ULB
Solvay Brussels School of Economics and Management

Advanced Corporate Finance
Exercises Session 4
« Options (financial and real)»

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This session

Options

1. Life is not black and white!
   - ➢ You have options

2. Not always need to make final decision today
   - ➢ Understand the existence of options
     - Waiting is also an option, and often practiced in reality
     - Strong CEO's understand options well
   - ➢ Example in portfolio management: (future) rebalancing

3. The opposite of thinking in options is static and myopic behaviour.
   - ➢ Static behaviour tends to deliver poor results

This session's Questions

Q1: Option Valuation: European call, binomial tree

- Starting without debt
- Introducing debt

Q2: Option Valuation: American put, binomial tree

Q3: Option valuation: Arbitrage

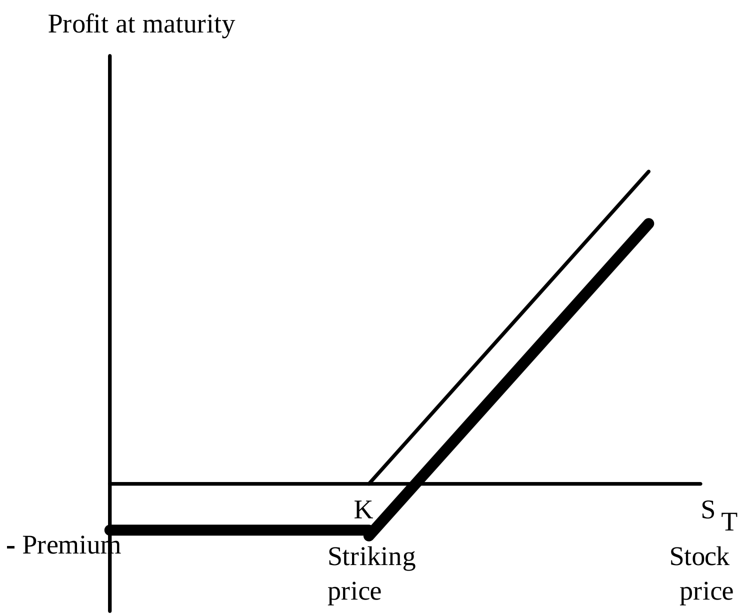
Q4: Option valuation (Black and Scholes)

Q5: Real Options

Q1 Option Valuation (European call, binomial tree)

Q1:

•Reminder: European call option: “Option that gives you the right to buy an asset at a determined price at a determined date”



Q1: data

- European call
- Maturity: 1 year
- Strike price K: 190€
- Spot price S: 200€
- Variance: 70% prob. to double, 30% prob. divided by 2
  - U = 2 and d = 0,5
- Risk free rate rf=4%

Q1.a) what is risk neutral probality?

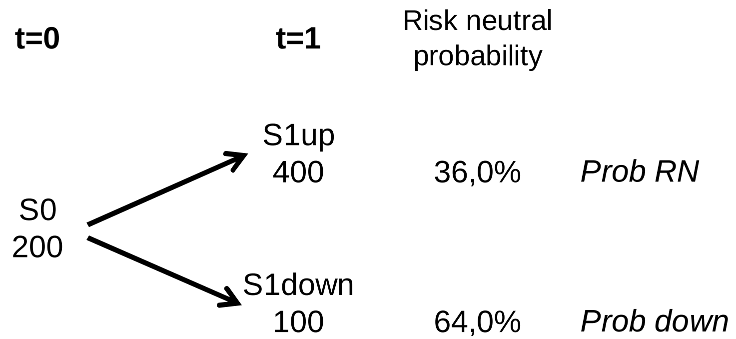
- Risk neutral probability:
  - ❖ “Probability that the stock rises in a risk neutral world” and
  - ❖ “where the expected return is equal to the risk free rate.

=> In a risk neutral world : $p \times uS + (1\text{-}p) \times dS = (1+r\Delta t) \times S$ => $Prob_{RN} = \frac{(1+rf-d)}{u-d}$

•Solving: with u = 2 and d = 0.5

$$\text{Prob}_{RN} = \frac{(1 + 0{,}04 - 0{,}5)}{2 - 0{,}5} = \frac{0{,}54}{1{,}50} = 36\%$$

•Or in a binomial tree



Q1.b) value of the call (use a one year binomial tree)?

