Predicting radar attenuation within the Antarctic ice sheet

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ABSTRACT

To better understand the ability of ice-penetrating radar to diagnose the subglacial environment from bed(returned power, we model the englacial radar attenuation of Antarctic ice. First, we use a one-dimensional thermo-mechanical model to evaluate the sensitivity of the depth-averaged attenuation rates to ice temperature as a function of surface accumulation rate, geothermal flux, and ice thickness. We find that attenuation is most sensitive to variations in geothermal flux and accumulation rate when the bed temperature is close to the pressure-melting point. But even if geothermal flux and accumulation rate remain fixed, attenuation can easily vary with ice thickness. Such high sensitivities show that one should not assume a uniform attenuation rate in the radar data analysis. Then, using ensembles of modeled ice temperatures with different boundary conditions, we generate multiple attenuation predictions for the Antarctic ice sheet and evaluate the resulting uncertainties. The largest contributor to uncertainty in these predictions is the geothermal flux. This uncertainty is localized within the deeper half of the ice sheet. By combining these temperature ensembles with ice-core chemistry data, we show that the sea salt adds little to the attenuation, but the contribution from acids accounts for ~29% (inland) to ~53% (coast) of the total attenuation. We conclude that improving radar diagnosis of the subglacial environment using bed(returned power requires both (1) better data-interpretation algorithms that account for attenuation variations and (2) better constraints of geothermal flux and bulk chemistry.

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1. Introduction

Satellite observations of anomalous changes in ice-sheet surface elevation led to a paradigm shift in our understanding of the Antarctic subglacial environment (Gray et al., 2005; Wingham et al., 2006; Fricker et al., 2007). Subglacial hydraulic networks are now understood to transfer water on the timescales of months to years, and active subglacial lakes have variable linkages between them (Smith et al., 2009). Most such lakes lie upstream of the fast-flowing glaciers that discharge much of the Antarctic ice into the oceans (Stearns et al., 2008; Rignot et al., 2011). Field observations of spatiotemporal variations of the subsurface of the ice sheets are thus crucial for understanding the dynamics and future sea-level contributions of the Antarctic ice sheet.

Ground-based and airborne ice-penetrating radar has been used widely to characterize the basal interface. Spatial variations of bed(returned power have been accepted as an indication of a distinct variation of the bed conditions; higher returned power is taken to indicate either a subglacial lake (e.g., Siegert et al., 2005; Carter et al., 2007, 2009) or a thawed bed (Bentley et al., 1998; Fujita et al., 2012; Rippin et al., 2004; Jacobel et al., 2009, 2010). However, the bed(returned power is attenuated by the ice overburden that the radio waves travel through, and recent studies have demonstrated that the attenuation rates in ice sheets are rarely uniform (Matsuoka, 2011; MacGregor et al., 2012). These studies explored attenuation rates for only a narrow range of glaciological conditions, so our understanding of radar attenuation across Antarctica remains incomplete. Thus, assessing the spatial variability of englacial radar attenuation is a central problem facing the use of radar for determining the nature of the subglacial environment.

Here, we first analyze the sensitivity of attenuation upon the ice-sheet boundary conditions and then make predictions of attenuation throughout the entire present-day Antarctic ice sheet. We examine the sensitivity of attenuation by first calculating just the pure-ice contribution using depth profiles of ice temperature from a one-dimensional model. We then predict the continent-wide spatial variation of the pure-ice contribution to the englacial attenuation using temperature outputs from a suite of thermo-mechanical model ensemble runs of the ice sheet. Finally,
we explore how the ice chemical composition affects ice-sheet-wide variations of the attenuation.

2. Background and methods

2.1. Effects of englacial attenuation on bed-returned power

The returned power from the bed $P$ depends on bed reflectivity $R$ as well as ice properties integrated along the two-way travel path to the bed. The three major travel-path effects on the radio-wave propagation are geometric spreading $G$, dielectric attenuation $L$ and signal reduction $B$ due to birefringence (Matsuoka et al., 2010). In the decibel scale ($|x|_{dB}=10 \log_{10}|x|$), the bed-returned power is

$$|P|_{dB} = |S|_{dB} - |G|_{dB} + |R|_{dB} - |L|_{dB} - |B|_{dB},$$

(1)

where $S$ represents instrumental factors.

