

# RELATIVE AMENABILITY, AMENABILITY, AND COAMENABILITY OF COIDEALS

BENJAMIN ANDERSON-SACKANEY

Abstract:

Amenability is a deeply studied property of groups, with many interesting reformulations and connections to the operator algebraic aspects of groups. For example, the reduced  $C^*$ -algebra  $C_r^*(G)$  of a discrete group has a unique tracial state if and only if there are no non-trivial amenable normal subgroups. This, among other related results, makes it apparent that the structure of the amenable subgroups of  $G$  contains important information about  $C_r^*(G)$ . For a quantum group  $G$ , an appropriate analogue of a subgroup is a coideal  $N \subseteq L^\infty(G)$ . We will present notions of relative amenability, amenability, and coamenability for coideals of discrete and compact quantum groups motivated by "relativizations" of amenability and coamenability of a subgroup of a group. We will discuss the known relationships between these formally distinct notions and their relevance to certain properties of the reduced  $C^*$ -algebras of discrete quantum groups.