Random Subsampling and Data Preconditioning for Ground Penetrating Radars

EDISON CRISTOFANI\textsuperscript{1,2}, MATHIAS BECQUAERT\textsuperscript{1,2}, S\'EBASTIEN LAMBOT\textsuperscript{3}, (Member, IEEE), MARIJKE VANDEWAL\textsuperscript{1}, JOHAN STIENS\textsuperscript{2}, AND NIKOS DELIGIANNIS\textsuperscript{2,4}, (Member, IEEE)

\textsuperscript{1}CISS Department, Royal Military Academy, Brussels, Belgium (e-mail: edison.cristofani@elec.rma.ac.be; mathias.becquaert@rma.ac.be; marijke.vandewal@rma.ac.be)
\textsuperscript{2}ETRO Department, Vrije Universiteit Brussel, Brussels, Belgium (e-mail: jstiens@etrovub.be; ndeligia@etrovub.be)
\textsuperscript{3}Earth and Life Institute, Catholic University of Louvain, Louvain-la-Neuve, Belgium (e-mail: sebastien.lambot@uclouvain.be)
\textsuperscript{4}imec, Kapeldreef 75, B3001, Leuven, Belgium

Corresponding author: Edison Cristofani (e-mail: edison.cristofani@elec.rma.ac.be).

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\textbf{ABSTRACT}  
Ground penetrating radars for mine detection can profit from the many advantages that compressed sensing can offer through random subsampling in terms of hardware simplification, reduced data volume and measurement time, or imagery simplification. An intrinsic antenna-ground model is used, canceling the undesired reverberation effects and the very strong reflection from the air-soil interface, producing higher detection rates or even unmasking shallowly buried mines. Extensive Monte-Carlo simulations on real GPR measurements (800-2200 MHz) show an increase in the probability of detection, yielding globally promising exploitable results, whenever the principal component analysis technique is used a as preconditioner, as well as providing lower random subsampling bounds for frequency and spatial measurements (cross-range), whether applied individually or combined.

\textbf{INDEX TERMS}  
basis pursuit denoising, compressed sensing, ground penetrating radar, landmines, principal component analysis, synthetic aperture radar, stepped-frequency continuous-wave

\section{I. INTRODUCTION}

The potential of radars in detecting buried objects or structures has been greatly explored in the past decades \cite{1}, \cite{2}, partially thanks to the readiness of affordable ground penetrating radar (GPR) systems. The ever-growing list of applications varies from military operations (mine detection \cite{3}, wiring or clandestine man-made tunnels), to civil engineering (road pavement maintenance \cite{4}, utility tunnel detection \cite{5}, agricultural and environmental engineering \cite{6}, or discovery and exploration of geological \cite{7} and archaeological structures). These radar systems are in contact with the ground and sometimes operate at short ranges above the ground (e.g., for road inspection). When compared to time-domain radars, GPR systems can be easily implemented using stepped-frequency continuous-wave (SFCW) radars which allow transmitting electromagnetic waves with larger bandwidths and lower power requirements. This implies that high-resolution radar measurements can be performed in a continuous fashion, producing very high data volumes which can pose a tremendous burden in further data transfer, processing and storage. From a practical point of view, data volume and measurement time become two constraints of utmost interest. Compressed sensing (CS), a recent advance in data sampling theory, can be used to reduce those constraints. CS reduces the amount of required measurements with respect to the conventional Shannon-Nyquist sampling theorem \cite{8} and can also simplify the resulting reconstructed reflectivity function by mathematically imposing sparsity in the recovered data and allowing only the strongest elements to be represented. After populating a CS over-determined dictionary with the adequate signal model, random subsampling is applied to achieve subsequent reflectivity
function—the scene—recovery by means of a CS algorithm, in this case basis pursuit denoising (BPDN) [9]. Given that SFCW radars are relatively slow compared to pulse radars, CS enables the interesting possibility of collecting fewer frequencies and measurement positions yet obtaining high probabilities of detection, ensuring the practical exploitation of these radars.

