**STEADY AND TURBULENT FLOW STABILIZATION BY SANDWICH-LIKE VISCOELASTIC COATING: A NATURE INSPIRED SOLUTION**

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**SUMMARY**

Fluid flows in distensible tubes is characterized by a number of unstable modes and flow stabilization is an important problem for the fluid motion in the tubes of the heat and mass exchangers; the artificial heart, oxygenators and other biomedical devices; microfluidic and nanofluidic cells [1-3]. Flow instability in compliant ducts can be the cause of flow-limitation and pressure-limitation effects, wall oscillations, damage of the innermost layer (endothelium) of the duct, and sound generation in veins, airways, larynx and glottis. Many problems of blood flow through stents and grafts, collapse of airways and apnea in snorers, speech generation and others are determined by the flow interaction with compliant walls and flow instability. Therefore, keen understanding of the mechanics of those instabilities and particularly their control is desired.

The system can be stabilized by a proper choice of the rheological parameters of the multilayered viscoelastic coatings [1,2]. The stability of the Poiseuille flow and a fully-developed turbulent flow of the viscous liquid through a thick-wall multi-layer viscoelastic tube with complex rheological parameters is considered. The temporal and spatial eigenvalues of the system are found. Influence of the material parameters of the separate layers and the regime (Reynolds number) on the spatial and temporal amplification rate of the most unstable mode is studied. It is shown that the system possesses both absolute and convective unstable modes. Effects of the rheological parameters of the layers on the amplification rate of the most unstable mode representing an absolute instability are examined. It is shown that the absolute instability of the system can be converted into a convective one, and in some cases the system can stabilized with an appropriate choice of the material parameters. The rheological parameters of the tube layers influence the system stability in different ways. While some of those parameters destabilize the system, others stabilize it. It is found that an anisotropic tube composed of layers possessing distinct rheological values can completely eliminate all absolute instability modes.

**PROBLEM FORMULATION**

The stability of the Poiseuille flow of a viscous incompressible fluid in a multilayered thick-walled viscoelastic tube has been studied by Hamadiche and Kizilova [20] for the no displacement boundary conditions at the outer surface of the wall. In this paper the flow of the viscous liquid through a long viscoelastic thick-walled tube with the inner radius $R$, thickness $h$ and length $L$ is considered at different boundary conditions. The wall is composed of three anisotropic layers with thicknesses $h_1, h_2, h_3$, where $h_1 + h_2 + h_3 = h$. The conservation equations for the fluid are the incompressible Navier-Stokes equations are

$$\nabla \cdot \mathbf{v} = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + \nabla p + \frac{1}{\rho_f} \nabla \cdot \mathbf{\sigma} = 0$$

and the mass and momentum conservation equations for the incompressible wall are

$$\nabla \cdot \mathbf{u}^{(j)} = 0, \quad \rho_w^{(j)} \frac{\partial \mathbf{u}^{(j)}}{\partial t} = -\nabla p^{(j)} + \nabla \cdot \mathbf{\sigma}^{(j)}$$

where $\mathbf{v}$ is the flow velocity, $\mathbf{u}^{(j)}$ is the wall displacement, $\rho_f$ and $\rho_w^{(j)}$ are the mass densities for the fluid and solid layers, $j = 1, 2, 3$ is the number of the layer, $p$ and $p^{(j)}$ are the hydrostatic pressures, $\mathbf{\sigma}$ and $\mathbf{\sigma}^{(j)}$ are the stress tensors for the fluid and the wall layers.

The viscoelastic body with parallel connection of the elastic and viscous properties (Voight model) has been considered for the solid layers

$$\sigma_i^{(j)} = A_{iik}^{(j)} \epsilon_k^{(j)} + \mu_w^{(j)} \frac{\partial}{\partial t} \epsilon_k^{(j)}$$

where $A_{iik}^{(j)}$ is the matrix of elasticity coefficients, $\mu_w^{(j)}$ are the viscosities of the layers, $\epsilon_k^{(j)} = \frac{1}{2} \left( \nabla \mathbf{u}_k^{(j)} + \nabla_k \mathbf{u}_l^{(j)} \right)$ is the strain tensor, $\mathbf{\sigma}^{(j)} = \{ \sigma_{11}^{(j)}, \sigma_{22}^{(j)}, \sigma_{33}^{(j)}, \sigma_{23}^{(j)}, \sigma_{31}^{(j)}, \sigma_{12}^{(j)} \}$ is the stress vector, and $\mathbf{\epsilon}^{(j)}$ is the similar strain vector.

The boundary conditions include the continuity conditions for the fluid velocity and displacement of the inner layer and normal and tangential stresses at the fluid–solid interface; the continuity conditions for the displacements and stresses at the interfaces of the layers respectively:

$$r = R : \quad v = \frac{dq^{(1)}}{dt}, \quad \sigma_n = \sigma_n^{(1)}, \quad \sigma_t = \sigma_t^{(1)}\ldots$$

$$r = R + h_1 : \quad u^{(1)} = u^{(2)}, \quad \sigma_n = \sigma_n^{(2)}, \quad \sigma_t = \sigma_t^{(2)}\ldots$$

$$r = R + h_1 + h_2 : \quad u^{(2)} = u^{(3)}, \quad \sigma_n = \sigma_n^{(3)}, \quad \sigma_t = \sigma_t^{(3)}$$

at the outer surface of the tube for the both no-stress

$$r = R + h : \quad \sigma_n^3 = 0, \quad \sigma_t^3 = 0\ldots$$
and no-displacement
\[ r = K + \pi h : \quad u^{(3)} = 0 \] boundary conditions have been considered, where \( n \) and \( \tau \) denotes the normal and tangential components of the stress tensor.

