Advanced Corporate Finance
Exercises Session 5
« Bonds and options»

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This session

Leaving the risk free debt world

Intro

- Options
  - Provide insight into value of debt
  - Have led to creation of different bond types

1. Risky debt

- Options provide insight into risky debt
  - The value of risky debt can be decomposed into an option

2. Bonds with embedded options

- Convertible bonds
- Callable bonds

This session's Questions

Q1: Risky Debt: the Merton Model

Q2: Risky Debt: the Merton Model in continuous time

Q3: Convertible bonds

Q4: Callable bonds

Q1: Risky Debt: the Merton Model

- Story
  - Unhappy client has a biotech (volatile!) company that has just done IPO
    - ➢ In Tongoland: no taxes
  - Company wants to change capital structure: issue debt to do buyback
  - You
    1. have to answer some questions on a zero-coupon bond and
    2. Advise your boss whether to accept client demands

Q1: Risky Debt: the Merton Model

- DATA
  - Company
    - IPO: 100 k shares currently @ 30 €
    - Volatility: U = 4 & D = 0.25
  - Tongoland
    - RFR = 3%
    - No tax
  - Bond
    - Zero coupon
    - T = 3 yr
    - Maturity = 1 Million €

Q1: Risky Debt: the Merton Model

- Questions

(a) Bond value using binomial tree with a one year step?

(b) Should your boss take the offer?

(c) What is the risk premium of the company?

(d) Broadly speaking which kind of rating could they expect with such a figure?

Q1.a: Value of the bond: steps

- Step 1: Risk neutral probability

- Step 2: Draw binomial trees

    1. Tree of company value: left to right ↗
    2. Tree of debt: right to left ↙

Q1.a.1 what is risk neutral probability?

- Risk neutral probability:
  - ❖ "Probability that the stock rises in a risk neutral world" and
  - ❖ "where the expected return is equal to the risk free rate.

=> In a risk neutral world : $p \times uS + (1-p) \times dS = (1+r\Delta t) \times S$ => $Prob_{RN} = \frac{(1+rf-d)}{u-d}$

•Solving: with u = 4 and d = 0,25

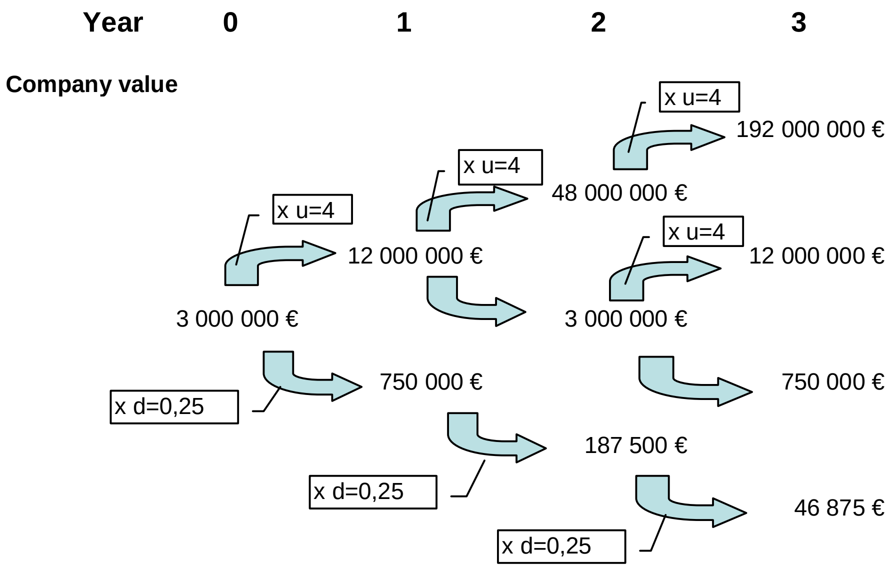
$$Prob_{RN} = p = \frac{1+ rf - d}{u - d} = \frac{(1+0{,}03 - 0.25)}{4-0{,}25} = \frac{0{,}78}{3{,}75} = 0{,}208 \quad \Rightarrow 1-p = 0{,}79$$