A. CONTRIBUTION

In this paper, a synthetic aperture radar (SAR) simulator replicating a SFCW GPR sensor, antenna and scanner is developed and complemented by real GPR measurements. The simulator is used for modeling point target responses in 2-D GPR measurements, i.e., in depth and cross-range. This simulator not only takes into account the theoretically expected returns from an ideal scene, but it is further improved and adapted to real measurements thanks to the additional modeling of antenna-to-ground electromagnetic interactions, which can become a limiting factor for detecting targets close to the surface. Echoes scattering back from the air-ground interface present relatively high amplitudes, which may mask small objects shallowly buried, such as mines. Filtering these first interface reflections as introduced by [10], [11] is of special interest since most of the anti-personnel (AP) mines are buried only several centimeters deep. To the best of the authors’ knowledge, this or any similar modeling has never been used to reduce the mismatch between the CS dictionary and real GPR measurements, hence yielding an improvement in the probability of detection while keeping a low false alarm rate.

In this paper, three data subsampling strategies, from which a SFCW GPR can benefit, are proposed: reducing the number of discrete frequencies collected directly from the radar (less data volume), reducing the scanning burden by exploiting the data redundancy in the scanning or cross-range dimension in SAR configurations (fewer scanning positions, therefore less data volume and faster measurements), and a combination of both. Additionally, the performance of the GPR can be slightly improved by imposing a data transformation or preconditioner that compacts the energy of high-dimensional data in fewer coefficients—as it is the case of principal component analysis (PCA) [12]—facilitating finding a solution to the minimization problem [13]. The performance assessment as of probability of detection and real measurements performed, whereas Section 4 provides details on how the antenna-ground interactions are modeled in order to reduce their negative effects in the target detection. Finally, the performances after applying several subsampling rates and PCA on real GPR data are shown and discussed in Section 5, from which conclusions are drawn in Section 6.

II. COMPRESSED SENSING

A. PROBLEM BACKGROUND AND FORMULATION

Let us assume that a one-dimensional, fully sampled signal according to the Shannon-Nyquist criterion [8], and of length $N$ is defined as $x \in \mathbb{C}^{N \times 1}$. The signal is said to be compressible if it can be unequivocally represented with $S \ll N$ non-zero coefficients. In any given representation domain $D$, if the condition $\|D(x)\|_0 \leq S$ is satisfied, vector $x$ is said to be $S$-sparse [18] and, therefore, compressible in...
The \( \ell_0 \) pseudo-norm can be used to encode the sparsity of \( x \) as \( \|x\|_0 = \{n : x_n \neq 0\} \).

Let us now define a linear system which describes \( y \in \mathbb{C}^{M \times 1} \) as a subsampled observation of vector \( x \):

\[
y = \Phi x,
\]

where \( \Phi \in \mathbb{C}^{M \times N} \) is the observation or sensing matrix relating both vectors. If \( M < N \) is true, the recovery of \( x \) from \( y \) yields infinite solutions since such system is said to be underdetermined or ill-posed. The recovery of sparse vectors is only feasible if this problem is treated as a minimization problem. A solution using a linear program can be obtained by solving the following problem:

\[
\hat{x} = \arg \min_x \|x\|_0 \quad \text{s.t.} \quad y = \Phi x,
\]

where the sparsest solution maximizing the number of zeros in the estimate of \( x \) is achieved. The implementation of the \( \ell_0 \) pseudo-norm in this context is problematic and reconstructing \( x \) becomes a non-convex, NP-hard problem to solve [19]. Hence, a relaxed version of the minimization problem is commonly proposed by using the \( \ell_1 \) norm instead:

\[
\hat{x} = \arg \min_x \|x\|_1 \quad \text{s.t.} \quad y = \Phi x.
\]

where \( \|x\|_p = (\sum_{i=1}^n |x_i|^p)^{1/p} \). This optimization problem is known as basis pursuit (BP) [9] and can be theoretically solved with high probability if the following conditions are satisfied: the null space property [20], the mutual coherence property [21] and the restricted isometry property (RIP) [22], [23] of full order \( Q \), by means of which the matrix \( \Phi \) and all possible \( \binom{N}{Q} \) submatrices \( \Phi_q \) must comply with the following expression for any given \( \delta_q \in (0, 1) \):

\[
(1 - \delta_q) \|x\|_2 \leq \|\Phi x\|_2 \leq (1 + \delta_q) \|x\|_2.
\]

For fairly large sensing matrices, testing the RIP condition for all submatrices \( \Phi_q \) becomes infeasible [24]. In order to circumvent the RIP condition, it is common to use a matrix populated following a random distribution [24]–[26], e.g., Gaussian, \( \phi_{m,n} \sim \mathcal{N}(0, 1) \), however this solution cannot be implemented for radar sensors. For a full-rate SFCW radar measurement, the sampling matrix \( \Phi \) is defined as the identity matrix \( I_N \), which will keep all \( N \) measurements. By applying stochastic sampling [27], that is, randomly selecting \( M \) rows in \( I_N \) and storing them in \( \Phi \), only \( M \) elements from \( x \) are kept in \( y \).

