralized architecture is very suitable due to an efficient use of control elements, lower cost, good coordination of alarms and event management and lower latency as a consequence of smaller number of PDCs, although it is exposed to a greater probability of failure than a distributed architecture.

Regarding transmission media for communication in smart grids, dependent media, i.e., those who are part of a power system and owned by independent system operator (ISO) like Power Line Communication (PLC), All-Dielectric Self Supporting (ADSS) and Optical Power Ground Wire (OPGW) are preferred due to lower latency in comparison with other medias [16]. However, OPGW stands out for the advantages of high channel capacity and transfer rate, low transmission losses and immunity to electromagnetic interference [9] [20]. Furthermore, the CI can be co-optimally designed in conjunction with power system planning problems.
This paper proposes to optimize costs on a centralized WAMS supported with OPGW transmission media. Additionally, it is considered costs of the switches. These are understood as electro-optical and/or optic-optical converters integrated into an Add Drop Multiplexer (ADM), which enable PMU connection to the OPGW communication network or allow the various OPGW branches arriving on a bus to form a communication network node.

The remaining of the paper is organized as follows. Section II gives an introduction of the fundamental concepts. Section III presents the OPP problem and a VNS-based heuristic to solve it. Section IV extends the problem to a total cost objective-function and details the algorithm implementation. Section V provides simulations results and, finally the conclusions are drawn in section VI.

2. Technical Background

2.1. Metaheuristic Variable Neighbourhood Search

The Variable Neighborhood Search (VNS) is a metaheuristic introduced by [2] and with several successful applications in solving combinatorial and global
optimization problems [21]. VNS relies on systematic changes in the vicinity for local search. This metaheuristic is characterized by simplicity and efficiency.

VNS is based on variable neighborhood method which exploits neighborhoods progressively further away from the current solution and moves to another solution if it is better than the previous one. This metaheuristic has two stages to search new solutions: (1) Shaking (or perturbation), which allows to vary progressively the neighborhood of current solution; and (2) Descent, which is a local search procedure. The shaking stage is necessary to move the actual solution away from local optimum and is parametrized by $k$ (shaking amplitude) which changes the current solution more and more as it increases. Each value of $k$ corresponds to a different neighborhood $N^k$. The search is increasingly global and diverse when there is no progress in the solution. If the local search can improve the solution prior to a perturbation then the new solution is adopted and the perturbation is resumed with value $k = 1$. This process is iterative and continues until a stopping criterion is reached.

The basic structure of the VNS is presented below although there are some variations of the algorithm [2]:

**Basic VNS Algorithm**

1. Function VNS $(x, k_{max}, t_{max})$
2. repeat
3. $k \leftarrow 1$
4. repeat
5. $x' \leftarrow$ Shake$(x, k)$
6. $x'' \leftarrow$ FirstImprovement$(x')$
7. $x \leftarrow$ NeighbourhoodChange$(x, x'', k)$
8. until $k = k_{max}$
9. $t \leftarrow$ CpuTime()
10. until $t > t_{max}$
2.2. Graph Theory Concepts

In this section we present some concepts of graph theory that will be important for a good performance of the algorithm proposed and based on VNS metaheuristics.

1) **Graph**: A finite graph \( G = (V, E) \) is defined as a set of \( |V| \) vertices and a set of \( |E| \) edges connecting those vertices. A subgraph is a subset of a graph obtained by removing any set of vertices or edges from the original graph.

2) The degree of a vertex is the number of edges with that vertex as an end-point.

3) **Path and cycles**: A walk is a ‘way of getting from one vertex to another’ and consists of a sequence of edges, one following after another. A walk in which no vertex appears more than once is called a path. A cycle is a path where the initial and terminal node is the same and that does not use the same edges more than.

4) **Connected graphs and trees**: A graph in which any two vertices are connected by a path is a connected graph. A tree is a connected graph with no cycles.

5) **Adjacency matrix**: A non-directed graph with \( N \) vertices can be represented by an adjacency matrix \( A = [a_{ij}]_{N \times N} \) whose elements are defined as follows:

\[
a_{ij} = \begin{cases} 
1, & \text{if vertex } i \text{ is connected to vertex } j; \\
0, & \text{otherwise.}
\end{cases}
\]  

6) **Betweenness Centrality**: Betweenness Centrality (BWC) or simply betweenness is a measure of how significant a node is in facilitating communication between any two nodes in the network. BWC for a node is the ratio of the proportion of shortest paths a node is part of for any source-destination node pair in the network, summed over all possible source-destination pairs. The betweenness of a vertex \( v \) in a graph \( G=(V,E) \) is computed as follows:
• For each pair of vertices \((s,t)\), compute the shortest paths between them 
  \(sp_{st}\).

• For each pair of vertices \((s,t)\), determine the fraction of shortest paths that
  pass through the vertex \(v\).

• Sum this fraction over all pairs of vertices \((s,t)\) - 
  \(sp_{st}(v)\).

