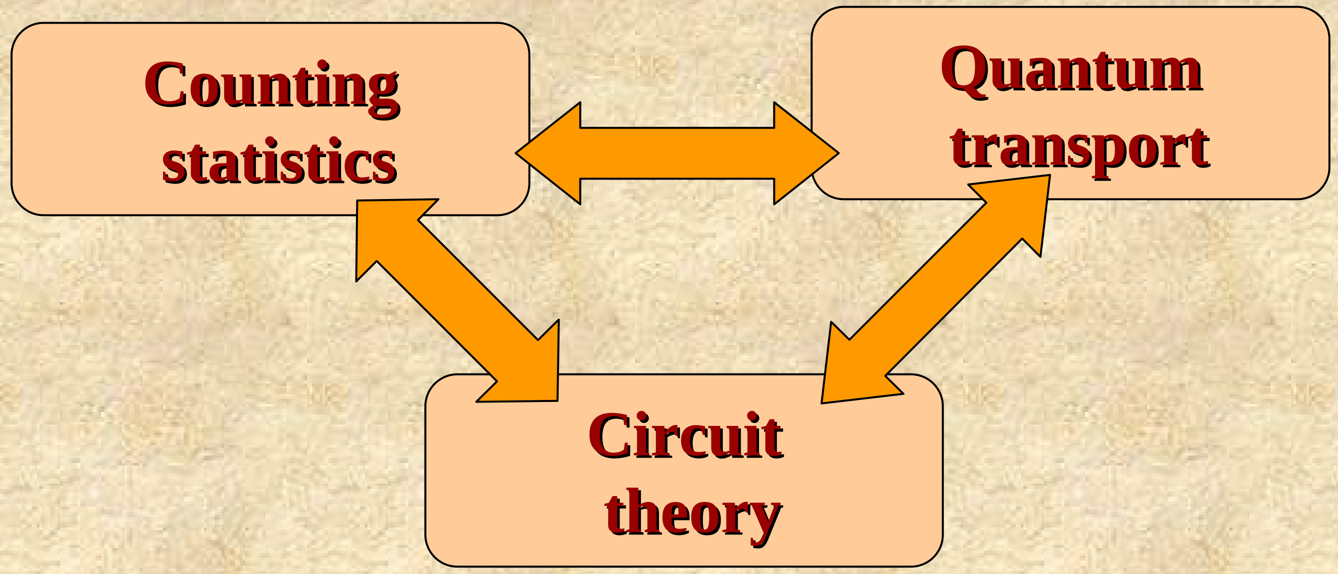
Electron counting statistics



Lecture #1

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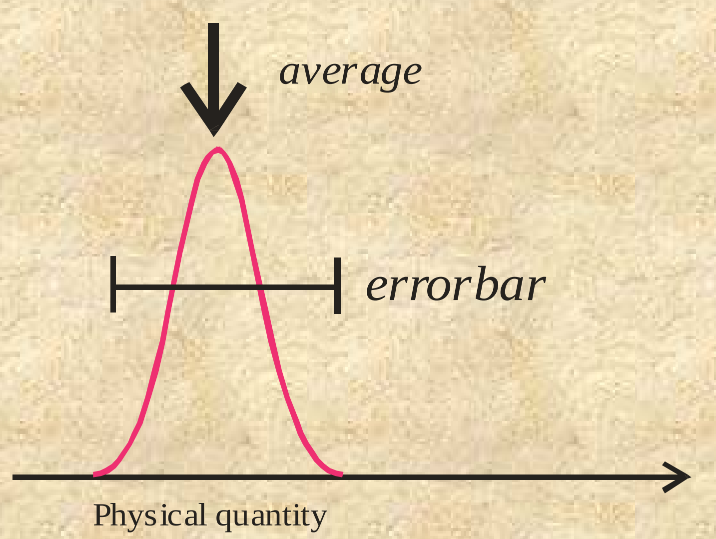
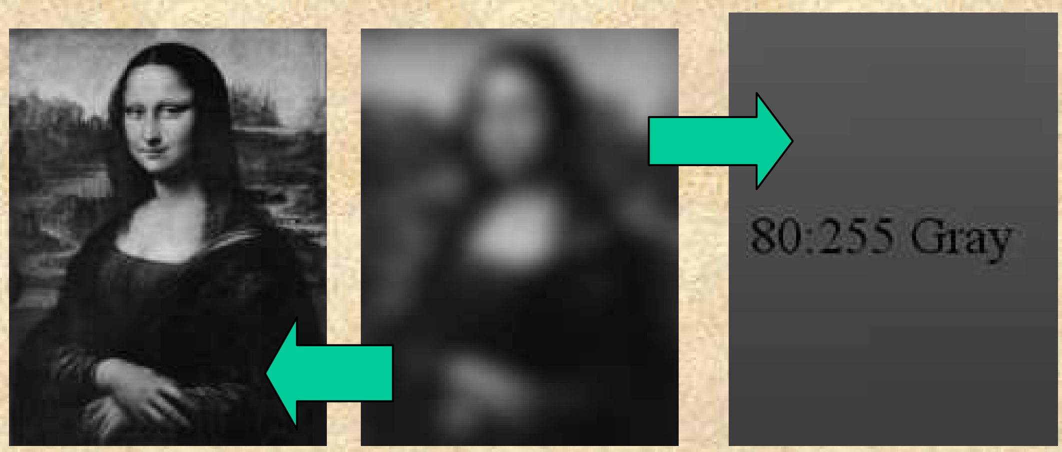
Outline

- Personal motivation
- What is counting statistics?
- What is quantum transport?
  - this lecture: scattering, scattering, scattering...
  - and statistics arising from this

First day at university

- Cosmic ray counter: 12 14 11 9 10 13 14 8
- Average: 11.9 Error: 0.01
- Physics concerns average values

