ULB

Solvay Brussels School
of Economics and Management

Advanced Corporate Finance

5. Options (a refresher)

Objectives of the session

1. Define options (calls and puts)
2. Analyze terminal payoff
3. Define basic strategies
4. Binomial option pricing model
5. Black Scholes formula

ULB
Solvay Brussels School of Economics and Management

Definitions

A call (put) contract gives to the owner

- the ***right*** :
  - to buy (sell)
  - an underlying asset (stocks, bonds, portfolios, ...)
  - on or before some future date (maturity)
    - on : "European" option
    - before: "American" option

- at a price set in advance (the exercise price or striking price)

- Buyer pays a premium to the seller (writer)

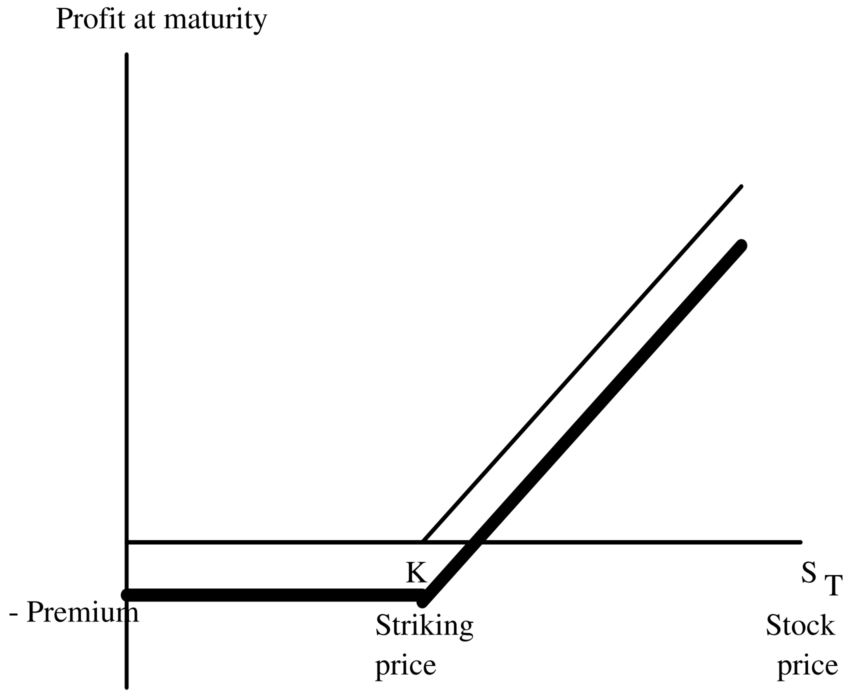
|3

Terminal Payoff: European call

- Exercise option if, at maturity:
  Stock price > Exercise price
  S_T > K

- Call value at maturity
  $C_T = S_T - K$ if $S_T > K$
  otherwise: $C_T = 0$

- $C_T = MAX(0, S_T - K)$

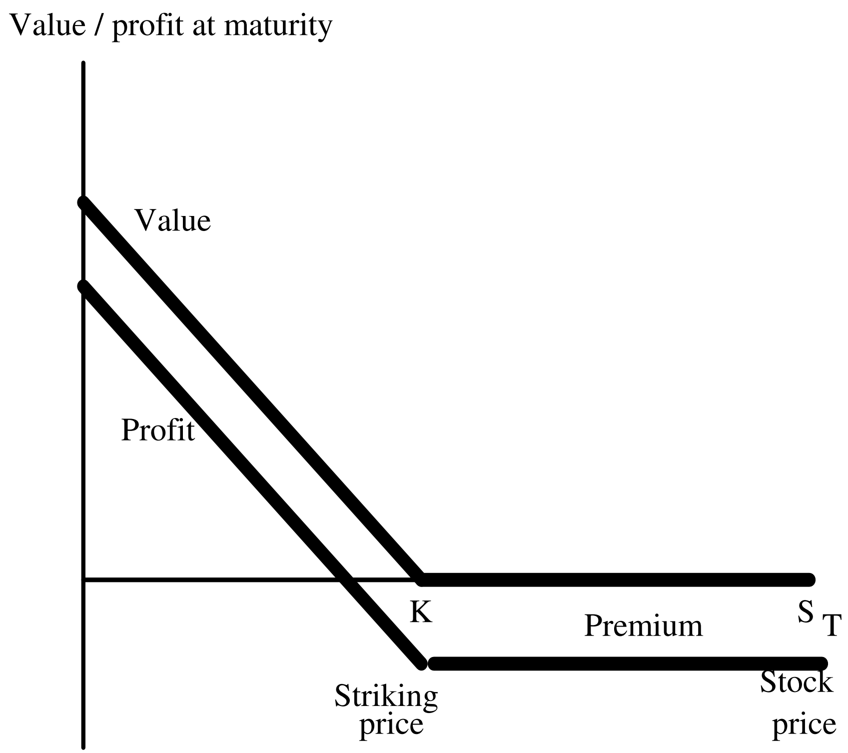


Terminal Payoff: European put

- Exercise option if, at maturity:
  Stock price < Exercise price
  S_T < K

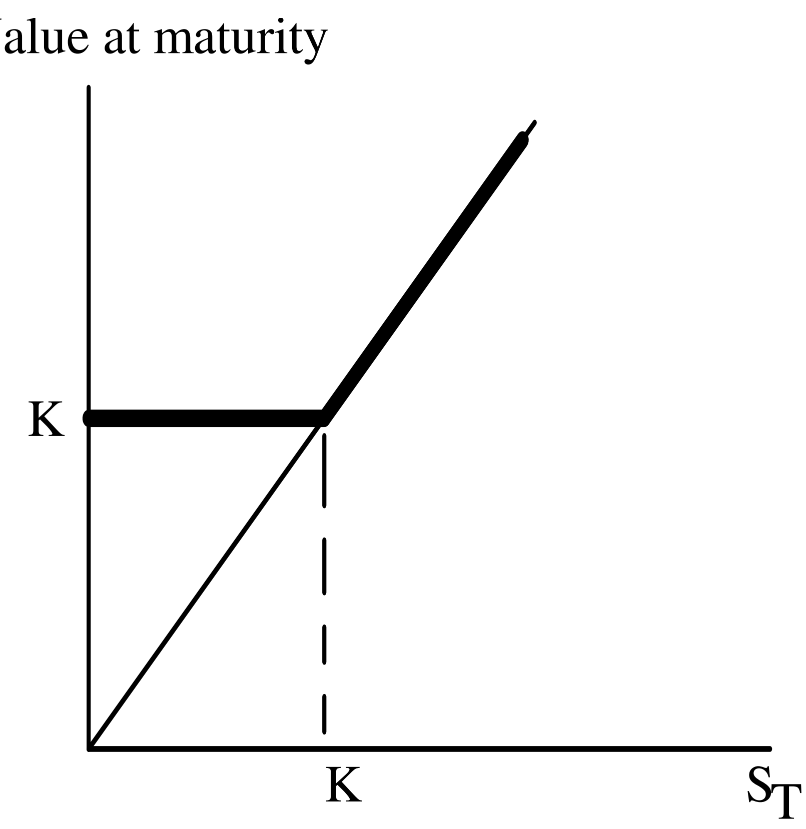
- Put value at maturity
  $P_T = K - S_T$ if $S_T < K$
  otherwise: $P_T = 0$

- $P_T = MAX(0, K - S_T)$



The Put-Call Parity relation (1/3)

- A relationship between *European* put and call prices on the same stock
- Compare 2 strategies:



- Strategy 1. Buy 1 share + 1 put

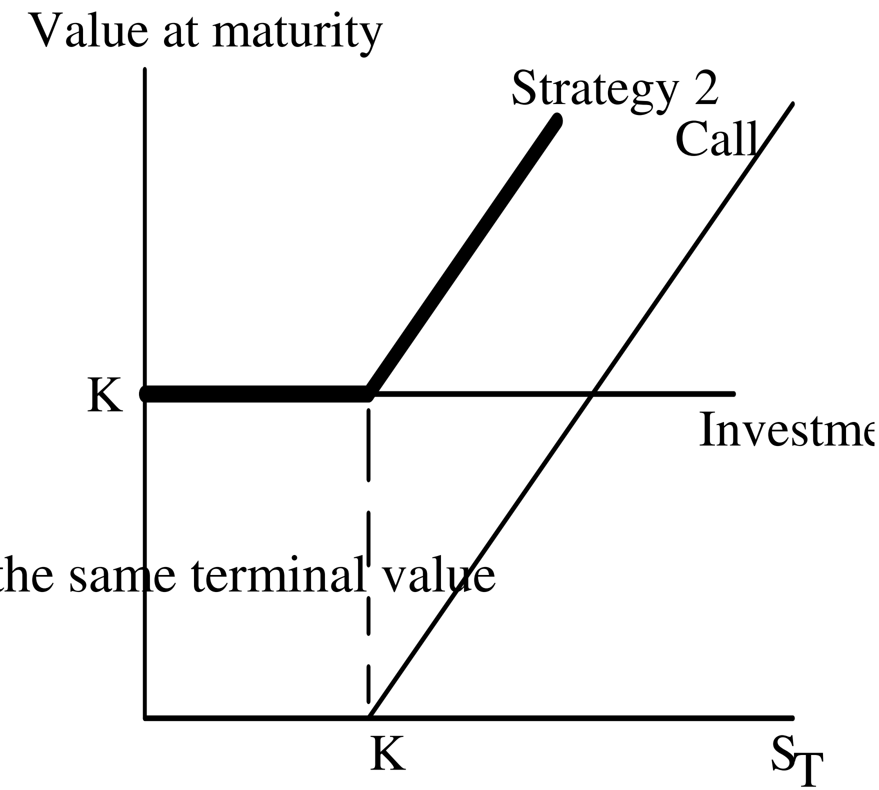
At maturity T:	$S_T<K$	$S_T>K$
Share value	S_T	S_T
Put value	$(K - S_T)$	0
Total value	K	S_T

- Put = insurance contract

Put-Call Parity (2/3)

- Consider an alternative strategy:
- Strategy 2: Buy call, invest PV(K)

At maturity T:	$S_T<K$	$S_T>K$
Call value	0	S_T - K
Investment	K	K
Total value	K	S_T



- At maturity, both strategies lead to the same terminal value

- Stock + Put = Call + Exercise price

Put-Call Parity (3/3)

- Two equivalent strategies should have the same cost

$$S + P = C + PV(K)$$

where S current stock price

P current put value

C current call value

PV(K) present value of the striking price

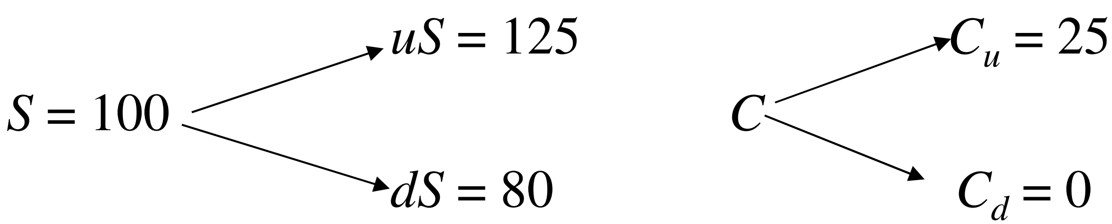
- This is the put-call parity relation
- Another presentation of the same relation:

$$\mathbf{C = S + P - PV(K)}$$

- A call is equivalent to a purchase of stock and a put financed by borrowing the PV(K)

Valuing option contracts

- The intuition behind the option pricing formulas can be introduced in a two-state option model (*binomial model*).
- Let S be the current price of a non-dividend paying stock.
- Suppose that, over a period of time (say 6 months), the stock price can either increase (to uS, $u>1$) or decrease (to dS, $d<1$).
- Consider a $K = 100$ call with 1-period to maturity.



Key idea underlying option pricing models

- It is possible to create a *synthetic call* that replicates the future value of the call option as follows:
  - Buy Delta shares
  - Borrow B at the riskless rate *r* (5% per annum – simple interest over a 6-month period)
- Choose Delta and B so that the future value of this portfolio is equal to the value of the call option.
  - Delta $uS - (1+r\,\Delta t)\,B = C_u$ — *Delta 125 – 1.025 B = 25*
  - Delta $dS - (1+r\,\Delta t)\,B = C_d$ — *Delta 80 – 1.025 B = 0*

- (Δt is the length of the time period (in years) e.g. : 6-month means $\Delta t=0.5$)