The Equation 2 summarizes the definition.

\[
BWC(v) = \sum_{s \neq v \neq t} sp_{st}(v) / sp_{st} \tag{2}
\]

7) Shortest paths and Dijkstra’s algorithm: Dijkstra’s algorithm is an algorithm for finding the shortest paths between vertices in a graph \(G = (V, E)\) with non-negative edge weights. There are many variants of this algorithm but this paper uses the single-source shortest path Dijkstra algorithm which fixes a single vertex as the source vertex and finds shortest paths from the source to all other vertices in the graph, producing a shortest-path tree. Dijkstra’s algorithm maintains a set \(S\) of vertices whose final shortest-path weights from the source \(s\) have already been determined. The algorithm repeatedly selects the vertex \(u \in (V \setminus S)\) with the minimum shortest-path estimate, adds \(u\) to \(S\), and relaxes all edges leaving \(u\). The runtime is \(O(V \log(V))\).

8) Minimum Spanning Tree and Kruskal algorithm: The minimum spanning tree [22] is a concept with a lot of applicability in several areas of knowledge.

The spanning tree consists of an acyclic subgraph \(A = (V,E')\) of a graph \(G = (V, E)\), which contains all the vertices of \(G\), that is, \(A\) is a maximum tree (containing all vertices of \(G\)) and \(E' \subseteq E\). Consequently we can obtain several generating trees of the same graph, considered other combinations of edges. In many graphs different weights are considered at the edges, which may be representative of costs, distances or other parameters. Thus, in many problems it is sought to find a generating tree whose sum of the weights of the edges is minimal.

Kruskal Algorithm is based directly on the generic minimum-spanning-tree algorithm. It grows a forest trying to connect any two trees by adding safe edges
(which do not form cycles). Kruskals algorithm is a greedy algorithm, because at each step it adds to the forest an edge of least possible weight. It can easily be made to run in time $O(E \log V)$.

9) Dominating Set and Connected Dominating Set: The problem proposed in this article is closely related to the minimum connected dominating set problem [3]. A dominant set (DS) of a graph $G = (V, E)$ is a subset $D \subseteq V$ such that each element of $V$ is either in the DS or is connected to a vertex in the DS (in the latter case, the vertex is said to be covered by a vertex in the DS). A connected dominant set is a connected subgraph induced by the vertices of $D$. The problem related to this concept is the cardinality minimization of the DS or the CDS, with some applications in wireless networks problems [23].

2.3. Topologic Analysis of Observability

In a topological observability analysis, a power system is represented by a topological graph. The graph vertices represent the network bus bar and the edges represent the branches of the network connecting the bus bars. Topological observability analysis is defined as the existence of at least one spanning measurement tree of full rank in a network [12]. This tree connects all observable vertices and branches which can be observed by direct measurements or calculations. The following rules are commonly used to assess the existence of this tree[24]:

1. Assign with direct voltage phasor measurement and direct current phasor measurement of incident lines to buses with PMUs.

2. If voltage and current phasors at one end of a line are known, then the unknown voltage phasor at the other end of the line can be calculated (called a pseudo-measurements) through Ohm’s Law.

3. A current phasor of a line can be calculated (pseudo-measurements) if voltage phasors of both ends this line are known.

4. If all line current phasors incident to a zero-injection bus (ZIB) are known except one, the current phasor of the unknown one can be calculated through Kirchhoff’s Circuit Laws equations. ZIB is a bus in which the net
power injection is zero. This is used as pseudo information to make system observable with less number of PMUs compared to the case when information of ZIBs is not considered.

### 3. The Optimal PMU Placement Problem

In this section we will present the OPP problem formulation considering only the PMU cost and the method to solve it. The formulation also takes into account the following restrictions: observability in normal condition and in N-1 contingency conditions, such us single PMU and branch outage.

#### 3.1. PMU Placement Problem Formulation

For an N bus system the optimal PMU placement problem is formulated as follows:

$$\text{Min} \left( \sum_{i=1}^{N} c_{pi} \cdot x_i \right)$$  \hspace{1cm} (3)

$$\text{s.t.} \quad A \cdot X \geq b$$  \hspace{1cm} (4)

Where,

- $c_{pi}$ is the PMU cost installation of the $i$th bus.
- $A$ is the adjacency matrix as defined in (1).
- $b=[1 \ 1 \ 1 \ ... \ 1]^T$ is a unit vector of length $N$.
- $X$ is the binary decision variable vector for PMU placement, whose entries $x_i$ are one if PMU is placed at $i$th bus and zero otherwise.

Observability requirement is attained by Constraint (4) under normal conditions. However, this constraint has to be replaced by Equation (5) or (6) if observability is imposed under single PMU or single branch outage conditions, respectively.

$$A \cdot X \geq [2 \ 2 \ 2 \ ... \ 2]^T$$  \hspace{1cm} (5)
In the latter equation, $A_j$ is a new adjacency matrix, slightly different from original matrix $A$, reflecting topology change inflicted by the branch outage. $N_{top}$ is the number of scenario for single branch outage.