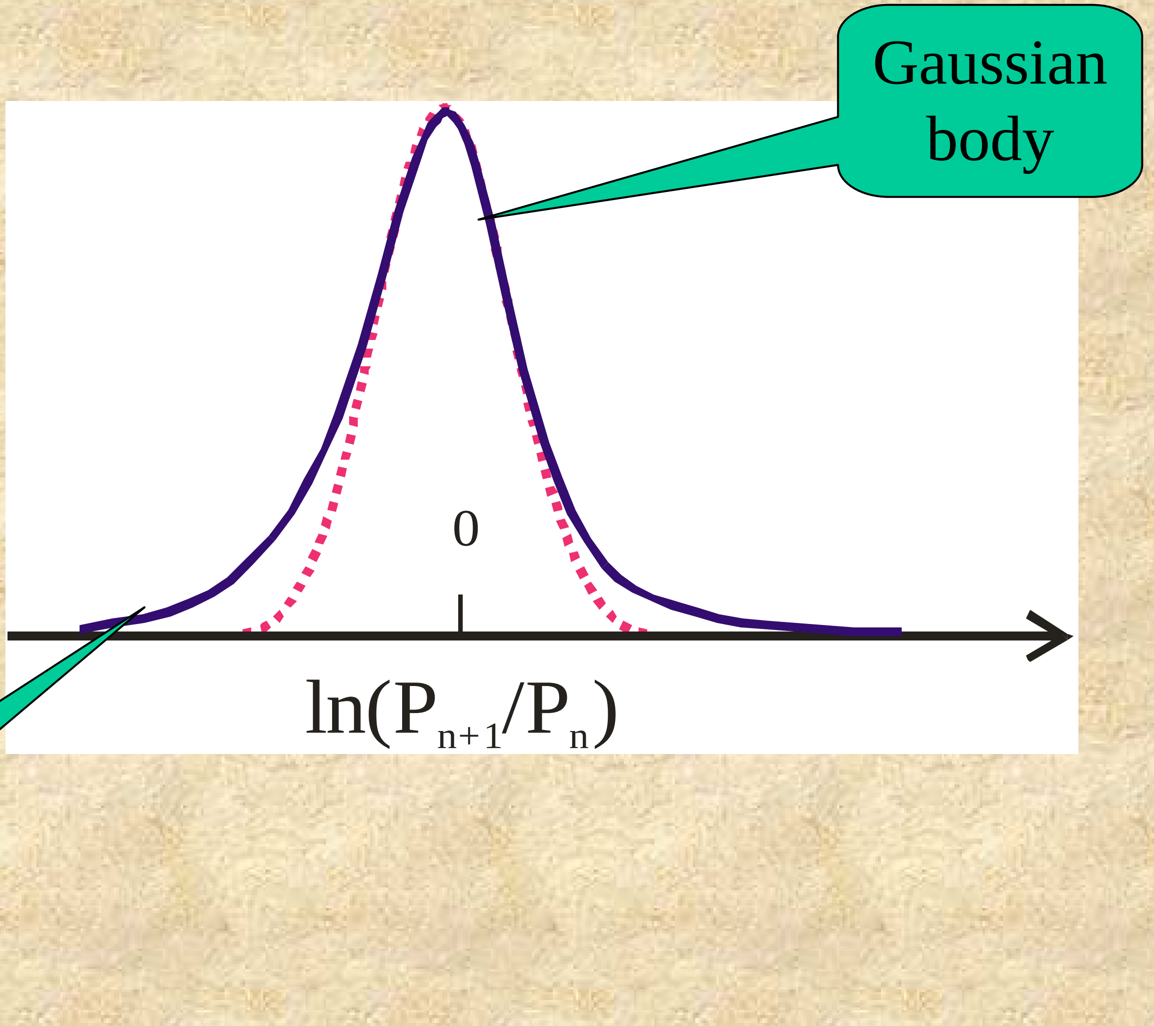




- And many be some gaussian(?) fluctuations
- Is this practical?