The term $G$ is a function of radar height $h$ above the surface, ice thickness $H$, and the depth-averaged relative permittivity $\varepsilon$ ($\sim 3.2$):

$$|G|_{dB} = 2h + H/\sqrt{\varepsilon_{dB}}.$$  

When $H$ greatly exceeds the firm-layer thickness ($\sim 100$ m), density variations which alter the permittivity affect $G$ by less than 0.05 dB (Matsuoka et al., 2010). Therefore, it is common to calculate the geometrically corrected returned power $P^{C}$ from the measured returned power by subtracting the geometric spreading for a uniform ice.

$$|P|^{C}_{dB} = |P|_{dB} + |G|_{dB} = |S|_{dB} + |R|_{dB} - |L|_{dB} - |B|_{dB}.$$  

(2)

Birefringent effects $B$ vary as a function of the magnitude of anisotropy in the ice-crystal alignments (ice fabrics), and the principal axes of ice fabrics. Over the full range of these variables, the probability that $B$ varies more than 5 dB and 10 dB is 12% and 3%, respectively (Fujita et al., 2006). Moreover, the principal axes of the crystal alignments have known relations to surface strain rates in central West Antarctica (Matsuoka et al., 2012a), so proper survey design can mitigate the confounding effect of birefringence. Therefore, $P^{C}$ is mostly a function of bed reflectivity and englacial attenuation.

When the bed is smooth and flat relative to the wavelength of the radio wave in ice ($\sim 1-100$ m), the bed reflectivity can be approximated by the Fresnel reflectivity. Contrasts in Fresnel reflectivity between thawed and frozen ice-sheet beds are $\sim 10-15$ dB (Peters et al., 2005). This range is thus an upper limit to the uncertainty in the englacial two-way attenuation $L$, with which $P^{C}$ can become a proxy for the bed reflectivity $R$. To constrain $R$ within 10 dB, the one-way depth-averaged attenuation rate $\langle N \rangle = |L|_{dB}/(2H)$ should be known to better than 2.5 or 1.7 dB/km when the ice is 2- or 3-km thick, respectively. Otherwise, variations in the bed-returned power begin to be dominated by uncertainty in attenuation.

2.2. Attenuation parameterization

The one-way depth-averaged attenuation rate $\langle N \rangle$ is determined from the depth profile of the one-way local attenuation rate $N(z)$:

$$\langle N \rangle = \frac{|L|_{dB}}{2H} = \frac{1}{2H} \int_{0}^{H} N(z) dz.$$  

(3)

The one-way local attenuation rate $N$ (dB/km) is proportional to local ice conductivity $\sigma$ (Winebrenner et al., 2003)

$$N(z) = \frac{1000(10 \log_{10} e)}{e_{0} \sqrt{\varepsilon_{0}}} \sigma(z) \approx 0.914 \sigma(z),$$  

(4)

where $e_{0}$ is the permittivity of free space.

Ice conductivity in polar ice sheets depends on contributions from pure ice (subscript $i=0$) and two chemical constituents: acidity ($i=1$) and sea salt ($i=2$). Each component has an Arrhenius-type dependence on ice temperature $T$ (Moore and Fujita, 1993; MacGregor et al., 2007):

$$\sigma = \sum_{i=0}^{2} \sigma_{i}^{0} C_{i} \exp \left[ - \frac{E_{i}}{k} \left( \frac{1}{T} - \frac{1}{T_{r}} \right) \right],$$  

(5)

where $\sigma_{i}^{0}$ is the pure-ice conductivity ($i=0$) or molar conductivities ($i=1, 2$) at the reference temperature $T_{r}=251$ K. Acid and sea salt contributions depend linearly on their radar-effective concentrations $C_{1}$ and $C_{2}$, which depend on in-situ conditions of soluble ions at the ice temperature (see MacGregor et al. (2007) for details). Here $C_{0}$ is defined as unity, $E_{i}$ are activation energies corresponding to individual components, and $k$ is the Boltzmann constant. $\sigma_{i}^{0}$ and $E_{i}$ are frequency independent over the HF and VHF ranges (Fujita et al., 2000). For this study, we used $\sigma_{0}^{0} = 9.2 \, \mu S/m$, $\sigma_{1}^{0} = 3.2 \, S/m/M$ (where M= mol/L), $\sigma_{2}^{0} = 0.43 \, S/m/M$, $E_{1} = 0.51 \, eV$, $E_{2} = 0.20 \, eV$, and $E_{2} = 0.19 \, eV$ (MacGregor et al., 2012). Fig. 1a shows the temperature dependencies of the pure-ice and acid contributions to the local attenuation rate $N$ calculated using Eqs. (4) and (5).