3.2. A new adjusted metric for DS

The OPP problem is well related to DS minimization problem of Graph Theory [3]. In [12, 6] the degree centrality was used as the main searching criteria for PMU placement in buses with higher connections in order to minimize the number of total PMU required for complete observability. However, this strategy is not so efficient to achieve a minimal number of PMU due to the fact that some vertices with high degrees could cover (or share) the same set of adjacent vertices. To avoid or diminish multiple covering probability, the searching process should consider vertices with high degree but connected to neighborhood vertices with low degrees. These informations are embedded together in a new metric called here Dominance Centrality ($dom$). The Dominance of a vertex $v_i$ is defined as follows:

$$dom(v_i) = deg(v_i) - adn(v_i)$$

where $v_i$ is the $i$th vertex, $deg(v_i)$ is the degree of the vertex $v_i$ and $adn(v_i)$ is the average degree of all the neighborhood vertices of $v_i$, which is formally defined in Equation (8) as follows:

$$adn(v_i) = \frac{\sum_{j=1}^{deg(v_i)} deg(v_j) \cdot a_{ij}}{deg(v_i)}$$

where each $v_j$ is a vertex adjacent to $v_i$. 

\[ A_j \cdot X \geq [1 \ 1 \ 1 \ ... \ 1]^T, \quad j = 1, 2, ..., N_{top} \]
3.3. *Optimal PMU Placement Algorithm*

In this section it is implemented a VNS-based heuristic for the OPP problem described in previous section. For comparison reasons with other works, PMU unit cost $c_p$, is taken as a constant value in the objective-function (3) stated previously, with the same constrained conditions stated as stated before.

The VNS is divided in three stages are: (1) initial solution, (2) Shaking and (3) local search (descent). These stages can be adapted to each formulation of the proposed problems. Some terms from graph theory are related to system observability and are exchanged in the algorithm:

- The vertices of a dominating set (DS) defines the buses where PMUs are allocated.
- The vertices of a DS is a feasible solution for PMU placement and guarantees system observability.
- A bus that is covered (observable) is a bus with allocated PMU or adjacent to a bus with PMU.

The method seeks to define a dominating set (DS) for the graph that represents the power network. This DS defines the vertices (or buses) to allocate PMUs on the network.

3.3.1. Normal operating condition

**Initial solution**: The adjacency matrix, degree and dominance of all the buses (vertices) of the network are calculated. It is selected 33% of the buses with the highest dominance value for PMU allocation (dominant vertex). The percentage value is related to the work in [12] in which they contend that about one-quarter to one-third of the bars of a system need to be allocated PMUs to meet observability criteria (although this is not true for many graphs, this range is used as a reference to build an initial solution). Additionally, PMUs are assigned to all buses that are adjacent to terminal buses. Finally, a check is
performed to analyze which buses are not yet observed by PMUs; in such cases PMUs are allocated to these buses.

**Shaking**: This stage consists of disturbing the current solution (moving away from local minima) by allocating more PMUs to other buses. In this way a new solution is obtained which can be improved with a subsequent local search. The number of PMUs to be introduced is determined by a factor that is proportional to the size of the network and the selection of the buses is done in parametrized ($k$) and probabilistic way. If the value of $k$ is low there is a greater probability that buses with higher dominance values are selected to allocate new PMUs; If $k$ increases, buses with lower dominance value are selected, that is, the solution (the neighborhood) is increasingly varied. The increase of $k$ always happens when the solution can not be improved in the previous local search iteration. When the local search can improve the solution, the disturbance resumes with $k$ equal to 1.

**Local Search**: The local search is performed after each shaking and seeks to reduce PMUs. PMUs are sequentially eliminated from buses with lowest to highest dominance or inversely in alternating iteration. A PMU is only removed from a bus if it does not violate the observability constraint $A \cdot X \geq b$. The new solution is assumed whenever it has at least the same number of PMUs and in this case perturbation restarts with $k = 1$.

### 3.3.2. Single PMU loss and branch failure conditions

Supporting simple PMU loss is one of the robustness requirements for a monitoring system. In this condition it is guaranteed that in case of failure of any single PMU the system remains observable. To meet this requirement a bus must be observed (or covered) by two PMUs, that is, be adjacent to two PMU buses or by one but must have another PMU allocated on it. This requirement does not introduce substantial changes in the base algorithm (observability under normal conditions) previously seen. For the initial solution it is allocated PMU in all the buses to assure observability for single PMU loss. The shaking stage remains the same from the previous algorithm but is not performed in the first
iteration (the initial solution do not allow any perturbation). The local search is performed in the same way but in the process it only accepts removing PMUs if all buses stay covered (or observable) by at least two PMUs ($A \cdot X \geq [2 \ 2 \ 2 \ldots 2]^T$).

For branch failure conditions, the implementation looks very similar, but the condition to remove PMUs in the local search stage is more relaxed since it depends on the branch failure scenario, i.e., a bus should be observed by two PMUs if a branch failure could affect connectivity to a bus with allocated PMU, otherwise one PMU is sufficient.