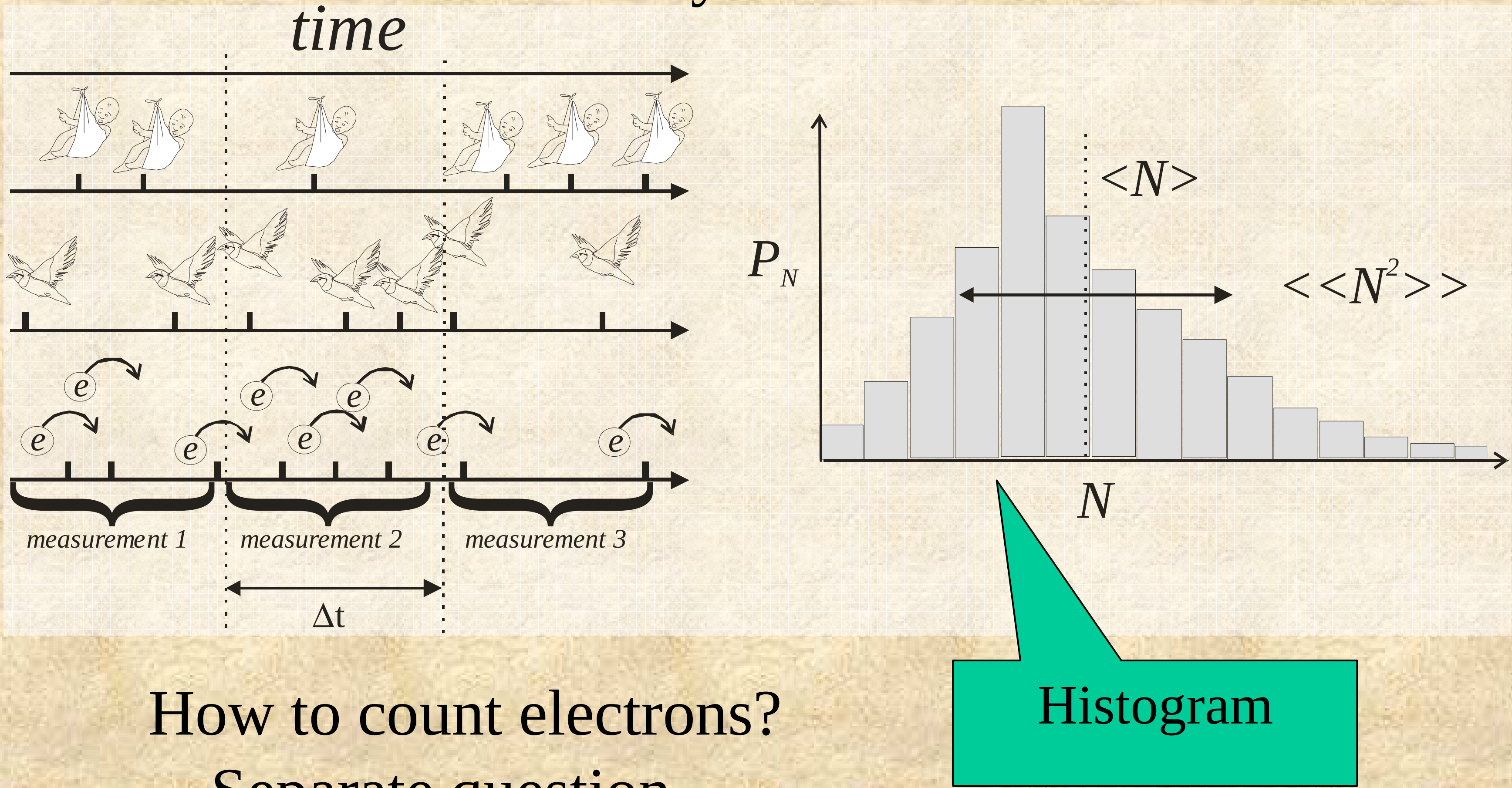
Rich statistics: practical example

- Distribution of daily dividents



Counting statistics

How to count correctly?



How to count electrons?
Separate question...

Characteristic function

$$\Lambda_t(\chi) = \sum_N \exp(i\chi N) P_t(N)$$

- Indepentent events: Λ factorizes
- Time property: $\ln \Lambda_t(\chi) \propto t$
- Elementary event analysis

$$\ln \Lambda_t(\chi) = t \ln(1 + W_1(\exp(i\chi) - 1) + W_2(\exp(i2\chi) - 1) + \ldots + W_M(\exp(iM\chi) - 1))$$

single

Couple

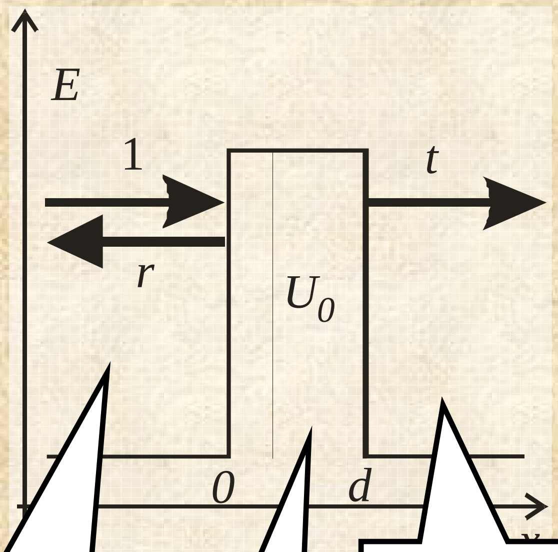
Flock

What is quantum transport?

- Natural sciences:
  - Everything consists of atoms
- Quantum transport
  - This is not important!
  - $$G \geq\leq e^2 / \pi\hbar \equiv G_Q$$

Potential barrier

Intersect the waveguide with a potential barrier U(x)



$$\psi(x) = \exp(ikx) + r\exp(-ikx)$$

$$\psi(x) = A\exp(i\kappa x) + B\exp(-i\kappa x)$$

$$\psi(x) = t\exp(ikx)$$

$$\frac{(\hbar k)^2}{2m} = E; \frac{(\hbar \kappa)^2}{2m} = E - U_0$$

4 unknown variables:

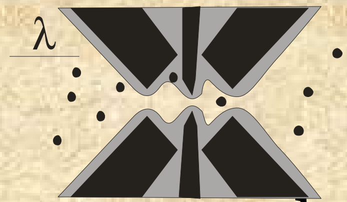
A,B,r,t

4 equations:

continuity of wavefunction and its derivative at two boundaries

Dictionary: reflection and transmission amplitudes

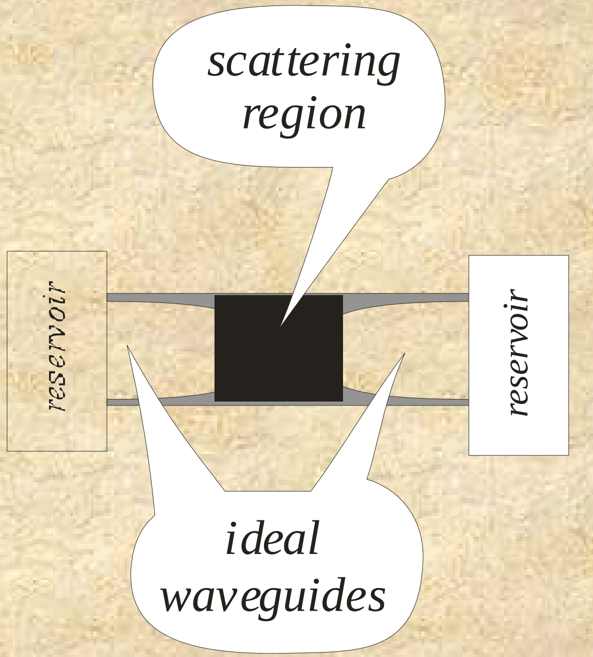
Nanostructure versus waveguide



Nanostructure: can be very complex

Can be modelled as: waveguide with transport channels + potential barrier

Essence: set of transmissions T_n



Enough to describe the transport!