SolvayBrusselsSchool of Economics and Management

No arbitrage condition

- *In a perfect capital market, the value of the call should then be equal to the value of its synthetic reproduction, otherwise arbitrage would be possible:*

$$\mathbf{C = Delta \times S - B}$$

- This is the Black Scholes formula
- We now have 2 equations with 2 unknowns to solve.
- [Eq1]-[Eq2] $\Rightarrow$ Delta $\times$ (125 - 80) = 25 $\Rightarrow$ **Delta = 0.556**
- Replace Delta by its value in [Eq2] $\Rightarrow$ **B = 43.36**
- Call value:
- C = Delta S - B = 0.556 $\times$ 100 - 43.36 $\Rightarrow$ **C = 12.20**

A closed form solution for the 1-period binomial model

- $C = [p \times C_u + (1\text{-}p) \times C_d\,]/(1+r\Delta t)$ with $p = (1+r\Delta t - d)/(u-d)$

- p is the probability of a stock price increase in a "risk neutral world" where the expected return is equal to the risk free rate.

  In a risk neutral world : $p \times uS + (1\text{-}p) \times dS = (1+r\Delta t) \times S$
- $p \times C_u + (1\text{-}p) \times C_d$ is the expected value of the call option one period later assuming risk neutrality
- The current value is obtained by discounting this expected value (in a risk neutral world) at the risk-free rate.

Risk neutral pricing illustrated

- In our example, the possible returns are:

  + 25% if stock up

  - 20% if stock down
- In a risk-neutral world, the expected return for 6-month is

  $5\% \times 0.5 = 2.5\%$
- The risk-neutral probability should satisfy the equation:

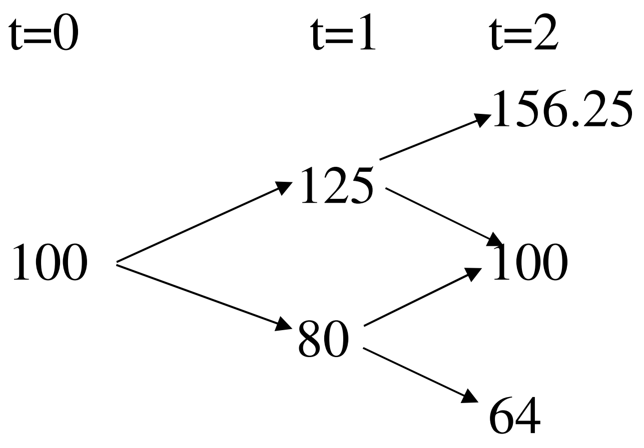
  $p \times (+0.25\%) + (1-p) \times (-0.20\%) = 2.5\%$
- $\Rightarrow p = 0.50$
- The call value is then: $C = 0.50 \times 25 / 1.025 = 12.20$

Multi-period model: European option

- For European option, follow same procedure
- (1) Calculate, at maturity,
  - the different possible stock prices;
  - the corresponding values of the call option
  - the risk neutral probabilities
- (2) Calculate the expected call value in a neutral world
- (3) Discount at the risk-free rate

An example: valuing a 1-year call option

- Same data as before: S=100, K=100, r=5%, u =1.25, d=0.80
- Call maturity = 1 year (2-period)
- Stock price evolution



Risk-neutral proba.	Call value
$p^2 = 0.25$	56.25
$2p(1-p) = 0.50$	0
$(1-p)^2 = 0.25$	0

- Current call value : $C = 0.25 \times 56.25 / (1.025)^2 = 13.38$

ULB

Solvay Brussels School of Economics and Management

Volatility

- The value a call option, is a function of the following variables:
  1. The current stock price S
  2. The exercise price K
  3. The time to expiration date T
  4. The risk-free interest rate r
  5. The volatility of the underlying asset σ

- Note: In the binomial model, u and d capture the volatility (the standard deviation of the return) of the underlying stock
- Technically, u and d are given by the following formulas:

$$u = e^{\sigma\sqrt{\Delta t}} \qquad d = \frac{1}{u}$$

Option values are increasing functions of volatility

- The value of a call or of a put option is an increasing function of volatility (for all other variables unchanged)
- Intuition: a larger volatility increases possible gains without affecting loss (since the value of an option is never negative)
- Check: previous 1-period binomial example for different volatilities
-

