

Present Value and Bond Math^{*}

- The risk-free interest rate is 6%. Consider the following risk-free bonds:
 - a 10-year zero-coupon bond that pays €100 at maturity;
 - a consol bond (a perpetuity) that pays 7€ each year;
 - a 10-year 7%-coupon bond with a face value of 100.
 - a 10-year indexed bond with a face value of 100. The next coupon is 7€. This coupon growth rate is equal to the inflation rate (2%)

For each bond, calculate:

- the present value
 - the expected value one year later
 - the expected return.
- Suppose that the interest rate is 11%. Calculate the future value (at the time of the last payment) of a series of 17 annuities of 680€.
- The current prices of €100 face-value zero-coupon bonds are:

Maturity	1	2	3	4
Market price	96.62	92.81	88.70	85.05

Consider a 4-year, 3-percent coupon bond with face value 100 that pays interest annually.

- Calculate the price of this bond.
 - Calculate the two-year spot interest rate.
 - How do you expect the price of this bond to change over time if interest rates do not change? Explain.
- Consider the three risk-free bonds in the table below. All the bonds have a face value of €100

	Bond Price	Year 1 Cash flow	Year 2 Cash flow
Bond A	€94.50	€100	€0
Bond B	€88.00	€0	€100
Bond C	€90	€8	€108

Assume that Bond A and Bond B are priced correctly

^{*} André Farber prepared this case as a base for class discussion.

1. What Bond C price would satisfy the law of one price? Are the bond prices consistent with the "law of one price"?
2. Assume that you can buy or sell any of the bonds at the price shown. How could you generate a profit?

5. Suppose that you observe the following bond price:

	Bond Price	Year 1 Cash flow	Year 2 Cash flow	Year 3 Cash flow
Bond A	€101.94	€105	€0	€0
Bond B	€103.85	€7	€107	€0
Bond C	€101.85	€8	€8	€108

Calculate the implicit discount factors and spot interest rates.

6. Calculate the Yield-to-Maturity (YTM) of the following bond:

Face Value: €100

Price: €98

Maturity: 2 years

Coupon rate (paid annually): 9 percent

7. Suppose that the interest rate is 5%. Consider the following zero-coupons:

	Maturity	Face value
Bond A	3-year	€100
Bond B	15-year	€100

1. Calculate the prices of these bonds.
2. Based on duration, by what percentage would you expect the price to change if the interest rate were to vary to 5.5%?
3. Check your result to the previous question based on the prices at the new interest rate. Explain why the results are different.

8. Consider a risk-free bond with the following characteristics:

Principal: €1,000

Term to maturity: 5 years

Coupon rate (paid annually): 7 percent

The risk-free interest rate is 5%

1. Calculate the price of this bond.
2. Calculate the expected return on the bond for the coming year.
3. Is the duration of these bonds greater than, equal to or lower than the duration of a 5-year zero-coupon? Explain.