Note: Direction of Arrows

- for S (underlying) from left to right ↗
- for C (option) from right to left ↙

- **Binomial tree:**

Draw binomial tree of possible spot prices => Draw NPV tree

t=0 **t=1**

S0 200 → S1up 400 $uS = u * S$

S0 200 → S1down 100 $dS = d * S$

=>

t=0 **t=1**

C0 **72,692** ← C1up 210 MAX(400-190;0) MAX (S_u - K ; 0)

C0 **72,692** ← C1down 0 MAX(100-190;0) MAX (S_d - K ; 0)

- **Solution**

Call Value = PV of Expected Cash flow

$C = [p \times C_u + (1-p) \times C_d]/(1+r\Delta t)$

➢ C = (210 * 36%)/(1+0,04) = 72,69

Note: I dropped C_d because here $C_d = 0$

Q1.c) how to replicate the call?

- **Put-Call Parity:**
  - A call is equivalent to a purchase of stock and a put financed by borrowing the PV(K)

    C = S + P - PV(K) OR **C = Delta × S - B**

    [with PV(K) = present value of the striking price]

- **Solution:**
  - Value of the call replicating the cash flows

What	**Price**	**u**	**d**
Gov Bonds	0.9615	1	1
Stock	200	400	100
Call	72.69	210	0

  - How to constitute a portfolio that replicates the CF of a call
    - **step 1 : calculate Delta (using Su and Cu)** $\Delta * 400 + B = 210$ => $\Delta = 0,7$ Delta uS - $(1+r\ \Delta t)\ B = C_u$
    - **step 2 :use Delta to calculate B** $\Delta * 100 + B = 0$ => $B = -70$ Delta dS - $(1+r\ \Delta t)\ B = C_d$
    - **step 3 :use B in formula to calculate C** $C = 0,7 * 200 - 70 * 0,9615 = 72,69$ => C = 0,7 Stock – 67,30 euro
  - **So buy 0,7 shares and borrow 67,30 EUR**

Q1.c) how to replicate the call? CHECK & d)

•**Checking the solution**

➢ Using the binomial tree

	t=0		t=1		
Replication					
Delta shares	0,7				
Borrow Beta	67,31				
			up		
			210	=0,7*400-67,31 * (1 +4%)	
Check value in 0	**72,69**	and in 1			
	OK!				
			down		
			0	=0,7*100-67,31 * (1 +4%)	

•**Q1.d) You have 200 EUR: what can you buy?**

- Shares = 200 EUR @ Spot price of 200 EUR = 1
- 200 EUR calls @ 72,69 EUR a piece = 200 / 72,69 = 2,75 calls

Q2. Option Valuation (American put, binomial tree)

Q2.a): Is an American put worth more than an European one? Don't crunch numbers!

- ❖ Owners of American-style options may exercise at any time before the option expires, while owners of European-style options may exercise only at expiration.

- ➢ **Answer: Intuitively American option should be worth more!**
  - ⇨ Theoretically, the American option will be worth more, as you have more opportunity to exercise the option
  - ⇨ In some cases however, early exercise is never interesting in this case the value is the same bur in any case **American option can never be worth less than European ones**

Q2.b. European put vs American put

Q2.b): He then would like to compute the value of :

b.1. a three months European put.

b.2. and a three months American put.

- **Data**
  - spot rice S = 100 €
  - strike price K = 100 €
  - U = 1,10 (per period!) and D =0,909
  - The continuous (!) risk free rate is worth 0.5% per month

➢**How to solve?**

Step 1: calculate risk-neutral probabilities (same for European and American)

Step 2: draw binomial tree of possible spot prices in different period & solve

➢Repeat for American option

Q2.b: Step1: calculate risk neutral probability

-Compute risk-neutral Probability

$Prob_{RN} = \frac{(1+rf-d)}{u-d}$ = 0,502 =>1-p=0,498

- Draw binomial tree of possible spot prices

S	100
K	100
Rfm	0.50%
u	1.1
d	0.91

- **CAUTION**: continuous rate
  - Use $e^{0,5}$, for discounting use $e^{-0,5}$
  - Or calculate monthly equivalent and then use 1+rf and 1 / 1 + rf