2.3. Ice-temperature modeling

Estimating depth-averaged attenuation rates $\langle N \rangle$ using Eqs. (3)-(5) requires estimates of depth profiles of ice temperature. For different purposes, we modeled ice temperature in two ways.

First, to examine the sensitivity of depth-averaged attenuation rates on ice-sheet boundary conditions, a steady-state one-dimensional heat flow model is coupled with a kinematic ice-flow model (Matsuoka, 2011; Morse et al., 2002). The kinematic model approximates the depth profiles of horizontal ice velocity with two piecewise linear functions. The thickness of the bottom
softer layer in the model is assumed to be $0.2H$, a value that gives a good approximation at flank sites at least one ice thickness away from an ice-flow divide (Neumann et al., 2008). This approximation is valid for most of inland Antarctica. Horizontal advection is not included, and all boundary conditions are kept steady in each experiment. To assess the spatial variations of attenuation rates across the Antarctic ice sheet, we use the temperature fields from Pattyn (2010), which had been calculated using a three-dimensional, hybrid, steady-state, thermo-mechanically coupled ice sheet model. The ice temperature field is calculated at 5-km horizontal resolution with 21 layers in the vertical. Within the ice shelves, ice temperature is determined analytically by accounting only for vertical heat conduction and advection, and by balancing the basal and surface mass-balance rates. For the remainder of the ice sheet, ice temperature is calculated numerically by accounting for heat advection, vertical heat diffusion and internal strain heating. Ice-velocity fields are set to maintain the balance fluxes, and depth profiles of the velocity fields are consistent with the dominant horizontal stress gradients over subglacial lakes and ice streams. The horizontal stress gradients are taken into account only in fast-flowing areas ($> 100$ m/a). Basal sliding over grounded ice is calculated when the bed is thawed. See Pattyn (2010) for more details.

2.4. Boundary conditions for continent-wide temperature fields

Englacial temperatures vary as a function of ice thickness $H$, surface accumulation rate $\dot{A}R$, surface temperature $T_s$, and geothermal flux $GF$. Example estimates of these parameters for the current Antarctic ice sheet are shown in Fig. 2. Except for the Antarctic Peninsula and the coastal area, $H$ generally exceeds 2 km. We examine attenuation mainly for such inland ice because spatial variations in attenuation rates are more likely to hamper diagnosis of bed conditions there (Eq. (3)).

For investigations using the one-dimensional model, we varied the ice thickness $H$ from 2 to 4 km, the surface accumulation rate $\dot{A}R$ from 0.02 to 0.5 m/a, and the geothermal flux $GF$ from 30 to 120 mW/m$^2$. The portion of Antarctica where the ice thickness exceeds 2 km is roughly the same as that where the surface temperature $T_s$ is below $-30$ $^\circ$C, so we assumed surface temperatures of $-30$ and $-40$ $^\circ$C.

The thermo-mechanically coupled ice sheet model was run for 24 sets of boundary conditions, keeping ice thickness and surface temperature at the current values (Fig. 2a). The predicted bed conditions for these model runs are presented by Pattyn (2010). Surface accumulation rate is obtained from Van de Berg et al. (2006), based on the output of a regional atmospheric model for the past 25 yr (accumulation rate dataset AR0). For this study, we also used two additional accumulation-rate datasets (AR1 and AR2) to account for decadal anomalies in regional patterns of accumulation (Monaghan et al., 2006); these datasets constitute the uncertainty in AR0. Because little is known about geothermal flux in Antarctica, we ran ensemble models with eight geothermal flux fields. The six geothermal flux fields are based on Fox-Maule et al. (2005) (FM), Shapiro and Ritzwoller (2004) (SR), and Pollard et al. (2005) (P); FM, SR, P. (FM + SR)/2, (SR + P)/2, and (FM + SR + P)/3. Because FM and P have similar spatial patterns, we do not consider (FM + P)/2. In addition, we used two uniform values (42 and 54 mW/m$^2$).