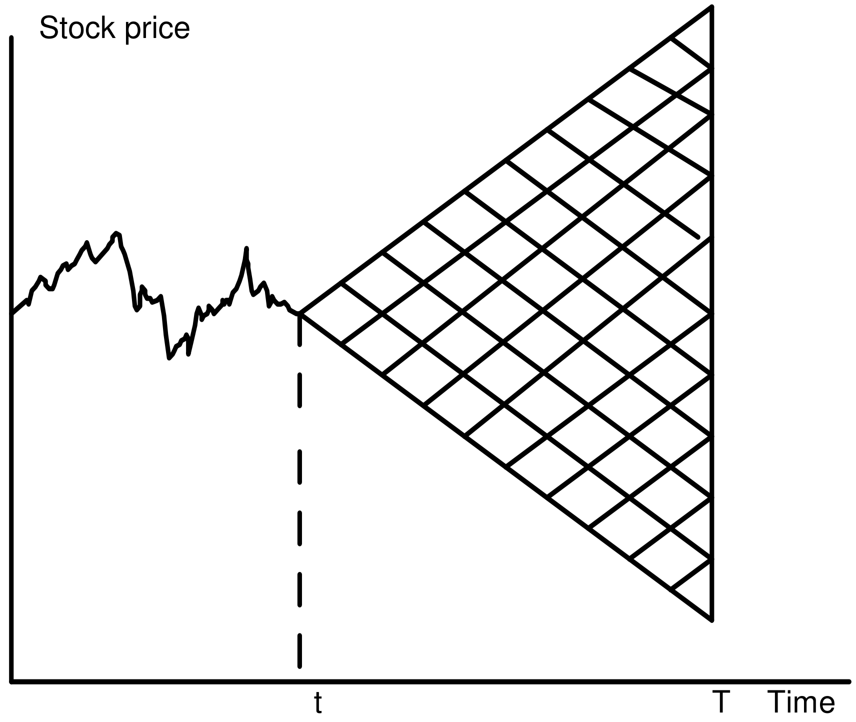
Volatility	u	d	C	P
0.20	1.152	0.868	8.19	5.75
0.30	1.236	0.809	11.66	9.22
0.40	1.327	0.754	15.10	12.66
0.50	1.424	0.702	18.50	16.06

($S=100$, $K=100$, $r=5\%$, $\Delta t=0.5$)