Q2 b.1. Value of European Put

Underlying price

t0	t1	t2	t3
100	110	121	133.10
			110.00
		100	110.00
			90.91
	90.91	100	110.00
			90.91
		82.64	90.91
			75.13

Step 1: Draw binomial tree of possible spot prices

S	100
K	100
Rfm	0.50%
u	1.1
d	0.91
p	0,502
1-p	0,498

European Put

t0	t1	t2	t3
6.36	2.23	0.00	0.00
			0.00
		4.50	0.00
			9.09
	10.60	4.50	0.00
			9.09
		16.86	9.09
			24.87

Step 2: Draw NPV tree of possible option prices

- Every T: you weigh next period by probability and you discount

16,86 =

(9,09 * 0,502 + (1-0,502) * 24,87) *EXP(-0,50%)

Q2 b.1. Value of American Put

Underlying price

t0	t1	t2	t3
100	110	121	133.10
			110.00
		100	110.00
			90.91
	90.91	100	110.00
			90.91
		82.64	90.91
			75.13

Step 1: Draw binomial tree of possible spot prices

S	100
K	100
Rfm	0.50%
u	1.1
d	0.91
p	0,502
1-p	0,498

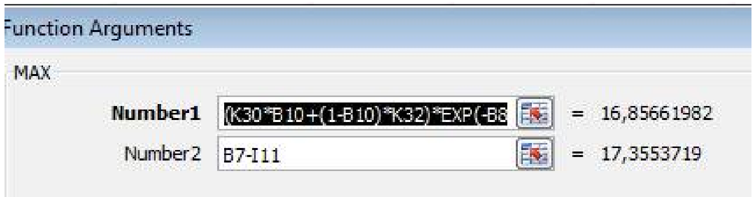
American Put

t0	t1	t2	t3
6.48	2.23	0.00	0.00
			0.00
		4.50	0.00
			9.09
	10.84	4.50	0.00
			9.09
		17.36	9.09
			24.87

Step 2: Draw NPV tree of possible option prices

Difference with EUR put: you add MAX on top

17,36 = MAX [(9,09*0,502 +(1-,0502)*24,87)*EXP(-0,5%); 100-82,64]



In other words, in the more negative scenario's here, you exercise the put sooner

Q3 Option Valuation Arbitrage (Call)

Q3: How can one seize an arbitrage opportunity for the following?

- Spot price=52$
- Strike price = 45$
- Maturity one year
- European call price = 53$

➢**Approach: calculate bounds**

1. Call value (53$) should be higher than Spot (52$) – Strike (45$) =7$:
   - ➢ IF NOT: Buy call and short share
2. Call value (53$) should be lower Spot (52$)
   - ➢ IF NOT: Sell call and buy share

➢**Here: case 2 = call is overvalued so sell it**

=> Sell a call at 53$ and buy a share at 52$: your arbitrage (immediate cash flow = 1$)

=> If not exercised at maturity CF = 1(+interests) +Spot price = immediate CF + future sale @ spot

=> If exercised at maturity CF = 1 (+interests) + Strike price = 1 (+interests) + 45$ due to exercising

Q4: Option valuation (Black and Sholes)

- Data
  - share price is 120$ = (S)
  - the strike price 100$ = (K)
  - the maturity one year => T= 1
  - the annual volatility of the share is 40% => U = 1,4 and D = 0,6
  - the continuous risk free rate is worth 5% = (Rf)

- **Reminder**: **continuous rate**
  - B&S uses the cumulative distribution function of the standard normal distribution
  - And thus also a continuous rate for discounting

- Questions

a) What would be the **value of a call** on the MekWhisky Cy

1. Use first a one year binomial tree and
2. then the Black and Scholes formula.

b) How can you **explain the difference**? (between binomial and B&S)

c) What would happen if you chose a **binomial tree with 6 months steps** (instead of one year)?