Fig. 2. Maps of (a) ice thickness, (c) surface accumulation rate (Van de Berg et al., 2006), and (e) geothermal flux (Fox-Maule, et al., 2005) in the present-day Antarctic ice sheet as compiled by Le Brocq et al. (2010). The geothermal flux excludes areas over ice shelves. The lower row shows normalized (%) histograms for (a), (c), and (e) together with the color legends. For (d) and (f), the histogram data are for areas where ice is thicker than 2 km only, which is bound by black contours in panels (c) and (e). The dashed curve in panel (f) shows the histogram for another geothermal flux dataset (Shapiro and Ritzwoller, 2004). In (a), contours show the surface temperatures of $-30$ $^\circ$C (red) and $-40$ $^\circ$C (white) (Comiso, 2000).
the numerical values of which are frequently used as references in continent-scale Antarctic and Greenland ice-sheet modeling studies (e.g., Payne et al., 2000; Huybrechts, 2002; Pattyn, 2003).

Boundary conditions are tuned so that the model predicts a thawed bed over known subglacial lakes and ice streams and also replicates depth profiles of the ice temperature measured at deep boreholes (Pattyn, 2010). For this tuning, Pattyn (2010) treated all ice streams and three lake types as thawed: definite lakes, which return a radar power at least 2 dB larger than their surroundings (Carter et al., 2007); fuzzy lakes, defined by high absolute and relative bed reflectivity but are not specular (Carter et al., 2007); and active lakes, which cause ice-surface vertical motions detected by the satellite altimetry (Smith et al., 2009). The model is run for steady-state conditions, but the real ice-temperature field may be transient. Adjusting the boundary conditions to replicate the ice temperature measured at deep boreholes also accounts for some of the effects of long-term evolution of the ice sheet. The ensemble mean of these 24 experiments shows that the base of 55% of the Antarctic ice sheet (excluding ice shelves) is at the pressure-melting point (Pattyn, 2010).

3. Results

Depth profiles of temperature and its ice-sheet-wide spatial variation are put into Eq. (5) so we can use Eqs. (3) and (4) to estimate local attenuation rates and depth-averaged attenuation rates. We first consider the pure-ice contribution only (i.e., we explicitly neglect the chemistry contributions), and later assess chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects. This approach is undertaken because the pure-ice contribution dominates the attenuation rates (Fig. 1a) and chemical effects.
3.2. Ice thickness dependence

that the bed is thawed (e.g., AR < 0.05 m/a when H = 2 km, GF = 60 mW/m²), and when AR is large enough to keep the bed well below the PMP (e.g., AR > 0.1 m/a when H = 2 km, GF = 60 mW/m²). Between these two regimes, ⟨N⟩ is at least ~1.5 times more sensitive to the surface accumulation rate. As an extreme example when H = 2 km and GF = 60 mW/m², ⟨N⟩ varies approximately 1 dB/km per 1 cm/a variation in the surface accumulation rate. This variation corresponds to variations of ~8 dB in the two-way attenuation L for 2-km-thick ice, when the surface accumulation rate varies between 6 and 8 cm/a.

Fig. 5 shows the geothermal flux dependence d⟨N⟩/dGF. ⟨N⟩ varies ~2 dB/km for a geothermal flux variation of 10 mW/m². And ⟨N⟩ varies more when the surface temperature is higher and the ice is thicker.

3.3. Continent-wide distribution

3.3.1. Ensemble-mean field

Using the mean ice temperature from the 24 model experiments, we estimated the spatial distribution of the pure-ice contribution to ⟨N⟩. (The mean basal temperature corresponding to this ice-temperature field is shown in figure 3a of Pattyn (2010)). Fig. 7 shows that ⟨N⟩ is more than ~30 dB/km in the Siple Coast and Antarctic Peninsula regions, as well as in several coastal regions in East Antarctica. Most of the coastal margin also has a similarly high attenuation rate (~25 dB/km), although the Ross and Ronne–Filchner ice shelves have smaller ⟨N⟩ values (15–20 dB/km) than most of the smaller ice shelves. The majority of the West Antarctic Ice Sheet has intermediate ⟨N⟩ values (10–15 dB/km). Variations in ⟨N⟩ over inland East Antarctica are noticeably smaller than the other regions (~10–15 dB/km). However, ⟨N⟩ can still vary more than 2–3 dB/km over a horizontal distance of 50–100 km. Such local variability agrees with more detailed attenuation modeling along the Vostok flowline (MacGregor et al., 2012). Since ice is thicker than 3 km in most of East Antarctic inland (Fig. 2a), these relatively small variations in ⟨N⟩ can easily result in more than 10-dB differences in predicted L over horizontal distances of 50–100 km. In the inland regions, ice flows more uniformly than that in the coastal regions, which may give the false impression that the attenuation rates vary little. However, the inland combination of a modest uncertainty in ⟨N⟩ with thick ice makes predictions of L challenging.
subglacial lakes and ice streams (Fig. 7b). In contrast, grounded ice that is neither fast-flowing nor over a subglacial lake has only
\[ 7.3 \text{ dB/km}, \]
with a distinct median value of 9.2 dB/km. For grounded ice thicker than 2 km, attenuation rates are even lower
\[ 3.0 \text{ dB/km}. \]