Q1.a.2: Binomial tree of the bond

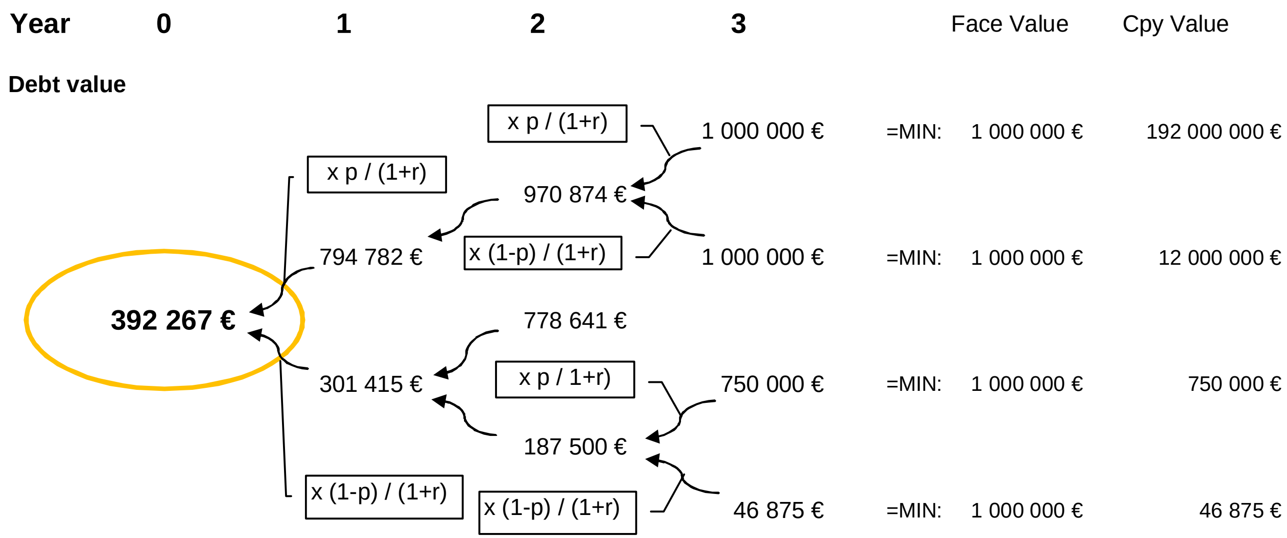
- drawing binomial trees

- Every T: you weigh next period by probability and you discount

Tree 1: possible company values



Tree 2: possible debt values



Q1.b: Should he (your boss) accept to issue this debt for a price of 300.000 € ?

- ANSWER: Yes! 392 k € Value received > 300 k € Cash demanded by client

Q1. c & d: risk premium and rating

(c) What is the risk premium of the company?

$$\text{Price} = \frac{\text{Face Value}}{(1+\text{yield})^{\text{maturity}}} = 392\,267\ € = \frac{1\,000\,000\ €}{(1+y)^{3}}$$

- Step 1: yield y = 36,61 %
- Step 2: **risk premium** : yield – risk free rate = 36,61% - 3% = 33,61% = **3361 bps**

(d) Broadly speaking which kind of rating could they expect with such a figure?

	Moody's	Standard & Poor's	
Highest Quality	Aaa	AAA	Investment Grade
High Quality	Aa	AA	
Upper Medium	A-1, A	A	
Medium	Baa-1, Baa	BBB	
Speculative	Ba	BB	Not Investment Grade
Highly Speculative	B, Caa	B, CCC, CC	
Default	Ca, C	D	

Source: Watson Wyatt Europe

- ANSWER:
  “Highly Speculative”
- Note: when yields very high, value is often quoted
  = 39 cents

Q2: Risky Debt: Merton in continuous time

- DATA
  - Company
    - Value = $ 1 million today; equity & debt
    - No dividends
    - Annual variance asset returns (continuous) = 0,16
    - Asset beta = 1
  - Bonds
    - Zero-coupon
    - T = 6 months
    - # 700
    - Face Value = $ 1000
  - Market
    - Continuous RFR for T= 6 months = 8%
    - Market risk premium = 6%

Q2: Risky Debt: Merton in continuous time

- Questions

(a) Use the Black-Scholes model to calculate the values of firm's
   1. debt and
   2. equity.

(b) Compute debt's
   1. yield and
   2. spread.

(c) Break up the debt value into
   1. put value and
   2. risk-free debt.

Q2: Risky Debt: Merton in continuous time

- Questions (continued)

(d) What's

1. the risk neutral default probability of this company and
2. the delta of its equity?