Q4 a.1 one-year binomial tree

$$1 + r_{yearly} = \exp(r_{cont})$$
$$r_{yearly} = 5{,}127\%$$
$$1 + r_{yearly} = (1 + r_{6month})^2$$
$$r_{6month} = 2{,}532\%$$

S	120
K	100
r(continued)	5%
σ_{yearly}	40%
u_{yearly}	Exp(40%)=1,492
d_{yearly}	1/u=0,670
p_{yearly}	0,464
$1\text{-}p_{yearly}$	0,536

$$u = e^{\sigma\sqrt{\Delta t}}$$

Step 1: Draw binomial tree of possible spot prices

Underlying price		
t0		t1
120	→	179.02
	↘	80.44

Step 2: Draw NPV tree of possible option prices

Call price		
34.86	←	79.02
	↖	0

❖ **How to discount?**

1. Linear = 1/ (1+r)^t: using 5,127%
2. Exponential: e^-t: using 5% continuous

Remarks

- **We have done this at start of session!**
- **Irrelevant whether American or European as only 1 period**

Q4 a.2 using B&S formula

S	120
K	100
r(continued)	5%
σ_{yearly}	40%
r_{yearly}	5,127%

- <u>Black and Scholes Formula</u>
- $C = S * N(d_1) - \text{PV(K)} * N(d_2)$
- $d_1 = \frac{\ln\left(\frac{S}{PV(K)}\right)}{\sigma\sqrt{T}} + 0{,}5\sigma\sqrt{T}$
- $d_2 = d1 - \sigma\sqrt{T}$

Step1: Calculate d's from formula above

- d_1= 0,781
- d_2=0,381

Step2: Lookup N(d)'s in N-table on next page

- $N(d_1)$ = 0,783
- $N(d_2)$ = ~~0,643~~ Correct N (d2) = 0,6483, or table approx = 0,648

- C = 32,234 **Step3: Calculate PV(K) & Plug everything in B&S formula and calculate**

PV(K) = 100 * $e^{-0,05}$ = 95,12 => C = 32,234 = (120 * 0,782) - (95,12 * 0,648)

Q4 a.2 using B&S Table

Table lookup: steps:

1 decimal in row (select)

2 hundreths in colum (cross)

Theoretically you can calculate it yourself without table…

- d_1= 0,781
- d_2=0,381

- $N(d_1) = 0{,}783$
- $N(d_2)$ = ~~0,643~~

Correct N (d2) = 0,6483, or table approx = 0,648

	0	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7703	0,7734	0,7764	0,7793	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389

Q4 c) 6-month Binomial tree

- **We will first solve c and then b**

Remarks:

- Assuming European
- In excel linear equivalent used

Underlying price		
t0	t1	t2
120.00 →	159.23 →	211.28
↘	↘	120.00
	90.44 →	120.00
	↘	68.16

Step 1: Draw binomial tree of possible spot prices

Call price		
33.26 ←	61.70 ←	111.28
↖	↖	20.00
	9.24 ←	20.00
	↖	0.00

Step 2: Draw NPV tree of possible option prices

S	120
K	100
r(continued)	5%
σ_{yearly}	40%
σ_{6month}	40%*0,5yr^0,5=28,3%
u_{yearly}	Exp(28,3%)=1,327
d_{yearly}	1/u=0,754
p_{yearly}	0,474
$1-p_{yearly}$	0,526