4. PMU and Communication Infrastructure Cost Optimization Problem

In this section it is presented the problem formulation to optimize both PMU placement and related communication infrastructure (CI) for a minimum total cost associated. Also is derived from the previous OPP algorithm a method to solve this problem.

4.1. Problem Formulation

The PMU and Communication Infrastructure Cost Optimization Problem is stated in the Equations 9-12:

\[
\begin{align*}
\text{Min} & \left( \sum_{i=1}^{N} cp_i \cdot x_i + \sum_{i=1}^{N} \sum_{j=1}^{N} (cf \cdot [d_{ij} + cr \cdot x_i] \cdot [y_{ij}]) \right) \quad (9) \\
\text{s.t.,} & \\
A \cdot X & \geq b \quad (10) \\
\sum_{ij \in E} y_{ij} & = n - 1 \quad (11) \\
\sum_{ij \in E, i \in S, j \in S} y_{ij} & \leq |S| - 1, \forall S \subseteq V \quad (12)
\end{align*}
\]

The objective-function aims to minimize the total cost related to PMUs (first term) and communication infrastructure (second term). The CI cost has two
components. One is related to active elements, such as switches and routers, and the other one is related to OPGW deployment which is proportional to the length of cables deployed.

Constraint (10) ensures power system observability under normal conditions if $b$ is a unit vector of length $N$. For N-1 contingency conditions (single PMU or branch outage) this constraint is replaced by respective Equations (5) or (6).

Constraints (11) and (12) ensure a tree topology for PMUs communication network. In this sense the Equation (11) imposes a number of links or edges $(y_{ij})$ with less one unit than the number of vertices $n$ and the Equation (12) ensures there is no subcycle by imposing that any subset of $S$ vertices must have at most $(|S|-1)$ edges connecting them.

It is considered in this optimization a centralized architecture of the WAMS and that the communication network with OPGW is optimized over the electric network.

In addition, the variables involved are defined as follows:

- $N$ is the number of buses in the network;
- $c_{pi}$ is the PMU installation cost on the $i$th bus;
- $cr$ is the cost of active devices, such as routers and switches, allocated in all buses with PMUs;
- $cf$ is the OPGW cost per km;
- $d_{ij}$ represents the distance between bus $i$ and $j$.

$y_{ij} \in \{0,1\}$. Assumes value 1 if an existing line $\{i,j\}$ connecting two buses is selected to integrate the OPGW communication network.

$X$ is the binary decision variable vector for PMU placement whose elements $x_i$ are defined as one if PMU is placed at $i$th bus and zero otherwise.

$A$ is the adjacency matrix stated in (1).

4.2. Proposed Method for Total Minimum Cost

The proposed problem involves allocating a minimum set of PMUs and also minimizing the distances involved in the fiber cable communication network.
This two-component objective is related to the Minimum Connected Dominating Set (MCDS) problem addressed in Graph Theory, which is in general NP-hard. Although there are methods to solve the MCDS problem directly, there are others that propose two-phase strategy by first building the DS and then connecting it (CDS) [25]. This two-phase strategy [26] is more appropriate to the optimization problem proposed here for two reasons: (1) a single phase CDS construction method does not identify a minimum DS which is necessary for PMU placement acknowledge; (2) the nature of the problem requires the DS (PMU configuration) to address different cases of operating conditions, such as robustness of the measurement system (PMUs and branch outage) and ZIB presence, which is easier to model in a method with two phases. In this way, the optimization is implemented to address two subsequent objective function rather than a dual optimization. The Equation (13) resumes this approach and replaces Equation (9) in the problem formulation.

\[
\text{Min} \left( \sum_{i=1}^{N} cp_i \cdot x_i \right) + \text{Min} \left( \sum_{i=1}^{N} \sum_{j=1}^{N} (cf \cdot [d]_{ij} + cr \cdot x_i) \cdot [y]_{ij} \right) \quad (13)
\]

The two stages of the proposed algorithm are enumerated below.

**Stage I**: a VNS heuristic is implemented to find the optimal bus locations for PMU placement.

**Stage II**: it finds the shorter communication network that links all the PMUs found previously, which means finding a MCDS. This MCDS is determined in a weighted graph, which is somewhat different from MCDS’s problems in the literature that aims to minimize the CDS cardinality. Therefore the proposed algorithm construct a MCDS but respecting this specificity.

4.3. **Stage I - Optimal PMU Placement**

The objective in this stage is not just minimize costs with PMUs but choose the best PMU configuration that can impact more effectively the CI optimization in the second stage and globally to a lowest total cost. Following the work
of [4] which showed the advantage of Betweenness Centrality in the determination of a MCDS, the present methodology proposes to combine both metrics, dominance and betweeness in the first stage for PMU placement. Dominance is used to minimize the number of PMUs, while Betweenness Centrality seeks, at the same time, to influence the best PMU configuration that provides the lowest communication network costs.