(e) Break up the debt value in

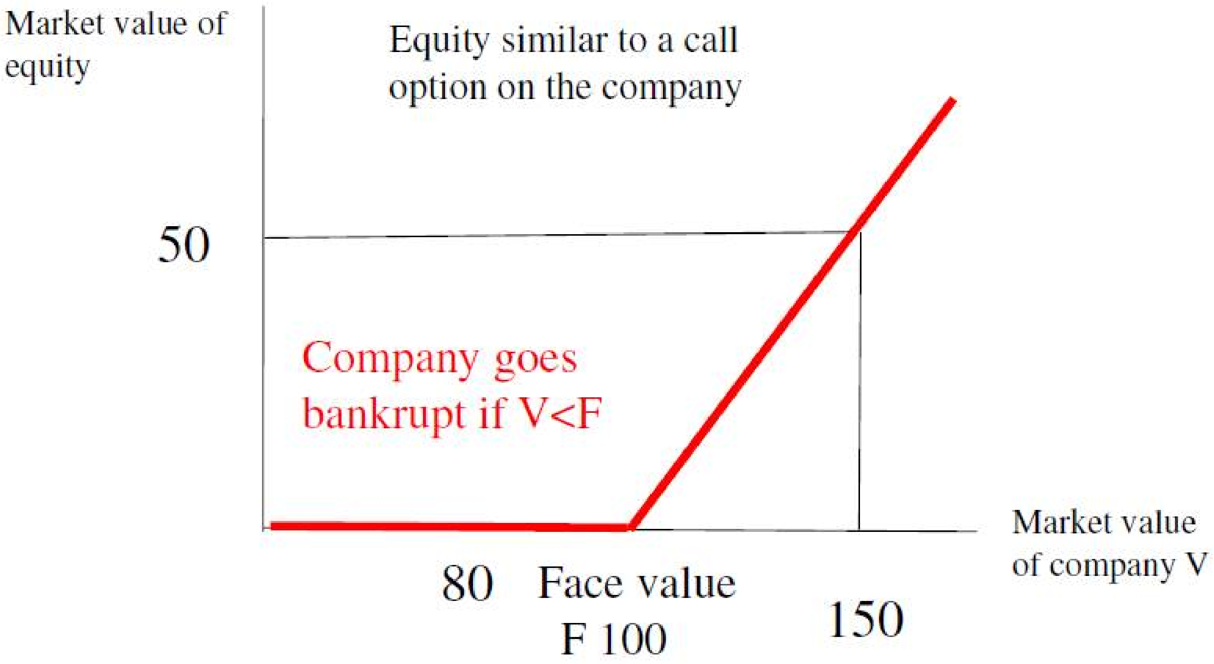
1. face value,
2. loss if no recovery, and
3. expected recovery given default.

(f) Compute

1. the beta debt,
2. the beta equity and
3. the WACC of the company

Q2.a: Risky Debt: using B&S formula

- Theory: part 1
  - Limited liability rules out negative equity
    - Equity ~ Call Option on company



Q2.a: Risky Debt: using B&S formula

- Theory: part 2 : calculating a call

  - For European call on non dividend paying stocks

Black & Sholes formula

$$C = S\,N(d_1) - PV(K)\,N(d_2)$$

$\uparrow$ Delta $\qquad \uparrow$ B

$$\text{Delta} = N(d_1)$$

$$B = PV(K)\,N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{PV(K)}\right)}{\sigma\sqrt{T}} + 0.5\sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

- Remarks
  - In BS: PV(K) present value of K (discounted at the risk-free rate)
  - N(): cumulative probability of the standardized normal distribution

Q2.a: Risky Debt: using B&S formula

- Input Data

Variable	Value		Comments
σ =	40,0%	=	Annual Volatility σ = √Variance =√σ2 = √0,16 =0,16^(1/2)
S =	1 000 000	=	Firm's Value where you have call on
K =	700 000	=	Debt = given = 700 * $1.000 per bond
r_f =	8,0%	=	given; continous rate
T =	0,5	=	Maturity = 6 months

- Step 1: Calculate d's from formula above

d_1 = [[ln (1,000,000 / PV (700,000)] / (0,16 x 0,5^1/2)] + 0,5 x 0,16 x 0,5^1/2 =