Volatility conversion

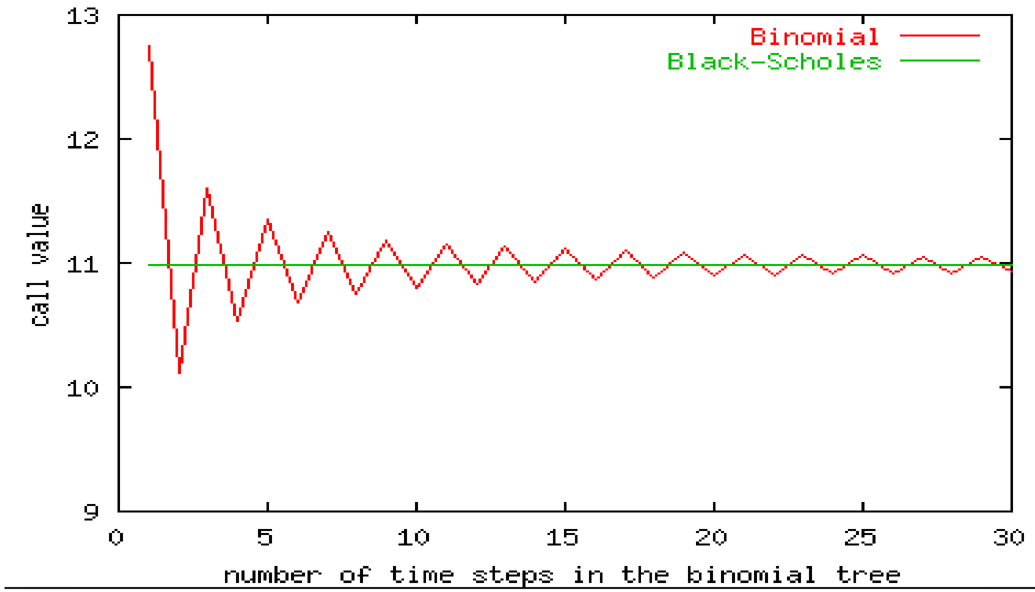
$\sigma_{annual} = \sigma$ bi-annual $\sqrt{(2)}$

σ bi-$_{annual} = \sigma$ annual / $\sqrt{(2)}$

σ bi-$_{annual}$ = 40% /(2^(1/2))

Q4 b. Comparing Black and Scholes and Binomial trees

	Call price	Compared to BS value
B&S	32.23	-
1 step	34.86	+8%
2 steps	33.26	+3%



Answer:

•Binomial converges with number of steps towards B&S, but in early steps moves around B&S

Remarks:

•B&S is lower here in both cases

•But 2 steps is lower than 1 step

•Maybe a 3rd step could be lower than BS, depending on variables

Q5: Real Options

- Story

  - The move from volatile but tax haven Tongoland to the more stable but taxing Bobland resulted in a lower market cap (2,325,000 $) for Freshwater (see Session 3).
    - Following the move, the stock traded at 23.25 $. => # shares = 100k

  - R&D partnership financing agreement with Bobland's main university Ewing State related to the potential development of a new energy drink 'Spirit of Southfork' => option value

  - The forecasts are very sensitive to a number of uncertain factors

Q5: Real Options

- Data
  - Agreement
    - a 2 year agreement (option)
    - Freshwater finances 100% of the partnership.
    - Every year Freshwater can terminate the partnership if they wish.
  - CF schemes

t	0	1	2
Investment	25,00	75,00	225,00

t	3	4	5
Net profits	50,00	55,00	60,00

Year 6 → ∞ : +2% annually

Thousands (k) $

- ➢ The volatility of the future business value is estimated to be 50%.
  - Other: Freshwater capital structure will remain constant & no change in working capital
  - Capital costs:
    - The project WACC is 17%
    - Freshwater WACC is 12%
    - The risk free rate is 2%

Q5: Real Options

- Questions

Stock finally lost 10% and closed down at 20.9$.

- The same evening John Ross III E. a well-known local investor and major Freshwater investor contacted you to know you what he should do with his participation.

a) calculate NPV of partnership

b) Calculate NPV of option value

c) Is the stock slide justified? Shareholder Ross asked you what to do!

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Q5: Real Options: Step 1: Cash flow statement

Rf	2%		
Total investment			
t	**0**	**1**	**2**
Investment	25,00	75,00	225,00
PV inv	25,00	73,53	216,26
PV (invest before launch)	-98,53		
PV (launch)	-216,26		
PV (total investment)	-314,79		

=>

Business future value					
Expected Return	17%				
Growth rate as of year 6				2%	
t	**2**	**3**	**4**	**5**	
Net profits		50,00	55,00	60,00	
FV	= CF1 / (r-g)			408,0	= 60*1,02 / (17%-2%)
DF (to T=2)		1,17	1,37	1,60	
PV		42,7	40,2	292,2	
Business future value	375,1				

