HOPF MONADS FROM LAX FUNCTORS

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Recently I constructed a monoidal bicategory $Span|V$ for any monoidal bicategory V. With the appropriate choice of V, many generalizations of Hopf algebras could be described as Hopf monads in $Span|V$. In the talk I will give a deeper explanation why this construction works.

Consider any category D as a bicategory with only identity 2-cells. For any bicategory V there is a bicategory of lax functors $D \to V$, lax natural transformations and modifications. We show that it naturally embeds into the bicategory of monads in $Span|V$. If moreover V is a monoidal bicategory then so is Span—V (but not the sub-bicategory of lax functors $D \to V$). Denoting by $OpMon(-)$ the bicategory of monoidales, opmonoidal 1-cells and opmonoidal 2-cells in a monoidal bicategory, $Span|OpMon(V)$ naturally embeds into $OpMon(Span|V)$. Thus a lax functor $D \to OpMon(V)$ induces a monad in $Span|OpMon(V)$ and therefore an opmonoidal monad in $Span|V$. We claim that the Hopf algebra-like structures which could be seen as Hopf monads in $Span|V$ all arise from suitable lax functors in this way.

For example, if D is an indiscrete category and V is a monoidal category (regarded as a bicategory with only one object) then a lax functor $D \to V$ is precisely a category enriched in V which (via the above embedding) can be seen as a monad in $Span|V$. If furthermore the monoidal category V is braided (that is, the corresponding one-object bicategory is monoidal), then the comonoids therein constitute a monoidal category $Cmd(V)$. The categories enriched in $Cmd(V)$ thus induce monads in $Span|Cmd(V)$ and therefore opmonoidal monads in $Span|V$. (This new point of view through lax functors does not seem to simplify, however, the proof that a $Cmd(V)$-enriched category is a Hopf category in the sense of Batista-Caenepeel-Vercrysse if and only if its image is a Hopf monad in $Span|V$.)