Stabilised profunctors between groupoids

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Abstract

We will examine the fundamental mathematical setting provided by the biequivalence between the compact closed bicategory **Prof** of profunctors (aka bimodules or distributors) between groupoids and natural transformations between them, and the 2-category of cocontinuous functors between categories of presheaves over groupoids (aka categories of groupoid actions) and natural transformations between them.

Our starting point is a basic representation theorem for presheaves over groupoids that leads to the consideration of groupoids with additional structure called *kits*. Kits have both combinatorial and logical character. From a combinatorial viewpoint, they serve to restrict presheaves to *stable* ones. From a logical perspective, we will consider a class of *Boolean kits*. These are drawn from Boolean algebras associated to groupoids by means of a general universal construction to be introduced and discussed. In this context, the dualities of profunctors and of Boolean algebras will be placed side by side to define a bicategory **SProf** of *stabilised profunctors* between Boolean kits and natural transformations between them. We shall show that **SProf** is $\star$-autonomous, with a projection onto **Prof** degenerating to its compact-closed structure, and that it is biequivalent to the 2-category of *linear functors* (those being left and right local adjoints) between categories of stable presheaves over Boolean kits and natural transformations between them.

The motivation and context for these investigations are developments in category theory (polynomial functors), structural combinatorics (species of structures), and theoretical computer science (linear logic). In particular, *symmetric-algebra* structure in this setting plays a fundamental role and, time permitting, will also be discussed.

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