=>

Net Present Value			
t	**0**	**1**	**2**
PV (invest before launch)	**-98,53**		
PV (launch)	**-216,26**		
PV (total investment)	**-314,79**		
Terminal value			375,12
PV (terminal value)	274,03		
NPV	**-40,76**		

Q5: Real Options: Step 2: Binomial tree

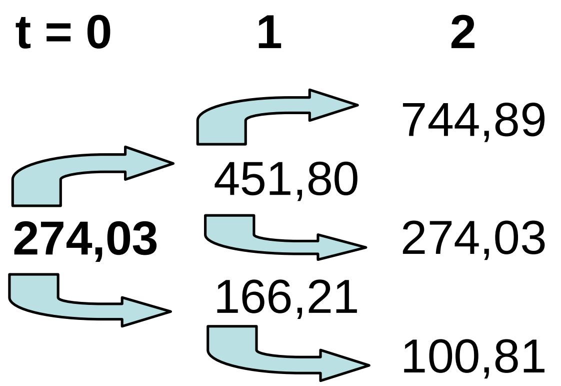
Step 2.1: Calculate binomial parameters:

vol = 0,5 => u = 1,65 = $e^{0,5}$ => p = 0,40
d = 0,61 = 1/d

$$Prob_{RN} = \frac{(1 + rf - d)}{u - d}$$

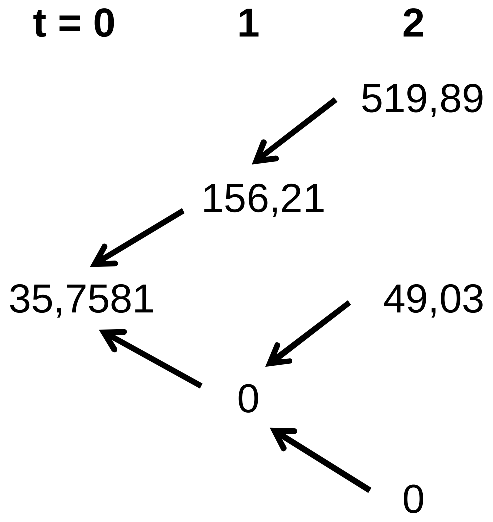
Step 2.2: Build trees

Binomial tree



=>

Real option NPV



Value at each node =

Max{0,[pVu +(1-p)Vd]/(1+rDt)-STRIKE}

1 / 1 + rDt = 0,98

Step 1: Draw binomial tree of possible spot prices => **Step 2: Draw NPV Option tree**

Q5: Real Options: Step 3: Evaluate & Answer

Step3: Evaluate pricing:

Real Option NPV	**35,76 k $**			
# shares	100 000			
per share	0,358 $ / share			
original share price	23,2500 $ / share			
theoretical price	23,6076 $ / share		**theoretical increase**	**1,54%**
actual closing price	20,9250 $ / share		**current mispricing**	**-11,36%**

Answer: Buy extra Freshwater shares as the option is mispriced / undervalued

- 10% stock price decline is not justified,
- based on the 'real option NPV' approach the stock price should have modestly (+1,5%) increased
- The current mispricing is -11,3% (too low)
- The NPV is effectively negative but ignores the option value

Concluding remarks

Wider context

•Scot Fitzgerald

- ➢ More than 60 years ago, F. Scott Fitzgerald saw “the ability to hold two opposing ideas in mind at the same time and still retain the ability to function” as the sign of a truly intelligent individual.

•Integrative thinking

- Succesfull leaders tend to share a somewhat unusual trait:
  - They have the predisposition and the capacity to hold in their heads two opposing ideas at once.
  - And then, without panicking or simply settling for one alternative or the other, they’re able to creatively resolve the tension between those two ideas by generating a new one that contains elements of the others but is superior to both.
- This process of consideration and synthesis can be termed integrative thinking.
- It is this discipline—not superior strategy or faultless execution—that is a defining characteristic of most exceptional businesses and the people who run them. The focus on what a leader does is often misplaced.
- ➢ **Evaluating Options is an essential part of Integrative Thinking**