3.3.2. Prediction uncertainty

The continental-mean value of \( \langle N \rangle \) is consistently \( \sim 15\)–16 dB/km for nearly all boundary conditions (Table 1). But when we applied a uniform geothermal flux of 42 mW/m², the value was only 14 dB/km. The standard deviation of \( \langle N \rangle \) is roughly 6 dB/km for all datasets.

To examine effects of uncertainty from the surface accumulation rate, separate from that of geothermal flux, we calculated the depth-averaged attenuation in the upper half of the ice sheet. In shallow ice, variations in the surface accumulation rate dominate the temperature variations. Over the entire ice sheet and ice shelves, the range of the attenuation rates \( \langle N \rangle \) varied from 0.2 to 0.3 dB/km (Fig. 8a). Even where the ice is 4-km thick, the variation over 0.52H yields variations in \( \langle N \rangle \) less than about 1 dB, which is much smaller than the other uncertainties. Therefore, these

subglacial lakes and ice streams (Fig. 7b). In contrast, grounded ice that is neither fast-flowing nor over a subglacial lake has only 10.7 ± 7.3 dB/km, with a distinct median value of 9.2 dB/km. For grounded ice thicker than 2 km, attenuation rates are even lower at 8.5 ± 3.0 dB/km.

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![Fig. 6. Relationships between depth-averaged attenuation rate \( \langle N \rangle \) and ice thickness. (a) \( \langle N \rangle \) for five cases of geothermal flux \( GF \) and surface accumulation rate \( AR \). (b) Values of \( GF \) and \( AR \) for the five cases and the range (maximum–minimum) of \( \langle N \rangle \) for \( H = 2\)–3 km. (c) Same as (b) but for the range of 2.5–3.5 km. The cases are as follows: (A) bed is always frozen, (B, C, D) bed alternates from frozen to thawed as ice becomes thicker, and (E) bed is always thawed. The white curves in (b) and (c) show the boundary between the following two domains: the bed is always frozen regardless of the ice thickness (left top); the bed becomes thawed at a certain ice thickness or is always thawed (right bottom). Surface temperature is \( \sim -40^\circ C \) for all cases.](image)

![Table 1 Mean and standard deviation of depth-averaged attenuation rates (dB/km, one way) over the Antarctic ice sheet and ice shelves.](table)

<table>
<thead>
<tr>
<th>Geothermal flux dataset</th>
<th>Surface accumulation rate dataset</th>
<th>AR0*</th>
<th>AR1</th>
<th>AR2</th>
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</tr>
<tr>
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<td>15.1 ± 6.0</td>
<td>15.1 ± 6.0</td>
<td></td>
</tr>
</tbody>
</table>

* See Section 2.4 for definitions of these surface accumulation rate and geothermal flux dataset labels.

![Fig. 7. Depth-averaged attenuation rates \( \langle N \rangle \) derived from the ensemble-mean ice temperature field. (a) Map. (b) Histogram and color code for (a). The histogram is divided into areas overlying subglacial lakes and ice streams (6% of the entire model domain), other grounded ice (83%), and ice shelves (11%). Only the pure-ice contribution is considered.](image)
3.4. Effects of ice chemistry

3.4.1. Relative importance of acidity and sea-salt contributions

We next consider the acid and sea-salt contributions to the attenuation. Activation energies for these contributions are roughly 40% of that of the pure-ice contribution, so their relative contributions are larger at lower temperatures (Fig. 1a); the ratio of the acid contribution relative to the sea-salt contribution can be described as

$$\alpha = \frac{\sigma_{0,\text{acid}}}{\sigma_{0,\text{sea-salt}}}$$

where $\sigma_{0,\text{acid}}$ and $\sigma_{0,\text{sea-salt}}$ are the activation energies for the acid and sea-salt contributions, respectively. The ratio $\alpha$ is a measure of the relative importance of the acid and sea-salt contributions to the attenuation. When $\alpha > 1$, the acid contribution is more important, and vice versa.