S / PV(K) =	1,487
σ x T^1/2 =	0,283
0,5 x σ x T^0,5 =	0,141

➢ d_1 = (LN(1,487) / 0,283) + 0,141 = 1,544

d_2 = d_1 - σ x T^0,5 = 1,544 - 0,40 * 0,5 ^0,5 = 1,261

Q2.a: Volatility calculation

- BS Model uses annual volatility
  - Variance => Volatility

$$\text{Volatility } \sigma = \sqrt{\text{Variance}} = \sqrt{\sigma 2}$$

  - Volatility Conversion

$$\sigma_{annual} = \sigma \text{ bi-annual } \sqrt{(2)}$$

$$\circ \sigma \text{ bi-}_{annual} = \sigma \text{ annual } / \sqrt{(2)}$$

Q2.a: Risky Debt: using B&S formula

- Step 2: Lookup N(d)'s in N-table

$N(d_1) = N(1{,}544) = 0{,}9387$

$N(d_2) = N(1{,}261) = 0{,}8964$

- Step 3: Calculate PV(K) & Plug everything in B&S formula and calculate

PV(K) = 700 000 $ * $e^{-0,08 * 0,5}$ = 672 553 $ =>

➢ **ANSWERS:**

Equity = Call = (1 000 000 $ *0,938) - (672 533 $ * 0,896) **= 335 847 $**

$$C = S\,N(d_1) - \mathrm{PV}(K)\,N(d_2)$$

↑ Delta ↑ B

Debt = Value - Equity = 1 000 000 $ - 335 847 $ = **664 153 $**

Q2.b: Risky Debt: debt's yield and spread

(b) Compute debt's yield and spread.

$$\text{Debt Value} = \text{Price} = \frac{\text{Face Value}}{e^{\text{yield} * \text{maturity}}} = 664\,153\ \$ = \frac{700\,000\ \$}{e^{\text{yield} * 0{,}5}}$$

- Step 1: yield= - LN(Debt/FaceValue) / T = = { -LN(664153 / 700000) } / 0,5

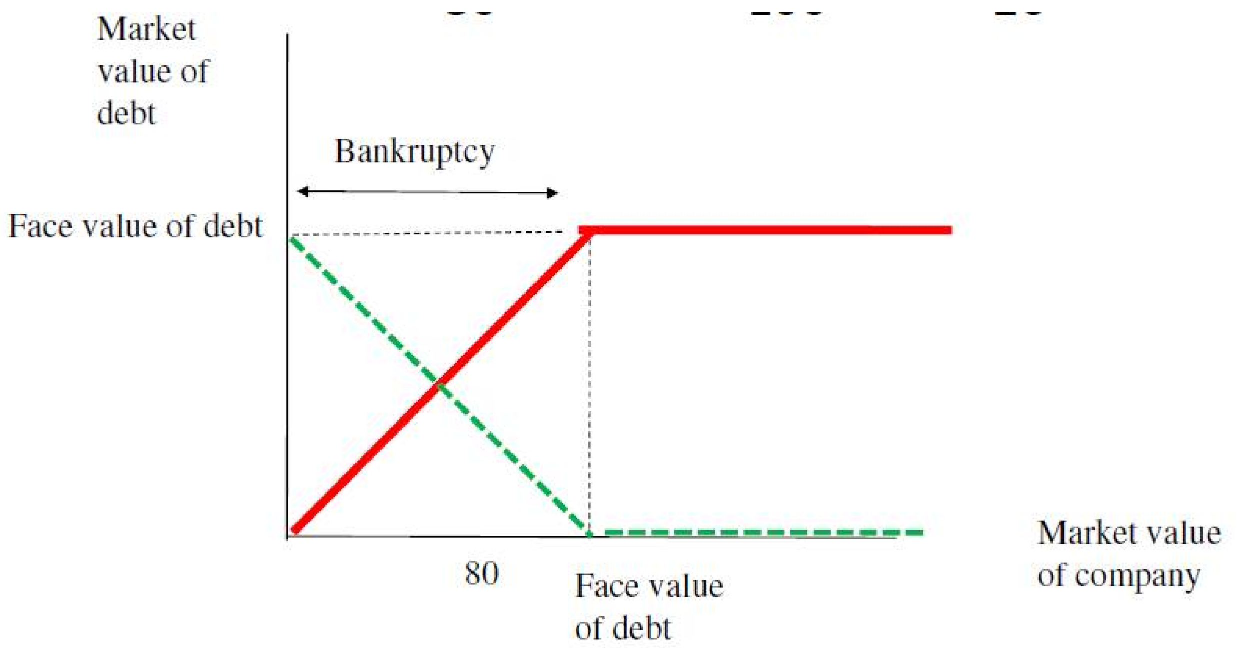
  yield = y = 10,51 %

- Step 2: **risk premium** : yield – risk free rate = 10,51% - 8,00% = 2,51% = **251 bps**