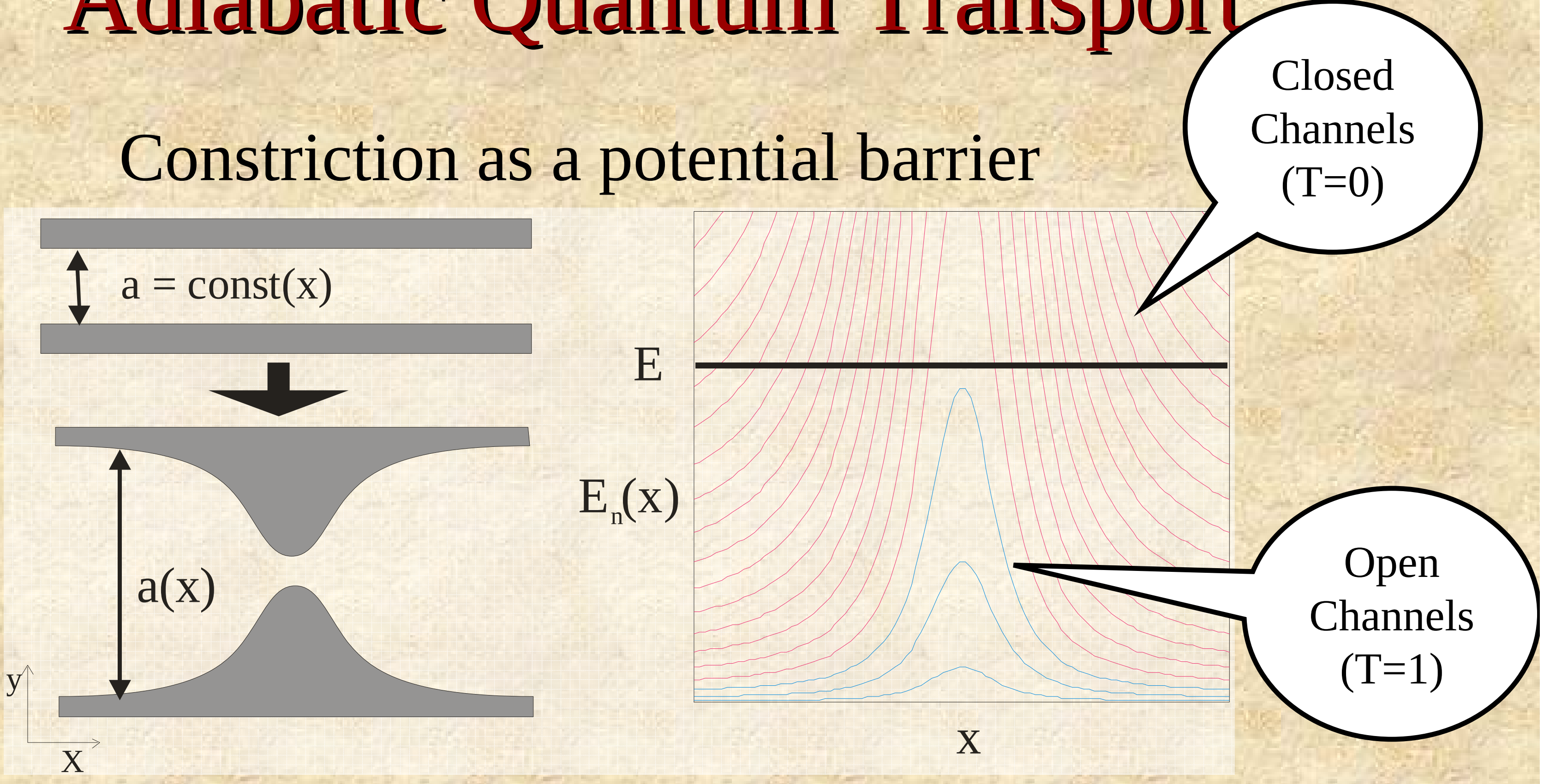
To see this: consider

Adiabatic Quantum Transport,

Quantum Point Contact

Adiabatic Quantum Transport

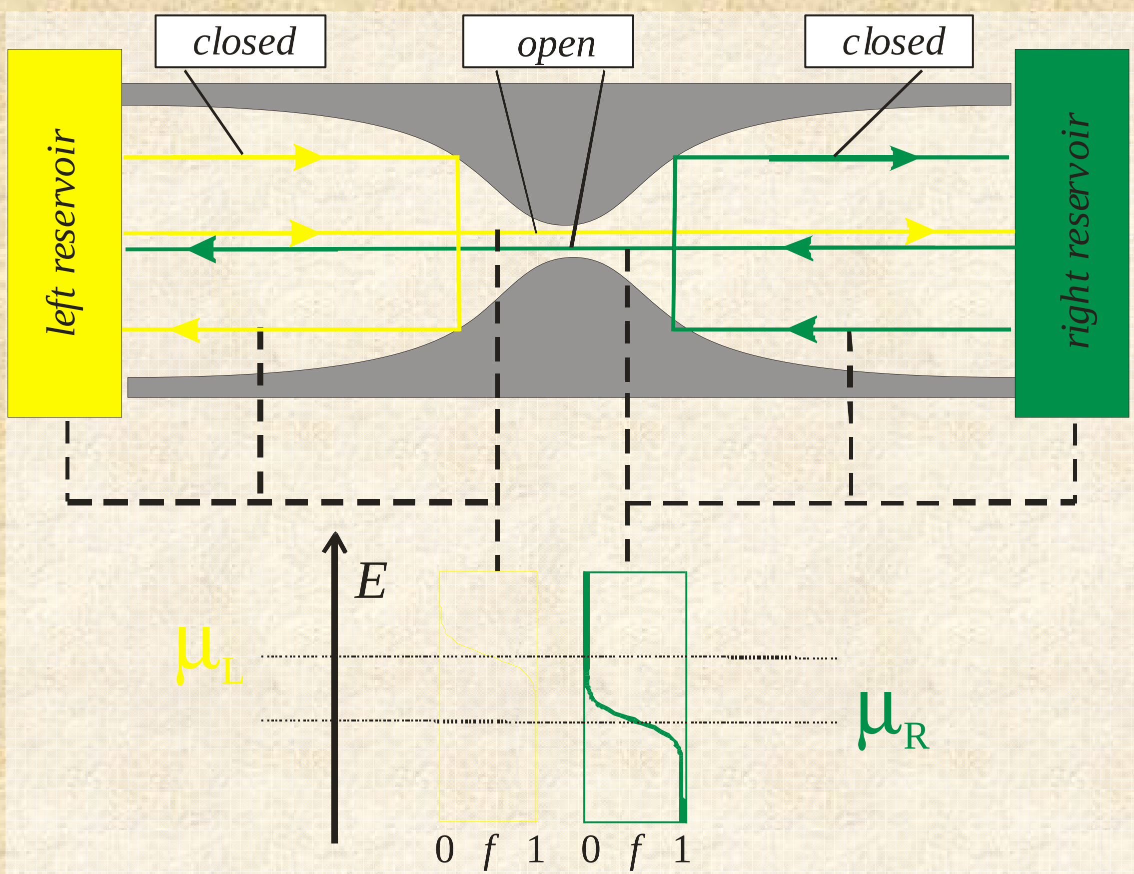
Constriction as a potential barrier



$$E_n(x) = \frac{\pi^2 \hbar^2}{2m}\left(\frac{n_y^2}{a^2(x)} + \frac{n_z^2}{b^2(x)}\right)$$