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The molar conductivity for acidity $\sigma_a$ is $\sim 7.4$ times larger than that for sea salt and activation energies for these two components are similar. So, $\alpha$ is roughly 7.4 over the full temperature range in the ice sheet (Fig. 1b). Therefore, for equivalent concentrations, the sea-salt contribution to the local attenuation rate $N$ is only $\sim 14\%$ of the acid contribution.

### 3.4.2. Continent-wide distribution of acidity and sea salt

To explore the effect of a spatially varying chemical composition on the attenuation rates, we examined the relationship between ice-core chemistry data and site conditions. We first derived radar-effective acidity and sea-salt concentrations using soluble-chemistry and ice-temperature data from the ice cores drilled at Siple Dome (MacGregor et al., 2007), South Pole (Cole Dai, 2004), Taylor Dome (Mayewski et al., 1996), and Vostok (MacGregor et al., 2009); See MacGregor et al. (2007, 2009) for more details on the temperature and chemical datasets for Siple Dome and Vostok, respectively, as well as methods to derive radar-effective values at all of these sites. Depth-averaged radar-effective acidities (salinities) at these sites are 1.2 $\mu$M (4.1 $\mu$M) at Siple Dome, 1.7 $\mu$M (0.6 $\mu$M) at the South Pole, 1.6 $\mu$M (0.7 $\mu$M) at Taylor Dome, and 0.5 $\mu$M (2.0 $\mu$M) at Vostok.

Ionic concentrations in the near-surface snow depend on surface elevation, distance from the coast, surface accumulation rate, and surface temperature (Bertler et al., 2005). We also found that the depth-averaged acidity and salinity at the ice-core sites depend on these parameters. Acidity correlates the strongest to the surface accumulation rate ($r = 0.41$), whereas salinity correlates the strongest to surface elevation ($r = 0.55$). We used these correlations to approximate the depth-averaged acidity and salinity across the Antarctic ice sheet. The resulting acidity ranges between 0 and 35 $\mu$M (contours for 1–3 $\mu$M in Fig. 9b), whereas the salinity ranges between 0.1 and 4.2 $\mu$M. In terms of attenuation rate, the salinity range is equivalent to acidities of only 0–0.6 $\mu$M. Moreover, the higher salinities are predicted at low surface elevations, where higher ice temperatures typically reduce the sea salt contribution to attenuation compared to that from pure ice (Fig. 1a). Therefore, we ignore the contribution to attenuation from sea salts and focus on the contribution from acids. This approximation is more appropriate in the inland elevated area with thicker ice, which is our region of primary interest.

More sophisticated methods have been developed to estimate the effects of chemistry upon attenuation, such as that applied to a Vostok flowline (MacGregor et al., 2012). Such methods require spatial patterns of (modeled or radar-observed) englacial isochrones and knowledge of the surface chemistry over space and time. However, englacial isochrones have not yet been mapped across the entire Antarctic ice sheet and relatively few ice cores exist. We therefore consider results from our present attenuation model to be a first-order prediction for the whole of the ice sheet.

### 3.4.3. Spatial pattern of the acid contribution

First, we assume that acidity is uniformly 1 $\mu$M and calculate the fractional contribution from $\langle N_{\text{acid}} \rangle$ to that of pure-ice $\langle N_{\text{pure}} \rangle$ for the ensemble-mean temperature field. Because acidity and pure-ice components have similar magnitudes at low temperatures (Fig. 1a), acids have a relatively high contribution in areas where the ice sheet is relatively cold and thin (Fig. 9a). In inland East Antarctica, shallow ice is cold (Fig. 2a) but the thickness of warmer, deep ice is significant, making the contribution from acidity relatively small.

Next, we calculated the ratio $\langle N_{\text{acid}} \rangle/\langle N_{\text{pure}} \rangle$ for the spatially variable acidity contribution estimated from ice cores and the ensemble-mean temperature field. In coastal areas, where acidity often exceeds 2–3 $\mu$M, the contribution from acid is predicted to be comparable to that from pure ice (Fig. 9b). For inland regions, the acidity contribution varies between 10% and 30% of the pure-ice contribution in East Antarctica, and 30% and 70% in West Antarctica. Comparison of Fig. 9a and b shows that both ice temperature and acidity distribution similarly affect the continent-wide distribution of the $\langle N_{\text{acid}} \rangle/\langle N_{\text{pure}} \rangle$ ratio. We also found that, in the upper half of the ice sheet where ice is relatively cold, $\langle N_{\text{acid}} \rangle/\langle N_{\text{pure}} \rangle$ is often as large as $\langle N_{\text{pure}} \rangle^{0.42}$ (not shown in figures), but the magnitude is much smaller than that at greater depths. On average, 16%–68% of acid-origin attenuation occurs in the deeper half of the ice sheet, whereas the remaining 32%–84% is localized in the shallower half of the ice sheet. Therefore, regardless of the depth, knowledge of englacial chemistry for the entire ice column is important.