Q2.c: Risky Debt: Break-up 1

- Theory: Risk free debt = Risky debt + put
  - **Put-Call Parity:** A call is equivalent to a purchase of stock and a put financed by borrowing the PV(K)

Call = S + Put - PV(K) OR **C = Delta × S – B** *[with PV(K) = present value of the striking price]*



Q2.c: Risky Debt: Break-up 1

- Solution: Risk free debt = Risky debt + put

Step 1: Calculate risk free debt

Risk free debt = F / $e^{r * T}$ = 700 000 $ / $e^{8\% * 0,5}$ = **672 553$**

Step 2: Calculate Put

Put = Call + PV(K) -S   *[with PV(K) = present value of the striking price ; Call = Equity; S = Cpy Value]*

Put = 335 847$ + 672 553$ - 1 000 000 $ = **8 400$**

Step 3: Calculate Risky debt = Risk free debt – Put

Risky debt = 672 553$ -8 400 $ = **664 153 $**

Q2.Risky Debt: d) Prob $_{RN}$ & Δ e) Break-up2 Recovery

(d) What's the risk neutral default probability of this company and the delta of its equity?

Probability of default = N(-d2) = 1-N(d2) = 1- 0,896 =10,36%

Delta of equity = N(d1) = 0,939

(e) Break up the debt value in

1. face value = 700$
2. loss if no recovery = 700$
3. expected recovery given default = 615,65$

Note: expected loss given default = 84,35$

$$D = e^{-rT}\left\{F - \underbrace{[1-N(d_2)]}_{\text{Prob. of default}} \times \underbrace{[\underbrace{F}_{\substack{\text{Loss}\\ \text{if no}\\ \text{recovery}}} - \underbrace{Ve^{rT}\frac{1\text{-N}(d_1)}{1\text{-N}(d_2)}}_{\substack{\text{Expected Amount}\\ \text{of Recovery given Default}}}]}_{\text{Expected Loss given Default}}\right\}$$

$$= e^{-0.08\times 0.5}\left\{700 - \underbrace{[1-0.8964]}_{0.1036} \times \underbrace{\left[700 - \underbrace{1000\times e^{0.08\times 0.5}\frac{0.0613}{0.1036}}_{615.65}\right]}_{84.35}\right\}$$

Q2.Risky Debt: f) Bd, Be and WACC

$$\beta e = \beta a * N(d1) * \left(1 + \frac{D}{E}\right)$$

$$\beta d = -\beta a * (1 - N(d1)) * \left(1 + \frac{E}{D}\right)$$

$$re = rf + rp * \beta e$$

$$rd = rf + rp * \beta d$$

$$WACC = rd * \frac{D}{V} + re * \frac{E}{V}$$

=>

$$\beta e = \beta a * N(d1) * \left(1 + \frac{D}{E}\right) = 2{,}795$$

$$\beta d = -\beta a * (1 - N(d1)) * \left(1 + \frac{E}{D}\right) = 0{,}092$$

$$re = rf + rp * \beta e = 24{,}77\%$$

$$rd = rf + rp * \beta d = 8{,}55\%$$

$$WACC = rd * \frac{D}{V} + re * \frac{E}{V} = 14{,}00\%$$

Q3: Callable Convertible bonds

- Story

Patient Mr D, CFO of Cpy X: “to call or not to call?” => you are Dr. Zoubowsky

- DATA

  - Company
    - Value = 360 million € today (equity & debt)
    - 6 million shares => value/share = 60 € /share
    - Volatility: U = 1,5 & D = 0.67
    - No dividends
  - Market
    - RFR = 4%

- Bonds
  - General
    - Callable, zero-coupon, convertible
    - Face = 100 million € @ 1 million bonds
    - T = 2 yrs = June 2009 => now = June 2007
  - Option details
    - Conversion
      - Conversion ratio = 1
      - Conversion price = 100 €
    - Call option
      - Call price = 70 €
      - Call dates: June 2007 & June 2008

Q3: Callable Convertible bonds

- Questions

(a) What are the possible values (in June 2009) of
  1. the company,
  2. the convertible issue and
  3. the equity at maturity

(b) Conversion option
  1. What are the possible values of the convertible issue in June 2008 if the issue was non callable?
  2. Would bondholders convert before maturity?