The solution of the fluid–structure interaction problems (1)–(7) and (1)–(6), (8) has been found as a superposition of the form of the normal mode:
\[
\begin{align*}
\{ \tilde{v}, p \} &= \{ v^0, p^0 \} + \left\{ v^\omega, p^\omega \right\} \exp \left( \pi t + \imath \pi z + \imath \pi \theta \right) \\
\{ u^{(i)}, p^{(i)} \} &= \{ u^{(i)}^\omega, p^{(i)}^\omega \} + \left\{ u^{(i)}^\omega, p^{(i)}^\omega \right\} \exp \left( \pi t + \imath \pi z + \imath \pi \theta \right)
\end{align*}
\]
where \( v^\omega, \ u^{(i)}^\omega, \ p^\omega, \ p^{(i)}^\omega \) are the amplitudes of the corresponding disturbances, \( k = k_p + \imath k_j \), \( s = s_p + \imath s_j \), \( s_i \) is the wave frequency, \( k_p \) is the wave number, \( s_p \) and \( k_j \) are spatial and temporal amplification rates. The steady part \( \{ v^0, p^0 \} \) of (9) is identified with Poiseuille flow. Both isotropic and transversely isotropic materials for the wall layers have been studied.

According to the experimental data [44, 45] the plane of isotropy of each layer is perpendicular to the radial axis and the matrix of elasticity coefficients is the following:
\[
\begin{bmatrix}
(E_i^{(i)})^1 & -v_i^{(i)}(E_i^{(i)})^1 & -v_i^{(i)}(E_i^{(i)})^1 & 0 & 0 & 0 \\
-v_i^{(i)}(E_i^{(i)})^1 & (E_i^{(i)})^1 & -v_i^{(i)}(E_i^{(i)})^1 & 0 & 0 & 0 \\
-v_i^{(i)}(E_i^{(i)})^1 & -v_i^{(i)}(E_i^{(i)})^1 & (E_i^{(i)})^1 & 0 & 0 & 0 \\
0 & 0 & 0 & (G_i^{(i)})^1 & 0 & 0 \\
0 & 0 & 0 & 0 & (G_i^{(i)})^1 & 0 \\
0 & 0 & 0 & 0 & 0 & (G_i^{(i)})^1
\end{bmatrix}
\]
where \( G_{i,j}^{(i)} \) are the shear modules, \( E_{i,j}^{(i)} \) are the Young modules and \( v_{i,j}^{(i)} \) are the Poisson ratios of the layers. The values \( E_{i,j}^{(i)} \) and \( G_{i,j}^{(i)} \) are different for different blood vessels (of elastic or muscle type) and for the normal vessel wall and at some pathological cases including hypertension, atherosclerosis and hyperlipidemia. The values \( v_{i,j}^{(i)} \) for the incompressible materials are close to zero. Since the blood vessel walls are filled with small vessels delivering the blood into the wall (vasa vasorum), some compressibility of the wall material can also be taken into account. The tubes of different technical devices composed from several layers with different viscoelastic parameters can be considered basing on (1)–(8).

The temporal and spatial eigenvalues of the systems (1)–(7) and (1)–(6), (8) have been computed using the technique described in [18, 19, 22]. The problems have a large number of the parameters, and we concentrated on the effect of the Reynolds number, viscous and elastic parameters and thicknesses of the wall layers on the system stability for the axisymmetric disturbances.

RESULTS AND DISCUSSION

The temporal eigenvalues of the system in the complex \( (s_i, s_p) \)-plane have been computed. The modes located near the real axis of the \( (s_i, s_p) \)-plane are solid based and those located near the imaginary axis are fluid based. The modes near the point of origin couple efficiently the solid and fluid motion. The distribution density of the eigenvalues along the real and imaginary axis depends on the elastic and fluid inertia forces determined by the values \( \Gamma \) and \( \Re \). For the cylindrical water column with free boundaries the fluid based modes only remain in the \( (s_i, s_p) \)-plane, whereas for the empty viscoelastic shell the fluid based modes disappear and we can find the solid based ones only. The modes involved efficiently in the fluid–solid interaction are placed near the origin of the complex plane where the unique unstable mode with \( s_p > 0 \) is located.

CONCLUSIONS

Stability analysis of a steady flow in thin- and thick-walled uniform and multilayered viscoelastic tubes revealed some ways to increase the system stability by targeting the most-unstable modes. It was shown that the system instability strongly depends on the rheological properties of the wall. Shear moduli and viscosities of the layers produce the most prominent effects on the temporal and spatial amplification rates and on the group velocity of the unstable mode. When the compliant tube is composed of three layers with the same material parameters (a single-layer, thick-walled tube), the system is found to be unstable. When the material parameters of the layers are different, the system may possess lower temporal amplification rates and can become stable.

The present results point to a novel strategy to eliminate flow-induced wall vibrations and to stabilize a fluid flow through a compliant tube by carefully choosing the values of the shear moduli and viscosities of the different layers. In that way optimal parameters of a composite wall that provide system stabilization can be used to construct sound absorption and vibration damping coatings in aerospace vehicles and noise-generating devices.

REFERENCES