It is used also a based-VNS heuristic but slightly different from the one proposed for minimizing PMUs in previous section. The operation of the VNS is detailed as follows:

**Previous Calculations:** from matrix A it is computed degree and dominance values for each vertex; betweeness of each vertex is computed from network distance and cable unit cost. Dijkstra algorithm is used to compute the single-shortest-paths from the bus with higher dominance value to all remaining buses.

**Initial Solution:** It is selected 33% of the buses with the highest dominance value for PMU allocation. Additionally, PMUs are assigned to all buses that are adjacent to terminal buses. Finally, a check is performed to analyze which buses are not yet observed by PMUs; in such cases PMUs are allocated to these buses. For single PMU outage and branch failure requirements the initial solution is formed by placing PMUs in all buses on both cases.

**Shaking:** The process to disturb the current solution is the same as described in section 3.3.1.

**Local Search:** The local search is performed after each shaking and seeks to reduce PMUs. PMUs are sequentially eliminated from buses with lowest to highest dominance or betweeness. These two metrics are used alternately from one iteration to another. A PMU is only removed from a bus if it does not violate the observability constraint $A \cdot X \geq b$. The new solution is assumed whenever its related total cost is lower or at least the same from previous solution. The total cost comprises costs with PMUs, switches and OPGW. This OPGW cost are determined using the shortest paths previously calculated. With a new solution a new perturbation restarts with k=1.

The flow chart of this algorithm is represented in Figure 2.
- Form system matrix $A$ and store distance between buses.
- Compute metric values: degree, dominance, and betweenness (BWC) for each bus.
- Compute PMU installation cost for each bus and OPGW cost for each branch. Compute BWC and single shortest paths.

**Initial solution:** Allocate PMUs in all buses.

**Shaking**

Allocate additional PMUs in some buses selected by a normal probability function. Higher $k$ parameter means less probability to select buses with higher dominance values.

**Local Search**

Check feasibility to eliminate PMUs from buses with lower to higher dominance (or BWC) value.

Compute estimated Total Cost

Discard the previous shaking and restart a new one with $k = k+1$. Does cost is lower?

Set new solution.

Stopping condition?

Yes

No, $k=1$

---

**Figure 2:** Flow chart of Stage I.

---

### 4.4. Stage II - Communication Network Optimization

After optimal PMU placement, it is optimized a network of communication between the buses with PMUs. This is translated as constructing a CDS from a pre-determined DS. Kruskal Minimum Spanning Tree algorithm is used together
with previous calculated Dijkstra’s Tree to find the smallest communication network. The following steps are implemented:

1. With the PMUs (DS vertices) and the single-shortest-paths previously calculated, it is identified the vertices which are not necessary to connect the PMUs. A subgraph of the original network is obtained by eliminating these vertices.

2. Kruskal algorithm is applied on this subgraph to set a minimum spanning tree structure to PMU communication network.

The flow chart of these steps is presented in Figure 3.

![Flow chart of Stage II.](image)

5. Simulations and Results

The algorithms has been simulated in the most common test transmission networks such as IEEE 14-bus, 24-bus, 30-bus, 118-bus. Additionally, test are also performed in IEEE 300-bus and in 5804-bus Brazilian transmission network in order to verify the algorithm scalability. The results of each algorithm simulated are exposed into two sections.
5.1. OPP Optimization Results

Results of simulations for optimal PMU placement problem under normal conditions are presented in Table 1. The optimal number of PMUs required for different systems and related bus locations are shown. The minimum number of PMUs allocated leveled the best results in literature [6, 27].

Simulations are also performed on larger networks such as IEEE 300-bus and Brazilian 5804-bus. For IEEE 300-bus the method allocated 87 PMUs which is compared to the best result on the literature [28]. Related to brazilian network, 34% of buses are equipped with PMUs, which is almost the same value for IEEE 30-bus (33%).

Table 1: Optimal PMU Placement Results for normal operating conditions without ZIB.

<table>
<thead>
<tr>
<th>System</th>
<th>Optimal PMU locations</th>
<th>No. of PMUs</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE14</td>
<td>2, 6, 7, 9</td>
<td>4</td>
</tr>
<tr>
<td>IEEE24</td>
<td>2, 8, 10, 16, 21, 23, 24</td>
<td>7</td>
</tr>
<tr>
<td>IEEE30</td>
<td>2, 4, 6, 9, 10, 12, 18, 23, 25, 29</td>
<td>10</td>
</tr>
<tr>
<td>IEEE57</td>
<td>1, 5, 9, 12, 15, 17, 21, 23, 28, 30, 36, 40, 44, 48, 49, 52, 56</td>
<td>17</td>
</tr>
<tr>
<td>IEEE118</td>
<td>1, 5, 9, 12, 15, 17, 20, 23, 28, 30, 36, 40, 44, 46, 50, 52, 56, 62, 63, 68, 71, 75, 80, 85, 86, 90, 94, 102, 105, 110, 115</td>
<td>32</td>
</tr>
</tbody>
</table>

The percentage of minimal buses equipped with PMUs needed for observability depends on network topology. Generally, a more connected graph needs less PMUs for observability requirements. Table 2 summarizes the percentual of PMUs allocated in the networks versus the average degree of the graph, which is one of the indicator of graph connectivity. It can be noticed that Brazilian
network has the highest percentage of buses with PMUs compared to other instances but it is also the lowest connected network (2.44 average degree).