Current in a QPC

Filling factors are brought from the reservoirs

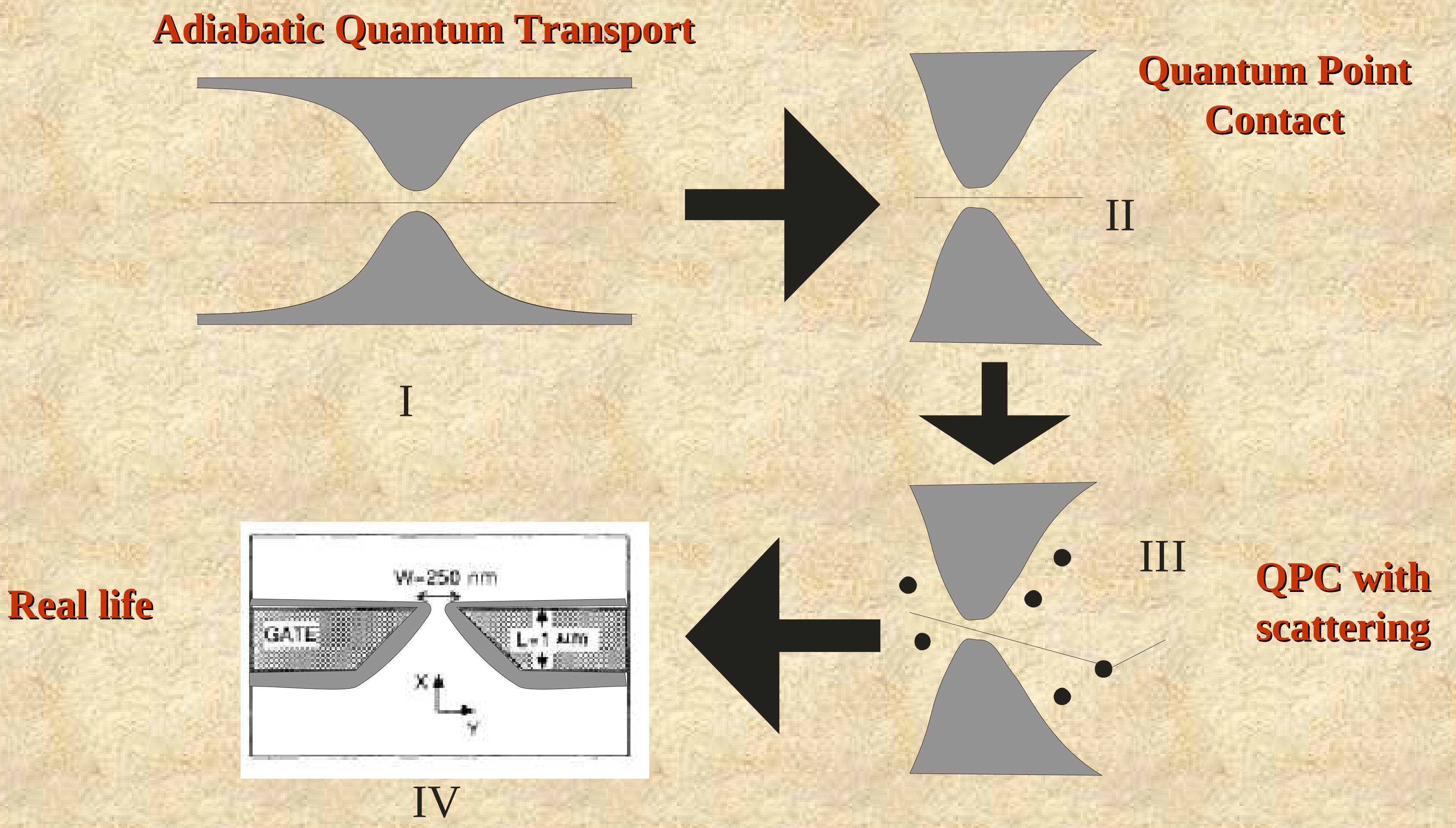


$$N_{att}(t) = = N_{open}\frac{2_s eVt}{2\pi\hbar}$$

Quantized conductance

$$I = \frac{2_s e}{2\pi\hbar}\sum_{n:open}\int dE\,[\,f_L(E) - f_R(E)\,] = \frac{2_s e}{2\pi\hbar}N_{open}(\mu_L - \mu_R) = G_Q N_{open} V$$