(c) Call option
  1. Should company X call the bonds in June 2008?
  2. How would bondholders react to a call decision?

(d) What decision should Mr D take in June 2007?

Q3.a: Possible values at maturity ~ Tree 1

- **STEP 1** = conversion claim:
  - ➢ If debtholders decide to convert they can claim
  $$\frac{\text{Shares}_{\text{Debtholders}}}{\text{Shares}_{\text{Total}}} = \frac{1 \text{ Million x } 1}{(1 \text{ Million x } 1) + 6 \text{ Million}} = 14{,}29\% \text{ of total Equity}$$
  - (a) What are the possible values (in June 2009) of
    1. the company, 2. the convertible issue and 3. the equity at maturity

ANSWER

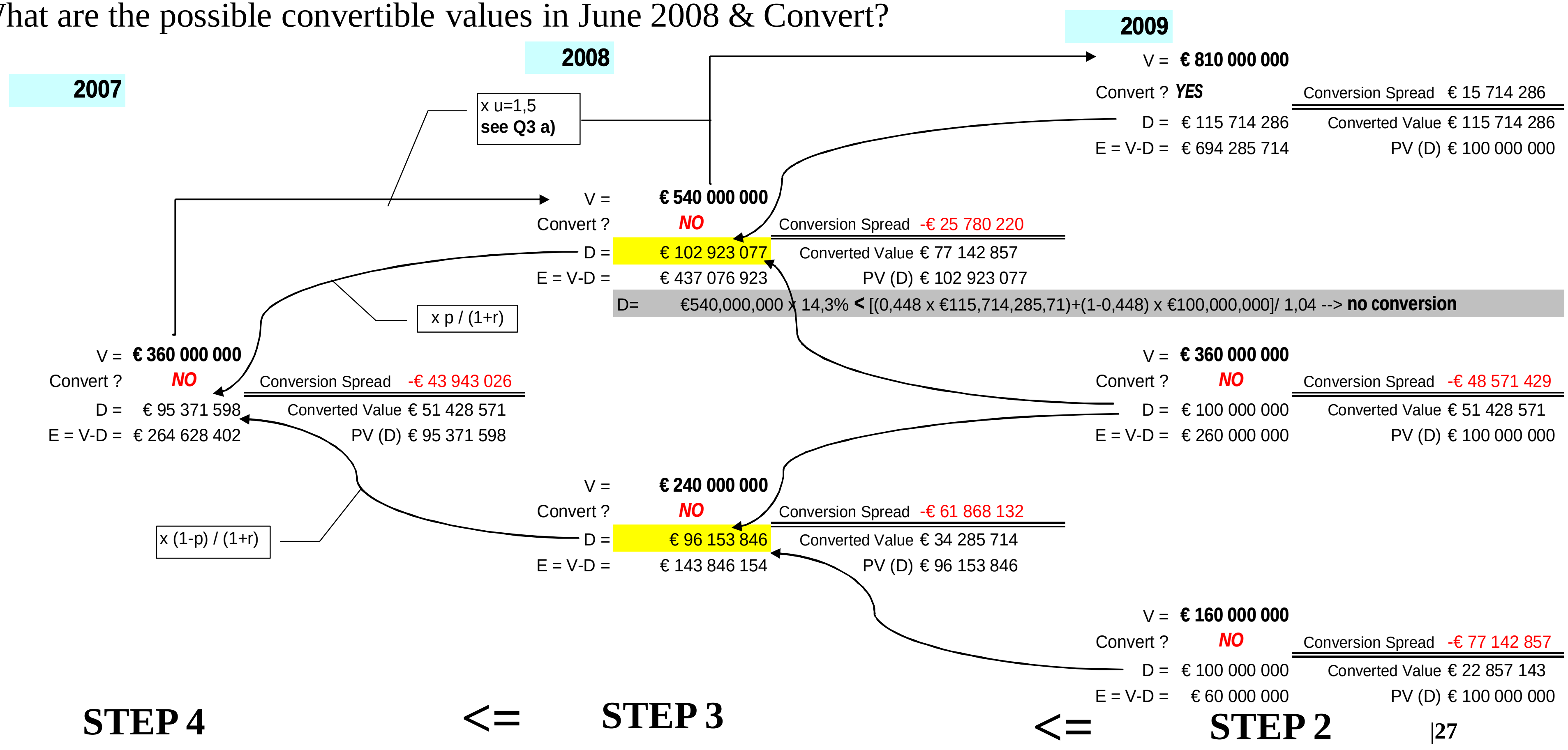
2007	2008	2009 (a) 1)				a) 2	a) 3				
Cpy Value = V	Cpy Value = V	Cpy Value = V	Converted Debt	Unconverted Debt = HOLD	Conversion?	Market Debt D	Equity = V - D	Extra			
	x u=1,5		Cpy Value x Claim ratio	Face Value	if Conv > Face	= MAX (Converted Value; Face Value)			Pre Conversion Eqty Value / Share	# Shares	Post Conversion Eqty Value / Share
x u=1,5		**€ 810 000 000**	€ 115 714 286	€ 100 000 000	**YES**	**€ 115 714 286**	**€ 694 285 714**		€ 101	7 000 000	€ 99
	€ 540 000 000										
€ 360 000 000		**€ 360 000 000**	€ 51 428 571	€ 100 000 000	**NO**	**€ 100 000 000**	**€ 260 000 000**		€ 43	6 000 000	€ 43
	€ 240 000 000										
x d=0,67		**€ 160 000 000**	€ 22 857 143	€ 100 000 000	**NO**	**€ 100 000 000**	**€ 60 000 000**		€ 10	6 000 000	€ 10
	x d=0,67										