In the real ice sheet, the total attenuation equals the sum of the pure-ice and chemical contributions: $\langle N_{\text{total}} \rangle = \langle N_{\text{pure}} \rangle + \langle N_{\text{acid}} \rangle$.
ments are needed especially to thoroughly constrain the pure-ice zone (Matsuoka et al., 2012b). Nevertheless, the three-field has been found not to vary abruptly across the shore of a temperature field and thus the attenuation field. The attenuation advection of heat reduces the horizontal gradients in the dimension of the attenuation to the boundary conditions when the bed is modest variability of Jacobel et al., 2009). Similarly, we estimated that, on average, $\langle N_{\text{Acid}} \rangle = 0.29 \langle N_{\text{Total}} \rangle$ in the inland area, where the acidity is less than 2 $\mu$M, and $\langle N_{\text{Acid}} \rangle = 0.53 \langle N_{\text{Total}} \rangle$ in the coastal area where the acidity exceeds 2 $\mu$M.

4. Discussion

The results presented here largely depend on the accuracy of the dielectric parameterization (Eq. (5), Fig. 1). Unfortunately, measurements are particularly sparse above $-10^\circ C$ and measured ice conductivities vary widely. A standard deviation between such measurements (table 1 in MacGregor et al., 2007) can result in attenuation difference equivalent to 2–4 $\mu$M variations in the acidity (Fig. 1a). Therefore, conductivity measurements are needed especially to thoroughly constrain the pure-ice contribution at temperatures above $-10^\circ C$.

Our one-dimensional modeling illustrates the high sensitivity of the attenuation to the boundary conditions when the bed is barely frozen (Figs. 3–5). In the real ice sheet, horizontal advection of heat reduces the horizontal gradients in the temperature field and thus the attenuation field. The attenuation field has been found not to vary abruptly across the shore of a subglacial lake (MacGregor et al., 2012) or across a grounding zone (Matsuoka et al., 2012b). Nevertheless, the three-dimensional modeling presented here shows that a combination of the modest variability of $\langle N \rangle$ and thick ice in inland regions can easily compromise bed diagnoses from radar (Fig. 7). Therefore, one can potentially interpret an abrupt change in the bed(returned power over distances of several ice thicknesses as delineating a boundary between a thawed and a frozen bed. But caution is needed especially when the bed is barely frozen, which can occur in the vicinity of ice streams, subglacial lakes, and grounding zones.

Gradual changes in the bed reflectivity can occur, for instance, when the basal-melt rate is small and melt water gradually accumulates through a subglacial hydraulic network (Wright et al., 2012). Describing such networks is crucial for understanding how subglacial water affects ice-sheet dynamics. When the bed(returned power varies more gradually over a longer horizon-This depth-averaged attenuation rate $\langle N \rangle$ by 2–8 dB/km (Fig. 4), thus changing $L$ by 12–48 dB for 3-km-thick ice. Such accumulation-rate variations can be inferred from depth patterns of englacial reflectors that have been widely accepted as isochrones (e.g., Waddington et al., 2007). Combined analysis of the radar returned power and radar stratigraphy can thus help avoid incorrect diagnosis of the bed conditions. However, geothermal-flux patterns remain largely unknown. When the geothermal-flux variations are large enough to compromise the bed diagnosis, associated basal melting rates are too small to clearly modify the depth patterns of the isochrones (Matsuoka, 2011). Therefore, it is virtually impossible to detect attenuation variations caused by geothermal flux except for extreme cases that have produced down-warping radar reflectors through anomalously large basal melting (e.g., Fahnestock et al., 2001).