Building a Landauer conductor



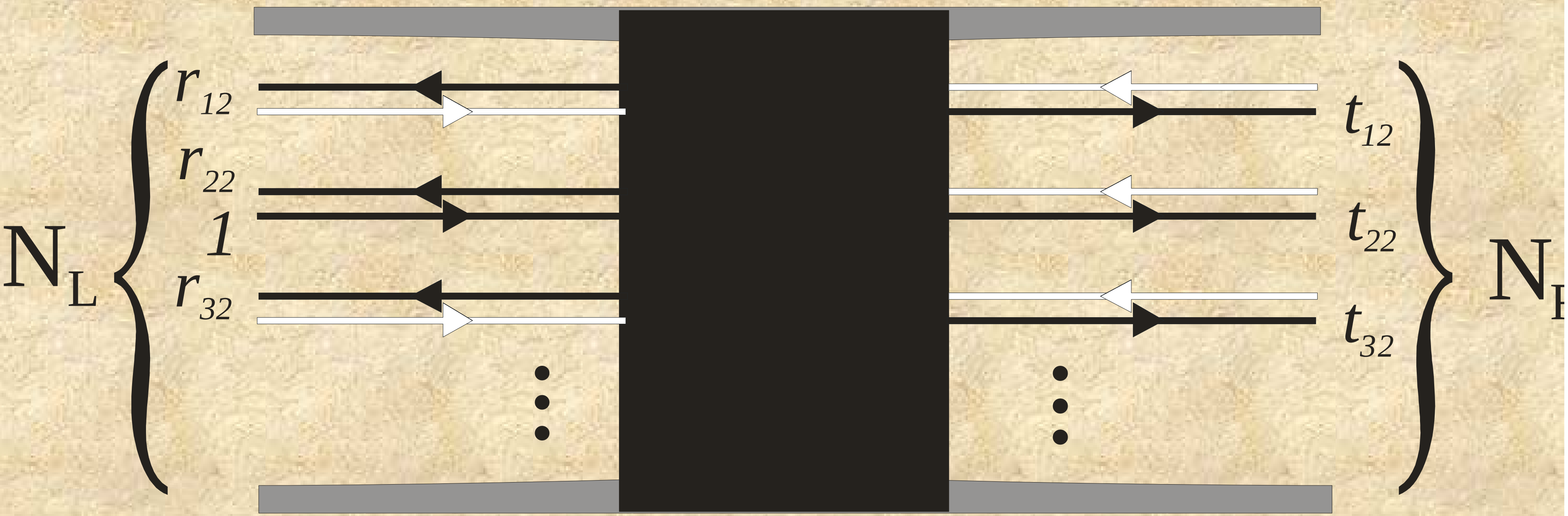
Scattering matrix

$N_R + N_L$ Incoming amplitudes $\vec{a}$

$N_R + N_L$ Outgoing amplitudes $\vec{b}$

$$\vec{b} = \hat{s}\vec{a}$$

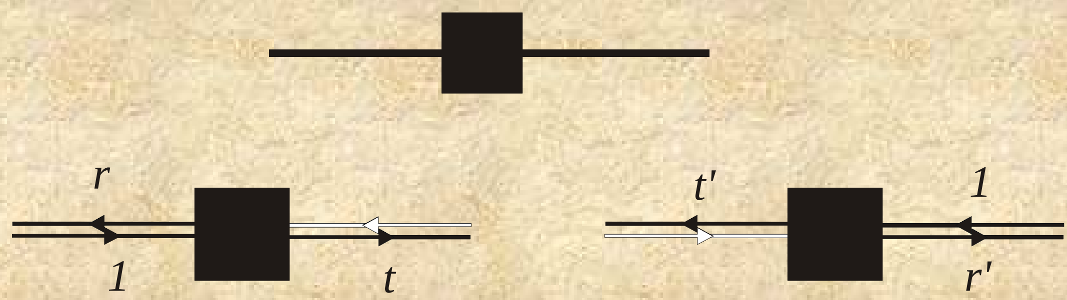
$$\begin{bmatrix} \vec{b}_L \\ \vec{b}_R \end{bmatrix} = \begin{bmatrix} \hat{r} & \hat{t}' \\ \hat{t} & \hat{r}' \end{bmatrix} \begin{bmatrix} \vec{a}_L \\ \vec{a}_R \end{bmatrix}$$



Scattering matrix:properties and example

$$\hat{s}^{+}\hat{s}=\hat{1}$$ unitarity $\Rightarrow$ $$\hat{r}^{+}\hat{r}+\hat{t}^{+}\hat{t}=\hat{1}$$

$$\hat{t}=\hat{t}'$$ Time reversability



One-channel scatterer

$$\begin{bmatrix} b_L \\ b_R \end{bmatrix} = \begin{bmatrix} r & t' \\ t & r' \end{bmatrix} \begin{bmatrix} a_L \\ a_R \end{bmatrix}$$

$$r=\sqrt{R}e^{i\theta};r'=-\sqrt{R}e^{i(2\eta-\theta)};$$

$$t=\sqrt{T}e^{i\eta};T=1-R$$

Restrictions and limitations

Restrictions:

•Elastic scattering: electrons pass without energy loss

•Electrons do not interact (*the same?)*

Limitations:

•Nature is generally merciful

•Electrons do not interact close to Fermi level

• Low temperature, voltage are good

• Short structures are good

• Limitations depend on quantity of interest

Landauer formula

Hermitian matrix $\hat{t}^+\hat{t}$ **Has a set of eigenvalues: transmissions** T_p At each energy E

The current reads:

$$I = G_Q \sum_p \int dE\, T_p(E)(f_L(E) - f_R(E))$$

$$I = G_Q \sum_p T_p \;\; V$$

Simple-minded derivation

One channel:

Reservoir biased at voltage *V* sends $N_{att}(t) = \frac{2_s eVt}{2\pi\hbar}$ **electrons**