STEP 2 => **STEP 3** => **STEP 4**

Q3.b: Possible value of convertibles @T1~ Tree 2

- **STEP 1** = Risk neutral probability: $\text{Prob}_{RN} = p = \dfrac{1+ rf - d}{u - d} = \dfrac{(1+0,04 -0,67)}{1,5-0,67} = \dfrac{0,373}{0,833} = 0,448 \Rightarrow 1-p = 0,55$

(b) What are the possible convertible values in June 2008 & Convert?



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Q3.c&d: Possible value of call & conversion option @T1&2

€ 70	= K
€ 70 000 000	Called debt

2009

V = **€ 810 000 000**
Convert ? ***YES*** see Q3.b)
D = € 115 714 286
Conversion V= € 51 428 571
E = € 694 285 714

2008

V = **€ 540 000 000**
1°Call ? 1,000,000 x €70 <€102,923,077 (see b)) --> **YES** **Payable Call Value < Market value**
2° Convert ? 14,3% x €540,000,000 > 1,000,000 x €70 --> **YES** **Converted Value > Receivable Call Value**
D = € 77 142 857 Converted debt
PV (D) € 102 923 077 *see Q3.b)*
Conversion V= € 77 142 857 *NO see Q3.b)*
E = € 462 857 143

2007

V = **€ 360 000 000**
1°Call ? 1,000,000 x €70 < €95,371,098 --> **YES** Mr D should call the bond
2° Convert ? 14,3% x €360,000,000 < 1,000,000 x €70 --> **NO**
D = € 70 000 000 Called debt
PV (D) € 95 371 598
Conversion V= € 51 428 571 **NO**
E = € 290 000 000

2009

V = **€ 360 000 000**
Convert ? ***NO*** see Q3.b)
D = € 100 000 000
Conversion V= € 51 428 571
E = € 260 000 000

2008

V = **€ 240 000 000**
1°Call ? 1,000,000 x €70 <€96,153,846 (see b)) --> **YES** **Payable Call Value < Market value**
2° Convert ? 14,3% x €240,000,000 < 1,000,000 x €70 --> **NO** **Converted Value < Receivable Call Value**
D = € 70 000 000 Called debt
PV (D) € 96 153 846 *see Q3.b)*
Conversion V= € 34 285 714 *NO see Q3.b)*
E = € 170 000 000

2009

V = **€ 160 000 000**
Convert ? ***NO*** see Q3.b)
D = € 100 000 000
Conversion V= € 22 857 143
E = € 60 000 000

STEP 3:

ANSWER c) =

- It can depend in 2008on scenario: up or down

1. **Up**: Issuer Calls, Holder converts
2. **Down**: Issuer Calls, Holders accept call

- Here in both cases Issuer calls

ANSWER d) = D should call in 2007

STEP 2…

1) Call? by Issuer
2) Convert? by Holder

- In 2008 and 2007

<= STEP 1

Q4: Callable bonds

- Story
  - Freshwater company History
    - Value in volatile tax haven Tongoland
    - Move to the more stable but taxing and neighbouring Bobland resulted in a lower market cap
    - R&D partnership financing agreement with Bobland's main university Ewing State related to the potential development of a new energy drink 'Spirit of Southfork' => option value
  - Today
    - Cash needed for capex (increase in production capacity)
    - Equity raise ruled out…for now
      - ➢ Issue bond
        - ➢ …but part of board convinced interest rates will drop in 1yr

          => issue Callable bond !