Table 2: Minimum number of PMUs versus graph average degree.

<table>
<thead>
<tr>
<th>System</th>
<th>IEEE30</th>
<th>IEEE57</th>
<th>IEEE118</th>
<th>IEEE300</th>
<th>BR 5804</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. PMU</td>
<td>10</td>
<td>17</td>
<td>32</td>
<td>87</td>
<td>1987</td>
</tr>
<tr>
<td>% bus with PMUs</td>
<td>33%</td>
<td>30%</td>
<td>27%</td>
<td>29%</td>
<td>34%</td>
</tr>
<tr>
<td>Average degree</td>
<td>2.73</td>
<td>2.74</td>
<td>3.03</td>
<td>2.73</td>
<td>2.44</td>
</tr>
</tbody>
</table>

For single branch outage operating conditions it was considered simulations on IEEE 30-bus and 118-bus with several branch failures scenarios. Results are in Table 3.

Table 3: Optimal PMU Placement Results for branch outage.

<table>
<thead>
<tr>
<th>System</th>
<th>IEEE30</th>
<th>IEEE118</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch outage</td>
<td>24 26 1215 2122</td>
<td>45 1112 1730 4849 5459 5659</td>
</tr>
<tr>
<td></td>
<td>2324</td>
<td>6162 70-74</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Optimal PMU locations</th>
<th>IEEE30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1, 5, 6, 9, 10, 12, 15, 19, 24, 25, 27</td>
</tr>
<tr>
<td></td>
<td>1, 5, 9, 11, 12, 15, 17, 20, 22, 25, 28, 30, 32, 36, 37, 41, 43, 46, 49, 52, 54, 56, 59, 62, 64, 68, 70, 71, 75, 77, 80, 85, 86, 90, 92, 96, 100, 105, 110, 115</td>
</tr>
<tr>
<td>No. PMUs</td>
<td>11</td>
</tr>
</tbody>
</table>

Simulations are also extended to PMU loss and optimal PMU placements are shown in Table 4 considering ZIB. The number of PMUs to assure a more robust monitoring system demands much more buses equipped with PMUs compared to normal conditions. Table 5 compares these results to other methods, which
shows better results in some networks and less efficient in others (IEEE 14-bus, 30-bus and 57-bus).

Table 4: Optimal PMU Placement results for single PMU loss condition considering ZIB.

<table>
<thead>
<tr>
<th>System</th>
<th>Optimal PMU locations</th>
<th>No. of PMUs</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE14</td>
<td>1, 2, 4, 6, 9, 11, 13</td>
<td>7</td>
</tr>
<tr>
<td>IEEE24</td>
<td>1, 2, 7, 8, 9, 10, 14, 16, 18, 19, 20, 21</td>
<td>12</td>
</tr>
<tr>
<td>IEEE30</td>
<td>2, 3, 4, 6, 7, 10, 12, 13, 15, 17, 18, 20, 24, 25, 27, 30</td>
<td>16</td>
</tr>
<tr>
<td>IEEE57</td>
<td>2, 3, 5, 7, 9, 10, 12, 15, 17, 19, 21, 23, 27, 28, 31, 35, 37, 38, 42, 46, 49, 50, 52, 53, 55, 56, 57</td>
<td>27</td>
</tr>
<tr>
<td>IEEE118</td>
<td>1, 2, 6, 7, 8, 9, 12, 13, 15, 17, 18, 20, 21, 23, 24, 27, 28, 29, 32, 34, 36, 37, 40, 41, 44, 45, 46, 49, 51, 52, 54, 56, 57, 60, 66, 68, 70, 72, 75, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 102, 105, 107, 109, 110, 111, 112, 115, 117, 118</td>
<td>63</td>
</tr>
</tbody>
</table>

Table 5: Comparison of optimal PMU placement results for single PMU loss condition considering ZIBs.

<table>
<thead>
<tr>
<th>Methods</th>
<th>IEEE14</th>
<th>IEEE24</th>
<th>IEEE30</th>
<th>IEEE57</th>
<th>IEEE118</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>7</td>
<td>12</td>
<td>16</td>
<td>27</td>
<td>63</td>
</tr>
<tr>
<td>[6]</td>
<td>7</td>
<td>13</td>
<td>15</td>
<td>26</td>
<td>64</td>
</tr>
<tr>
<td>[27]</td>
<td>7</td>
<td>N/A</td>
<td>17</td>
<td>26</td>
<td>65</td>
</tr>
</tbody>
</table>
5.2. **Optimization of Total Cost**