Two algorithms have been used to estimate englacial attenuation and then bed reflectivity using radar-profiling data. The more widely-used algorithm assumes that the geometrically corrected returned power $P^2/Km$ linearly depends on ice thickness (e.g., Jacobel et al., 2009). Half of the least-square-sense gradient, $\langle P^2/Km \rangle/2$, is then interpreted as the regional-mean attenuation rate. The other algorithm uses spatial variations in the returned power of both the single- and the double-bounced echoes (MacGregor et al., 2011). For both methods, the ice-thickness range should be large enough that the two-way attenuation $L$ is significantly larger than both the noise and spatial variation in the bed reflectivity (Eq. (2)). For ice thicker than 2 km, $\langle N \rangle$ averages roughly 10 dB/km (Fig. 7), implying that the ice thickness must vary by at least several hundred meters to ensure a robust attenuation estimate. However, for an ice-thickness change of 0.5 km, $L$ in 3-km-thick ice could easily vary by 3–4.5 dB when the bed is either completely frozen or thawed and $-9$ dB when the bed is barely frozen (Fig. 6). Also, the bed is more likely to be thawed under thicker ice, so both the englacial attenuation and basal reflectivity are inherently related to the ice thickness, which further complicates robust estimation of attenuation using these methods.

Attenuation estimates based on the three-dimensional ice-temperature fields (Figs. 7–9) can be used to examine the first-order, continent-wide attenuation distributions. However, this method cannot be used alone to deduce the bed conditions from radar data, because the method that predict englacial attenuation good enough should adequately predict the basal conditions as well (so radar measurements are no longer necessary to constrain the bed conditions). In reality, the model predictions have significant uncertainties due to poor knowledge of both the boundary conditions and of the ice-sheet evolution. A recent observation in central West Antarctica shows that the depth-averaged attenuation rates in the top one third to half of the ice sheet vary by 5 dB/km over 120-km-long radar profiles across an ice-flow divide (Matsuoka et al., 2010). In that area, the surface accumulation rate has a large lateral gradient (Morse et al., 2002) and both ice-sheet evolution and ice chemistry probably add further uncertainty to the attenuation field.

Ultimately, understanding the subglacial environment in Antarctica will require a combination of methods. Common-midpoint radar surveys can be used to constrain ice temperature (Winebrenner et al., 2003). Seismic data have similar potential (Peters et al., 2012). Use of a rapid ice drilling system should provide opportunities to measure geothermal flux underneath the ice sheet and to potentially log depth profiles of low-resolution acidity using a wall-attached device, which are two of the primary contributors to uncertainty in attenuation predictions. Currently, radar stratigraphy and returned power are interpreted separately, but we have shown that there is a motivation to conduct these interpretations simultaneously. In practice, they can be made iteratively towards the best possible estimates of the glaciological conditions in space and time. In the future, we expect that formal inverse problems will be formulated to simultaneously interpret all aspects of the radar data as well as other geophysical and geochemical data.

5. Conclusions

The depth-averaged attenuation rate $\langle N \rangle$ is most sensitive to the surface accumulation rate and geothermal flux when the ice-sheet bed is barely frozen (i.e. basal temperature is close to the pressure melting point). Specifically, $\langle N \rangle$ varies by up to 1 dB/km for either a surface accumulation rate variation of 1 cm/a or a geothermal flux variation of 5 mW/m$^2$. These sensitivities are even higher when both the surface accumulation rate and geothermal flux are low. In contrast, when the bed is thawed, $\langle N \rangle$ becomes independent of geothermal flux and insensitive to the surface accumulation rate. As the ice thickens, $\langle N \rangle$ increases if the bed is frozen but decreases when the bed is thawed. These
trends can result in large local variations in $\langle N \rangle$, which can jeopardize the accuracy of estimates of regional-mean $\langle N \rangle$ from radar data.

Uncertainty in $\langle N \rangle$ over present-day Antarctica is dominated by uncertainty in the geothermal flux. In inland East Antarctica, $\langle N \rangle$ uncertainties can lead to more than 10-dB variations in the two-way attenuation to the bed, a value roughly equal to the expected range of bed reflectivity. These predictions are based on the pure-ice contribution only; the spatially variable acidity and salinity adds further uncertainty. Our estimates show that the acid contribution to attenuation equals $-$29% (inland) and $-$53% (coast) of the total attenuation, whereas the sea-salt contribution is negligible. The acid contributes nearly equally at all depths, so low-resolution ice chemistry data for the entire ice thickness is important for attenuation modeling. All of these attenuation predictions depend critically on the accuracy of conductivity parameterization; uncertainty in this parameterization results in uncertainty in the attenuation. More studies of dielectric measurements are needed, especially at temperatures above $-10^\circ C$, because warm ice contributes greater uncertainty to attenuation predictions than cold ice.

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