QUANTUM ROOT VECTORS AND A DOLBEAULT DOUBLE COMPLEX FOR THE A-SERIES QUANTUM FLAG MANIFOLDS

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Abstract:
In the 2000s a series of seminal papers by Heckenberger and Kolb introduced an essentially unique covariant q-deformed de Rham complex for the irreducible quantum flag manifolds. In the years since, it has become increasingly clear that these differential graded algebras have a central role to play in the noncommutative geometry of Drinfeld-Jimbo quantum groups. Until now, however, the question of how to extend Heckenberger and Kolb's construction beyond the irreducible case has not been examined. Here we address this question for the A-series Drinfeld-Jimbo quantum groups $U_q(\mathfrak{sl}_{n+1})$, and show that for precisely two reduced decompositions of the longest element of the Weyl group, Lusztig's associated space of quantum root vectors gives a quantum tangent space for the full quantum flag manifold $\mathcal{O}_q(F_{n+1})$ with associated differential graded algebra of classical dimension. Moreover, its restriction to the quantum Grassmannians recovers the q-deformed complex of Heckenberger and Kolb, giving a conceptual explanation for their origin. Time permitting, we will discuss the noncommutative Kähler geometry of these spaces and the proposed extension of the root space construction to the other series. (Joint work with P. Somberg.)