After validations of the first algorithm, simulations were also extended to total cost optimization problems in WAMS and results are presented in this section. To be able to compare with related works, two cases are presented in following table 6 and are respectively used in previous works; these are inputs data for simulations. It was also assumed the same distance matrix between buses from previous works, i.e. that all transmission lines have the same conductors with the same configurations and relative distances between system buses were extracted from the system admittance matrix [29]. The total length of the transmission lines are shown in the same table.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>References</td>
<td>[16]</td>
<td>[9]</td>
</tr>
<tr>
<td>Network Total Distance, km</td>
<td>3.000 (IEEE 30-bus), 5.712 (IEEE 57-bus), 9.884 (IEEE 118-bus), 25.129 (IEEE 300-bus)</td>
<td></td>
</tr>
<tr>
<td>Unit costs</td>
<td>PMU - $40.000; OPGW - $10.000/km;</td>
<td>PMU - $40.000 (with one voltage and two current measuring modules) $4000 extra fee for any additional current measurement channel needed; OPGW - $4.000/km; switches - $4.000;</td>
</tr>
<tr>
<td>ZIB</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>
5.2.1. Case A

Simulations for this case are resumed in table 7. It can be notice from this results that the optimal PMU configurations which results in lower costs and assures system observability have in general more number of PMUs when compared to simple OPP algorithms (Table 1). Additionally, having more PMUs is better for redundancy measurement and state estimation. This conclusion was also stated in previous works. However, this methodology, when compared to related work in Table 8 shows a better efficiency in lowering meter planning cost on both IEEE 30-bus (-20.4%) and IEEE 118-bus (-32.5%). The resulting communication network is also smaller.

Table 7: Total Cost (PMU e CI) Optimization for Normal Operating Conditions - Case B

<table>
<thead>
<tr>
<th>System</th>
<th>No. PMUs</th>
<th>PMUs</th>
<th>Nr. Switches</th>
<th>km (OPGW)</th>
<th>US$ (x10⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-14</td>
<td>5</td>
<td>4, 5, 6, 7, 9</td>
<td>6</td>
<td>199</td>
<td>2.2</td>
</tr>
<tr>
<td>IEEE-30</td>
<td>10</td>
<td>2, 4, 6, 9, 10, 12, 15, 20, 25, 27,</td>
<td>11</td>
<td>625</td>
<td>6.7</td>
</tr>
<tr>
<td>IEEE-57</td>
<td>18</td>
<td>1, 4, 9, 15, 18, 21, 24, 28, 29, 31, 32, 36, 38, 39, 41, 46, 51, 54</td>
<td>35</td>
<td>1481.2</td>
<td>15.5</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>36</td>
<td>2, 5, 9, 11, 12, 17, 21, 27, 30, 31, 32, 34, 37, 40, 45, 49, 50, 51, 54, 59, 65, 66, 68, 70, 71, 75, 77, 80, 83, 86, 89, 92, 96, 100, 105, 110</td>
<td>51</td>
<td>1991.5</td>
<td>21.4</td>
</tr>
</tbody>
</table>
Table 8: Comparison of results under normal conditions - Case A

<table>
<thead>
<tr>
<th>System</th>
<th>Methods</th>
<th>No. PMU</th>
<th>No. Switch</th>
<th>km</th>
<th>US$ x10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-30</td>
<td>proposed</td>
<td>10</td>
<td>11</td>
<td>625</td>
<td>6.7</td>
</tr>
<tr>
<td></td>
<td>method [16]</td>
<td>10</td>
<td>15</td>
<td>804.6</td>
<td>8.4</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>proposed</td>
<td>36</td>
<td>51</td>
<td>1991.5</td>
<td>21.4</td>
</tr>
<tr>
<td></td>
<td>method [16]</td>
<td>39</td>
<td>76</td>
<td>3012.6</td>
<td>31.7</td>
</tr>
</tbody>
</table>

5.2.2. Case B

This case considers different cost structure for PMU and communication media. Simulations were tested for normal operating conditions and PMU loss requirement and are presented in tables 9 and 10.

The results confirm previous trends that the number of PMUs for lower cost do not mean minimum number of PMUs. Comparing these results to related work [9] in Table 11 that the proposed method was more effective to get a lower total costs for a meter planning system in IEEE 118-bus. The cost obtained by the algorithm managed to lower costs by 14.9% under normal operating condition constraint and 21.9% for PMU loss requirement. Yet, the proposed algorithm obtained more PMUs for normal conditions, which is better for measures redundancy and state estimation process. The communication network and number of nodes involved are smaller in this methodology.