Q4: Callable bonds

- DATA
  - Company
    - Freshwater
  - Callable Bond features
    - Coupon = 4,5%
    - T = 2 years
    - Amount = 100 million €
    - Callable in year 1 @ 101 €
    - 1 Yr rate = 4% and its volatility =35%
      - Binomial Node 1: try 2,5%
        => lower so bottom node
  - Market
    - Profile similar to Freshwater
    - Bond06: T 2; 6% coupon; p = 104,01 €

Bond 06		Freshwater	
Maturity	2	Maturity	2
Coupon	6,00%	Coupon	4,50%
Face Value	100,00	Face Value	100,00
Price	104,01	Price	
Call price @ T1	N/A	Call price @ T1	101,00

Q4: Callable bonds

- Questions

➢ Based upon Binomial tree

(a) What would be the **value of an option-free bond** taking into account your interest rate binomial tree?
(b) What is the value of the **callable bond**?
(c) What is the value of the **embedded call option**?

➢ Other

(d) Why is the value produced by a binomial model referred to as an **'arbitrage free model'?**
(e) What would happen to the value of the callable bond **if the expected volatility was higher**?

Q4: Construction of Binomial interest tree

- you have to take a guess for the first node.
  - Asked to try 2.50%

Year	0	1
		5,03% $= 2{,}50\% \times e^{2\sigma}$
	4,00%	
		2,50%

$\sigma = 35\%$

=>

	Year 1	Year 2	Comment
Bond 006 cash-flows	**6**	**106**	
DF @ node r1,H		1,0503	
PV @ node r1,H		100,92	
Bond 006 value @ node r1,H		106,92	= 100,92 + 6
DF @ node r1,L		1,0250	
PV @ node r1,L		103,41	
Bond 006 value @ node r1,L		109,41	=103,41 + 6
Value in 0	104,01		

= 0,5x(106,92/1,04) + 0,5x(109,41/1,04)

--> OK the tree generates a value for the on-the-run issue equal to its market price.

Or alternatively

=>

Bond 006	Comment	Yr 0	Yr 1	Yr 2
CF			6,0	106,0
PV if high IR	Yr2 50% @ 5,03%		100,92	
PV if high IR	Yr 50% @ 2,50%		103,41	
PV	PV Yr1 @ 4,0%, add C!	104,01		

Q4: Binomial tree of Callable Bond

a) **Option-free bond value**

Year	0	1	2	Comments	Check
			100,00	Face	
			4,50	Coupon Yr 2	
		99,49		Face + Coupon in yr2 discounted to yr1	
		4,50		Coupon Yr 1	
	101,17			***PV in 0 of the bond expected V in 1***	**Bond Value**
		101,95		= 0,5x[(99,49+4,5)/1,04]+0,5x[(101,95+4,5)/1,04]	101,17
		4,5			
			100,00		
			4,50		

b) **Callable bond value**

K = 101

Year	0	1	2	Comments		Check
			100,0			
			4,50			
		99,49		also MIN		
		4,5		***PV in 0 of the bond expected V in 1***		**Bond Value**
	100,72			= 0,5x[(99,49+4,5)/1,04]+0,5x[(101+4,5)/1,04]		100,72
		101,00		=Min (Call price,Bond value)		**=Min (101;101,95)**
		4,5		***PV in 1 of the bond expected V in 2 (see a.)***		101,95
			100,0		Call price	101,00
			4,50			

c) **Value of the call** **0,457** = Option-free bond value - Callable bond value

Q4: Callable bonds: Other Questions d) and e)

(d) Why is the value produced by a binomial model referred to as an **‘arbitrage free model’?**

- because the model built produces the same values as the market.
- = the i rate tree is constructed so that the value produced by the model when applied to an on the run issue is equal to its market price.
- It is also said to be **'calibrated to the market'**.

(e) What would happen to the value of the callable bond **if the expected volatility was higher**?

- Callable bond value = option free bond value - option value
  - If volatility increases, option value increases
    - Callable bond value decreases (as option free remains stable)