Chance to pass: T_0 Charge passed: $Q = e\, T_0\, N_{att}$

Average current:

$$I = Q/t = G_Q\, T_0\, V$$

Many channels: sum over channels

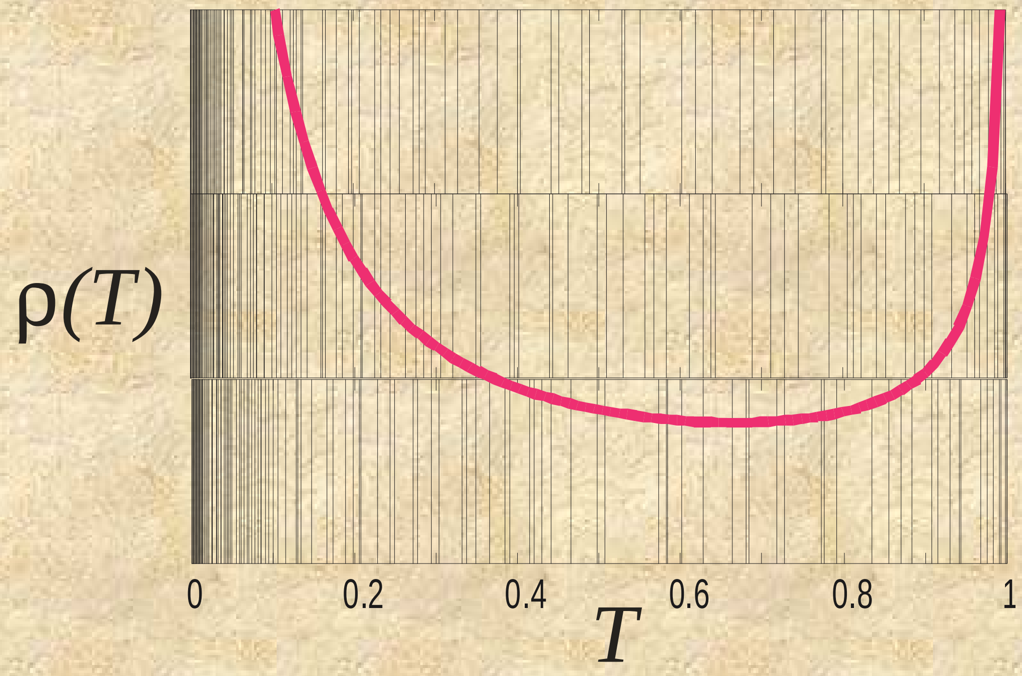
$$I = G_Q \sum_p T_p\, V$$

Transmission distribution

Person ⇒ pin-code ⇐ ATM machine

Quantum Quantum

Contact ⇒ $\boldsymbol{T_p}$ ⇐ Transport



$$\rho(T) = \left\langle \sum_p \delta(T - T_p) \right\rangle$$

Diffusive contact:

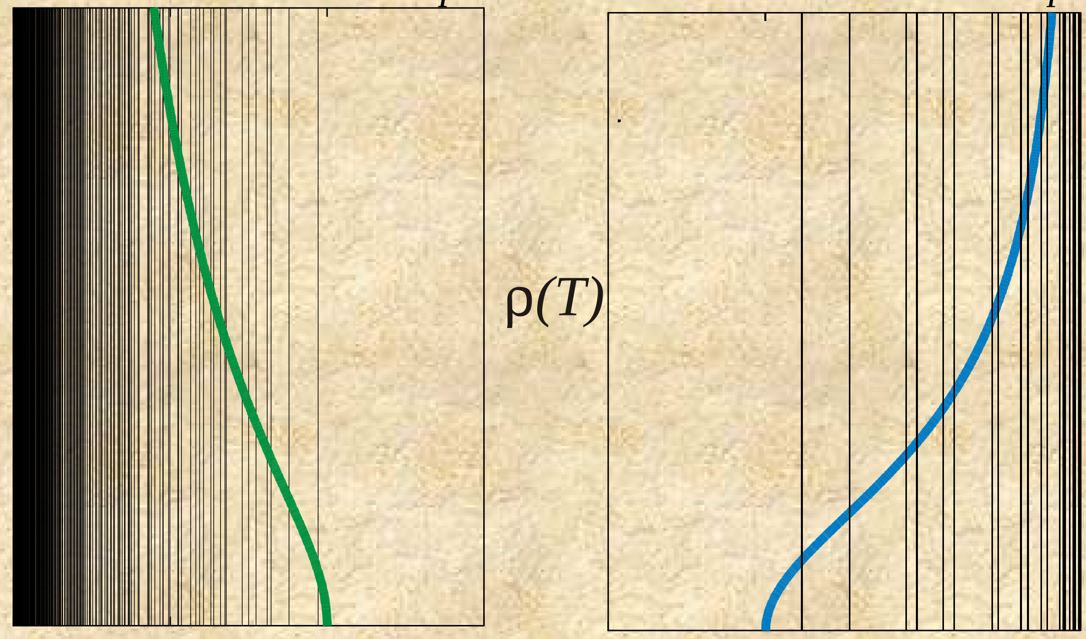
$$\rho(T) = \frac{\langle G \rangle}{2G_Q} \frac{1}{T\sqrt{1-T}}$$

Types of quantum contacts

Quantum contact: an individuality: pin-code $\boldsymbol{T_p}$

Type of a quantum contact:= shape of transmission distribution

Tunnel junction: $T_p \approx 0$ Realistic QPC $T_p \approx 1$



Turn to statistics: Two simple limits

Tunnel junction: $T_n << 1$: rare=independent electron transfers

$$\Lambda_t(\chi) = \exp(\tilde{N}(e^{i\chi} - 1)); \quad \tilde{N} \equiv t < I > / e$$

