Aunt Agatha*

Aunt Agatha, an energetic British old lady, has two passions in live : flowers (as might be expected) and finance (this is more unusual). Besides her cottage, she owns a portfolio, the total value of which is Euro 100,000. Following the advice of her late husband, she has decided to restrict her investment to European securities. John Smith, the charming young man from Royal Corner Bank, has convinced her some time ago to adopt an asset allocation of 80% in a money market fund (invested in TBills) and of 20% in a unit trust invested in European common stocks, the Royal Unit Trust. This mutual fund follows a passive management strategy; its risk-return characteristics are representative of the European stock market as a whole.

Unfortunately, Aunt Agatha is no longer sure that this portfolio allocation is the best for her. She estimates that the current risk-return characteristics of the stock and TB markets are the following:

	Expected Return	Standard Deviation
European Stocks	14%	16%
Tbills	6%	0%

In order to identify her preferences, Aunt Agatha did a lot of reading. She first discovered that her reluctance for risk was shared by many investors and even had a name, risk aversion. Moreover, she was surprised to learn that a mathematical expression of her preferences was possible. She became an ardent admirer of Mrs Markowitz and Sharpe and rejoiced when they were awarded the Nobel prize in 1990.

She, of course, eagerly read their Nobel lectures[1]. She is now convinced that the utility function that she wishes to maximize can be expressed as:

$$U = \overline{R}_P - a\sigma_P^2$$

where $\overline{R}_P$ represents the expected return on her portfolio, σ_P is the standard deviation of her portfolio and *a* is her risk aversion. She interprets *U* as a risk-

* André Farber prepared this case as a base for class discussion.

[1] Reproduced in *The Journal of Finance* June 1991

adjusted expected return or, alternatively, as a certainty-equivalent return since a portfolio with an expected return of *U* and zero risk would have the same utility[2].

After much thought, Aunt Agatha has come to the conclusion that her risk aversion is equal to 2. As a consequence, she is now able to calculate the utility for any portfolio. For instance, a portfolio with a 10% expected return and 10% standard deviation would provide a utility equal to:

$$U = 0.10 - 2\,(0.10)^2 = 0.08 = 8\%$$

In other words, she would be indifferent between this portfolio and a riskless investment with a 8% return.

Aunt Agatha is now impatient to check whether John Smith was a good adviser. She has written down the questions to which she is eager to get an answer:

1. What are the risk-return characteristics of her current portfolio ?
2. Is her current portfolio preferable to a portfolio totally invested in Treasury bills ?
3. What is her investment opportunity set ? She would love to see, on a diagram, the different risk-expected returns that could be achieved by changing the asset allocation. She is also willing to find the equation of this line. What is its slope ? How to interpret this number ?
4. What should her risk aversion be in order for her current portfolio to be optimal? (hint: to answer this question, the slope of her indifference curve should be set equal to the slope to her investment opportunity set at the location of her current portfolio)
5. Given her risk aversion, what is her optimal asset allocation ?

Some more questions come to her mind. Why avoid bonds in her portfolio? The expected return on bonds is 8% with the standard deviation is 6%. After some calculations, Aunt Agatha found that the correlation coefficient between the returns on stocks and bonds is 0.2.

6. Suppose that she invests 25% in bonds and 75% in stocks. What would be the expected return and the risk on her portfolio?
7. Redo the same calculations with 50% and 75% invested in bonds.
8. Given the risk free interest rate, which combination of stocks and bonds should be choosen by Aunt Agatha?
9. Given her risk aversion, what is her optimal asset allocation, taking bonds into account?

[2] To learn more about this representation of preferences, read Sharpe, W. and Alexander, G. *Investments*, 4th ed., Prentice Hall 1990 (chap 22) or Sharpe, W. 1991, Capital Asset Prices with and without Negative Holdings *Journal of Finance*,46 489-509