5.3. Optimization of Total Cost in Large Networks

The algorithm also is simulated to optimize total cost in large networks, such as IEEE 300-bus and BR 5804-bus. Distances between buses on IEEE 300-bus are set following the same procedure stated for smaller IEEE networks in pre-
<table>
<thead>
<tr>
<th>System</th>
<th>No. PMUs</th>
<th>PMUs</th>
<th>No. Switches</th>
<th>km (OPGW)</th>
<th>US$ (x10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-14</td>
<td>4</td>
<td>4, 5, 6, 9</td>
<td>5</td>
<td>199</td>
<td>1.0</td>
</tr>
<tr>
<td>IEEE-30</td>
<td>7</td>
<td>2, 4, 10, 12, 15, 20, 27</td>
<td>10</td>
<td>546.3</td>
<td>2.5</td>
</tr>
<tr>
<td>IEEE-57</td>
<td>16</td>
<td>1, 4, 10, 15, 20, 23, 28, 29, 31, 32, 36, 39, 41, 47, 49, 54</td>
<td>33</td>
<td>1476.7</td>
<td>6.5</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>32</td>
<td>3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 51, 54, 56, 61, 66, 70, 71, 75, 77, 80, 83, 86, 89, 92, 96, 100, 105, 110</td>
<td>49</td>
<td>1983.3</td>
<td>9.7</td>
</tr>
</tbody>
</table>

Previous section. For larger Brazilian network BR5804-bus the distance network is built in MATLAB® using a random number generator (rgn) seeded to a unit value and a randi function with mean equal to fourteen in order to get a total length network equal to 100,000 km, which is near the real value [30]. Results obtained have confirmed the influence of communication cost on resulting number of PMUs, who is in generally higher for cost optimization problem than for simple minimization OPP approaches (Table 2).

It must be remarked that different PMU and communication equipment costs could result in different results.
Table 10: Total Cost (PMU e CI) Optimization for PMU loss requirement, considering ZIB - Case B.

<table>
<thead>
<tr>
<th>System</th>
<th>Nr. PMUs</th>
<th>PMUs</th>
<th>Nr. Switches</th>
<th>km (OPGW)</th>
<th>US$ (x10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-14</td>
<td>7</td>
<td>2, 4, 5, 6, 9, 11, 13</td>
<td>8</td>
<td>376.7</td>
<td>1.8</td>
</tr>
<tr>
<td>IEEE-30</td>
<td>16</td>
<td>2, 3, 4, 6, 7, 10, 12, 13, 15, 17, 19, 20, 21, 24, 27, 30</td>
<td>19</td>
<td>1026.8</td>
<td>4.6</td>
</tr>
<tr>
<td>IEEE-57</td>
<td>30</td>
<td>1, 3, 4, 8, 9, 10, 12, 15, 19, 20, 22, 24, 27, 29, 30, 31, 32, 33, 35, 36, 41, 45, 46, 47, 49, 51, 53, 54, 56, 57</td>
<td>44</td>
<td>2347.4</td>
<td>10.3</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>64</td>
<td>1, 3, 5, 6, 8, 9, 11, 12, 15, 19, 21, 22, 24, 25, 27, 29, 31, 32, 34, 36, 37, 40, 42, 44, 45, 48, 49, 51, 52, 54, 56, 57, 59, 62, 65, 66, 69, 70, 71, 75, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 102, 105, 107, 109, 110, 111, 112, 115, 117, 118</td>
<td>80</td>
<td>3016.0</td>
<td>14.5</td>
</tr>
</tbody>
</table>
### Table 11: Comparison of results in IEEE-118-bus - Case B

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th>Methods</th>
<th>No. of PMU</th>
<th>No. of Switch</th>
<th>Km (OPGW)</th>
<th>US$ (x10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>proposed</td>
<td>32</td>
<td>49</td>
<td>1983,3</td>
<td>9,7</td>
</tr>
<tr>
<td></td>
<td>method [9]</td>
<td>30</td>
<td>60</td>
<td>2428,0</td>
<td>11,4</td>
</tr>
<tr>
<td>Loss of PMUs</td>
<td>proposed</td>
<td>64</td>
<td>80</td>
<td>3016,0</td>
<td>14,5</td>
</tr>
<tr>
<td></td>
<td>method [9]</td>
<td>64</td>
<td>88</td>
<td>3799,7</td>
<td>18,5</td>
</tr>
</tbody>
</table>

### Table 12: Total cost optimization in large networks

<table>
<thead>
<tr>
<th>System</th>
<th>No. of PMU</th>
<th>No. of Switch</th>
<th>Km (OPGW)</th>
<th>US$ (x10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 300</td>
<td>91</td>
<td>152</td>
<td>7862.4</td>
<td>35.9</td>
</tr>
<tr>
<td>BR 5804</td>
<td>2004</td>
<td>2887</td>
<td>18,220</td>
<td>156.7</td>
</tr>
</tbody>
</table>
6. Conclusions

A new methodology for total cost optimization in WAMS considering PMUs allocation and communication infrastructure costs was presented in this paper. Firstly, it was validated a simple optimal PMU placement algorithm based on Variable Neighborhood Search metaheuristic, which was never been used before in related PMU placement problems.

A new graph theory metric proposed has played a fundamental role.

This first algorithm proved to be flexible, efficient and scalable, in which a new graph theory metric proposed has played a fundamental role.

Next, a total cost optimization problem was introduced and the previous algorithm was adapted and extended to solve this problem.

The algorithm proved its simplicity and flexibility to consider several operating conditions and real-life cost parameters for PMU and communication infrastructure, could manage simulations in large and complex networks and ended up with good solutions, compared to previous works.

Acknowledgment

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References