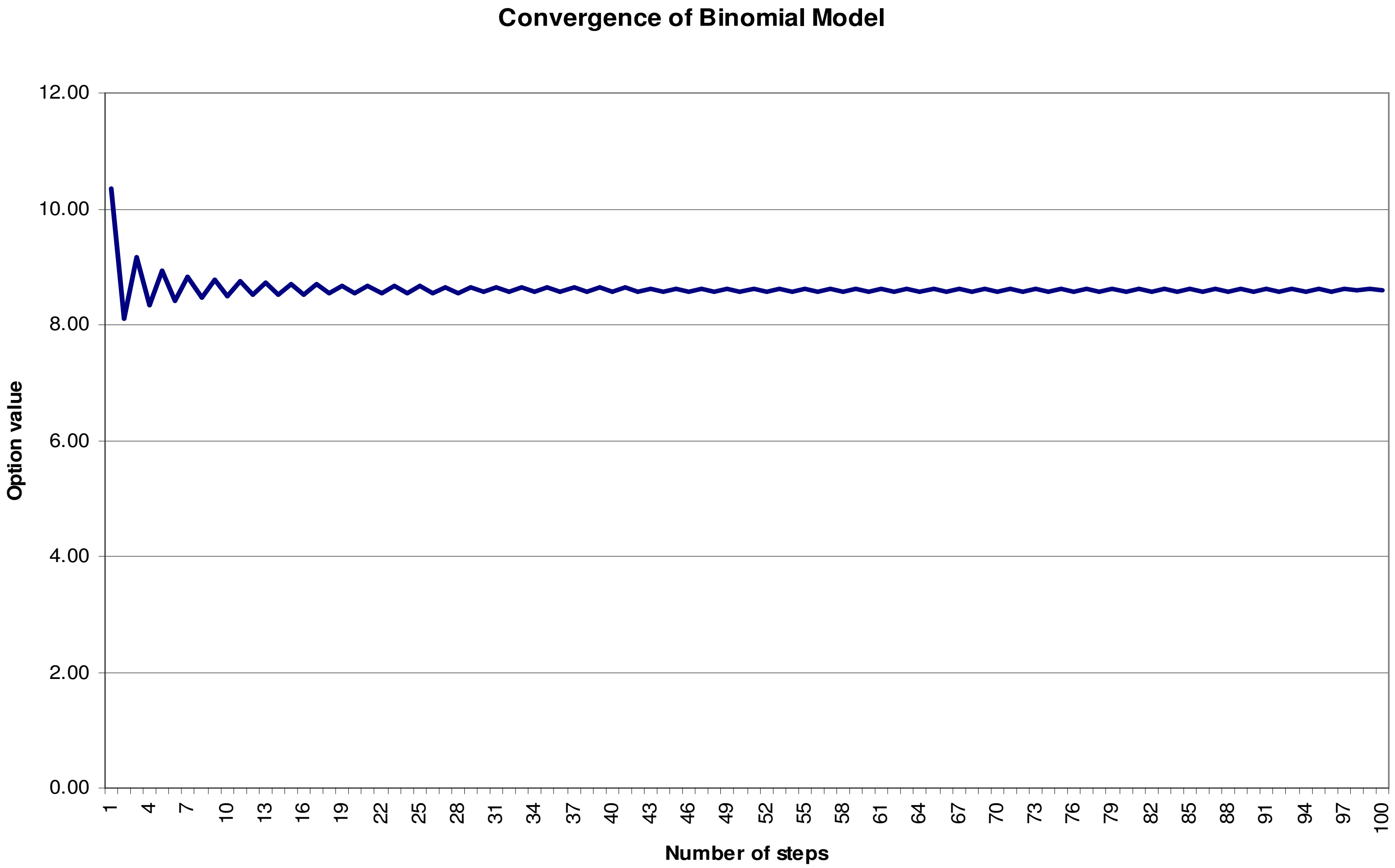
From binomial to Black Scholes

- Consider:
  - European option
  - on non dividend paying stock
  - constant volatility
  - constant interest rate

- Limiting case of binomial model as $\Delta t \rightarrow 0$



Convergence of Binomial Model



Black-Scholes formula

- For European call on non dividend paying stocks
- The limiting case of the binomial model for Δt very small

$$\boldsymbol{C = S\, N(d_1) - \mathrm{PV}(K)\, N(d_2)}$$

↑ Delta ↑ B

- In BS: PV(K) present value of K (discounted at the risk-free rate)
- Delta = $N(d_1)$

$$d_1 = \frac{\ln\left(\frac{S}{PV(K)}\right)}{\sigma\sqrt{T}} + 0.5\sigma\sqrt{T}$$

- $N()$: cumulative probability of the standardized normal distribution
- B = PV(K) $N(d_2)$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Black-Scholes: Numerical example

- 2 determinants of call value:

  "Moneyness" : S/PV(K) "Cumulative volatility": $\sigma\sqrt{T}$

- Example:

  S = 100, K = 100, Maturity T = 4, Volatility σ = 30% r = 6%

  "Moneyness"= $100/(100/1.06^4)$ = 100/79.2= 1.2625

  Cumulative volatility = 30% x √4 = 60%

- d_1 = ln(1.2625)/0.6 + (0.5)(0.60) =0.688 ⇒ N(d_1) = 0.754
- d_2 = ln(1.2625)/0.6 - (0.5)(0.60) =0.089 ⇒ N(d_2) = 0.535

- C = (100) (0.754) – (79.20) (0.535) = 33.05

Cumulative normal distribution

- This table shows values for $N(x)$ for $x \geq 0$.
- For $x<0$, $N(x) = 1 - N(-x)$
- Examples:
- $N(1.22) = 0.889$,
- $N(-0.60) = 1 - N(0.60)$
- $= 1 - 0.726 = 0.274$

- In Excell, use Normsdist()
- function to obtain $N(x)$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.3	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.8	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1.0	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.1	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.2	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.901
1.3	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.4	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.5	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.6	0.945	0.946	0.947	0.948	0.949	0.951	0.952	0.953	0.954	0.954
1.7	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.962	0.963
1.8	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
1.9	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977
2.0	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981	0.982
2.1	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986
2.2	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989	0.989
2.3	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.992
2.4	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994
2.5	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995
2.6	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996
2.7	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
2.8	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
2.9	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999

Black-Scholes illustrated