QPC: T_n=1: electrons are waves: current does not fluctuate

$$\Lambda_t(\chi) = \exp(i\chi\tilde{N}); \quad \tilde{N} \equiv t < I > / e$$

Levitov formula

$$\ln \Lambda(\chi) = 2_s \Delta t \int \frac{dE}{2\pi\hbar} \sum_p \ln \left\{ \begin{aligned} 1 + T_p(e^{i\chi} - 1) f_L(1 - f_R) + \\ T_p(e^{-i\chi} - 1) f_R(1 - f_L) \end{aligned} \right\}$$

Electron transfers

- Independent in different energy strip
- Independent in different channels
- To the left and to the right: are dependent!
- Transfers at negative energy are blocked!

blocking factor

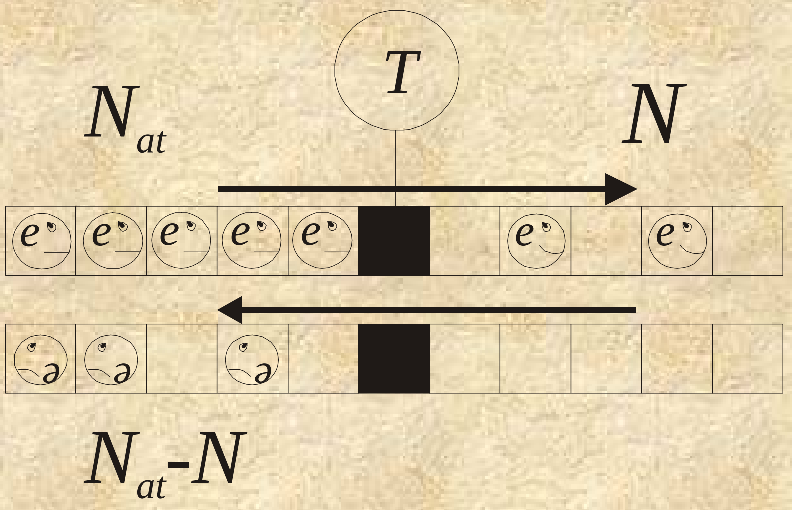
$f_L(1 - f_R)$

Simple example: electrons transferred in one direction

Electrons gambling

$$\ln \Lambda(\chi) = N_{at} \ln\left\{1 + T_0(e^{i\chi} - 1)\right\}$$

$$N_{at} = \frac{2_s eVt}{2\pi\hbar}$$



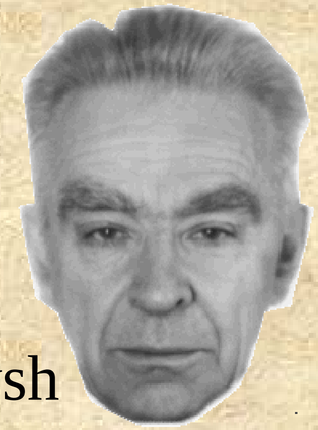
N_{at} = number of game slots

T_0 = winning chance

N = number of games won

$$P_N = \binom{N_{at}}{N} T_0^{\,N} (1 - T_0)^{N_{at} - N}$$

The method: Keldysh



- *N.,'99:* Extension: Generating function = Keldysh action with unequal Hamiltonians
- Difference: counting field

- *Q. M:* Measuring = disturbing (with counting field)
- Remarkable symmetry between usual and counting fields

$$\hat{\tilde{\rho}}(t_1) = \hat{U}^{(+)}(t_0, t_1)\rho(t_0)\hat{U}^{(-)}(t_0, t_1)$$

$$\hat{U}^{(+)} = \mathrm{T}\exp\{-\frac{i}{\hbar}\int_{t_0}^{t_1} dt\ \hat{\mathcal{H}}^{(+)}(t)\}$$

$$\hat{U}^{(-)} = \tilde{\mathrm{T}}\exp\{\frac{i}{\hbar}\int_{t_0}^{t_1} dt\ \hat{\mathcal{H}}^{-}(t)\} \neq (\hat{U}^{(+)})^{-1}$$

$$\hat{\mathcal{H}}^{(\pm)} = \hat{\mathcal{H}} \pm \hat{\mathcal{I}}\chi$$

- Counting statistics of $\hat{Q} = \int_{t_0}^{t_1} dt\ \hat{\mathcal{I}}(t),$

$$\Lambda(\chi; t_1 - t_0) = \mathrm{Tr}\left(\hat{\tilde{\rho}}(t_1)\right)$$

Keldysh Green functions

- Two contours = 2x2 matrix

$$G = -i < T(\psi^{+}(r_1, t_1)\psi(r_1, t_1)) >$$

- Generally: 8x8 (Keldysh x spin x Nambu)
- Reservoirs: isotropic:no coor.dependence

$$\overset{\frown}{G}(t_1, t_2) \Rightarrow \hat{G}$$

$$\boxed{\hat{G}^2 = 1}$$

Conform to Landauer-Buttiker approach: To cancel inside

- Greens –everywhere in the structure
- Inside: irrelevant within the scattering approach
- From coordinate-dependent Greens
- To Greens fixed in the reservoirs
- Answer in terms of scattering matrix

Keldysh action

- Two-terminal

$$A = \frac{1}{2}\sum_{p} Tr\ln\left[1+\frac{T_p}{4}(\hat{G}_L\hat{G}_R+\hat{G}_R\hat{G}_L-\hat{2})\right]$$

Green's with counting field

- Counting field = vector potential in a given cross-section

$$H_{count} = \frac{1}{2}\tau_z \chi \hat{I}$$

- Gauge transform -> to reservoir

$$transform = \exp(i\tau_z \chi)$$

- Common Greens

$$\check{G}_{usual} = \begin{bmatrix} 1 - 2f(E) & -2f(E) \\ -2 + 2f(E) & 2f(E) - 1 \end{bmatrix}$$

- With counting field

$$\check{G} = e^{-i\chi\tau_3/2}\check{G}_{usual}e^{i\chi\tau_3/2} = \begin{bmatrix} 1 - 2f(E) & -2fe^{i\chi} \\ (-2 + 2f(E))e^{-i\chi} & 2f(E) - 1 \end{bmatrix}$$