The observation vector \( x \) might not be sparse by itself but it may be described sparsely by a set of coefficients \( \sigma \in \mathbb{C}^{K \times 1} \) if a sparsifying representation matrix \( \Psi \in \mathbb{C}^{N \times K} \) is available, therefore decomposing vector \( x \) as:

\[
x = \Psi \sigma.
\]

The matrix \( \Psi \) can be interpreted as a dictionary and includes all possible dictionary atoms \( \Psi_k \) describing the measurement \( x \). By substituting (5) in (3), one can obtain:

\[
\hat{\sigma} = \arg \min_\sigma \|\sigma\|_1 \quad \text{subject to} \quad y = \Phi \Psi \sigma.
\]

A new matrix \( \Theta = \Phi \Psi \), with \( \Theta \in \mathbb{C}^{M \times K} \), is defined as a subsampled dictionary of \( K \) atoms, or a dictionary in which only \( M \) samples per atom have been randomly kept.

An error tolerance threshold \( \epsilon > 0 \) is introduced whenever noise is to be considered as a part of the minimization problem. This modification to BP is known as basis pursuit denoising [9] and allows reconstructing noisy measurements within the imposed tolerance:

\[
\hat{\sigma} = \arg \min_\sigma \|\sigma\|_1 \quad \text{subject to} \quad \|y - \Theta \sigma\|_2^2 \leq \epsilon.
\]

Nowadays, available algorithms and computing power allow for using BPDN implementations while offering higher performance rates [28]–[30] than the simpler and faster OMP [31], which is preferable for very large data sets. Given the possibility of subdividing GPR measurements in smaller patches, we use the BPDN algorithm in all the experiments presented.

### B. THE SFCW SIGNAL MODEL

A SFCW ultra-wideband (UWB) radar sequentially synthesizes and transmits \( N_f \) pulses consisting of continuous waves at discrete frequencies \( f_n \) for a transmission duration \( T_f \) per frequency and, eventually, covering the system full bandwidth \( B \) in a complete period \( T_p = N_f \cdot T_f \) (see Fig. 1). The frequency step size is defined as \( \Delta f = B / (N_f - 1) \), whereas each \( n \)-th discrete frequency is \( f_n = f_0 + n \Delta f \), with \( f_0 \) being the minimum frequency in \( B \). SFCW systems are considered to be extremely tunable and because of their narrow-band frequency operation, their hardware is relatively simple and affordable.

The resolution in distance within any material can be computed as \( \Delta d = c / 2(N - 1) \Delta f = c / 2B \), with \( c = \ldots \)
The speed of the waves in a given medium, $c_0$, and in vacuum, $c_0$, is proportional to the frequency $f$, and the relative permittivity of the medium, $\epsilon_r$.

A general expression for a transmitted signal using SFCW is:

$$s_T(t) = \sum_{n=0}^{N_f-1} \text{rect} \left( \frac{t - nT_p}{T_p} \right) \cdot \exp (j2\pi [f_0 + n\Delta f] t),$$

where $\text{rect}(\cdot)$ is the rectangle function. For the sake of simplicity, neither the variations in the antenna gain for each of the discrete frequencies nor their interactions with the ground or inspected materials are considered here. Indeed, these effects can be filtered out from the radar data [10], as mentioned in Section 4. However, the signal model at reception does not neglect the antenna radiation diagram $G(\beta_p)$, squared for the round-trip case, and the impinging angle $\beta_p$ of the $p$-th element in the illuminated region, limited by the antenna opening angle $\theta$:

$$s_R(t) \propto \sum_{p=0}^{P-1} G^2(\beta_p) \cdot s_T(t - \tau_p).$$

In the detection stage, the received and transmitted signals are mixed as in $s_p(t) = s_T(t) \cdot s_R^*(t)$, with $s_p^*(t)$ denoting the complex conjugate of $s_R(t)$, creating beat signals with frequency components proportional to the round-trip distance $\tau_p$ of the $P$ received echoes. An equivalent result is obtained if a vector network analyzer (VNA) is used in one-port configuration to compute the reflection or $S_{11}$-parameters for each $N_f$ discrete frequency. Taking into account the transmitted and received complex amplitudes in the frequency domain, or $S_R(f_n)$ and $S_T(f_n)$ respectively, the reflection $S$-parameters are:

$$S_{11}(f_n) = \frac{S_R(f_n)}{S_T(f_n)}.$$  

Finally, because a wide beamwidth antenna is used, a unique phase history for every target is created during a measurement in which the SFCW sensor moves along the scene (along-track). This history links adjacent measurements in the along-track dimension with each other as described by SAR theory [32], making possible the concept of random subsampling in along-track positions. The cross-range resolution of the sensor can be derived as $\Delta_{\text{cr}} \approx D/2 = \alpha/(2f_c \sin \theta)$, where $D$ is the aperture length in the scanning dimension and $f_c$ is the center frequency.

C. ADAPTING CS TO GPR

The motion of the scanner where the VNA is mounted on generates 2-D GPR data at full sampling rates. For each of the $N_{\text{cr}}$ along-track steps in the measurement, a total of $N_f$ frequencies is acquired. These matrices of size $(N_{\text{cr}} \times N_f)$ are vectorized and used to populate the full-observation vector $x$ of size $N$. The full observation is then subsampled in software emulating a CS receiver by using $y = \Phi x$. The dictionary matrix $\Psi$ is populated column-wise using a SFCW SAR simulator as done in [15], with $\psi_{q,r}$ being the frequency values of the expected returns from a given point target in a known position $(q, r)$ in the scene, with $q = 1, 2, \ldots, N_{\text{cr}}$, $r = 1, 2, \ldots, N_d$, and $N_d$ the number of depth positions considered in the dictionary:

$$\Psi = [\psi_{1,1}, \psi_{1,2}, \ldots, \psi_{1,N_d}, \ldots, \psi_{N_{\text{cr}},N_d}] = \begin{bmatrix} f_{\psi_{1,1}}^{\psi_{1,1}} & f_{\psi_{1,1}}^{\psi_{1,2}} & \ldots & f_{\psi_{1,1}}^{\psi_{1,N_d}} \\ f_{\psi_{2,1}}^{\psi_{1,1}} & f_{\psi_{2,1}}^{\psi_{1,2}} & \ldots & f_{\psi_{2,1}}^{\psi_{1,N_d}} \\ \vdots & \vdots & \ddots & \vdots \\ f_{\psi_{N_{\text{cr}},1}}^{\psi_{1,1}} & f_{\psi_{N_{\text{cr}},1}}^{\psi_{1,2}} & \ldots & f_{\psi_{N_{\text{cr}},1}}^{\psi_{1,N_d}} \\ f_{\psi_{1,1}}^{\psi_{2,1}} & f_{\psi_{1,1}}^{\psi_{2,2}} & \ldots & f_{\psi_{1,1}}^{\psi_{2,N_d}} \\ \vdots & \vdots & \ddots & \vdots \\ f_{\psi_{N_{\text{cr}},1}}^{\psi_{N_{\text{cr}},1}} & f_{\psi_{N_{\text{cr}},1}}^{\psi_{N_{\text{cr}},2}} & \ldots & f_{\psi_{N_{\text{cr}},1}}^{\psi_{N_{\text{cr}},N_d}} \\ \end{bmatrix}$$

(11)

The reflectivity values of every possible point target in the scene are defined as a vector

$$\sigma = [\sigma_{1,1}, \sigma_{1,2}, \ldots, \sigma_{1,N_d}, \ldots, \sigma_{N_{\text{cr}},N_d}]^T,$$

known as the reflectivity function of the scene. The total size of the scene to reconstruct is defined by the number of range-cross-range cells, each one of them containing one point target and of size equal to $\Delta_{\text{cr}} \times \Delta_{\text{cr}}$. The assumption that the scene is physically sparse — only few bright objects are expected to be in the scene and scattering energy back to the sensor — implies that vector $\sigma$ is sparse as well.

D. SUBSAMPLING STRATEGIES AND IMPLICATIONS

For an application like GPR, in which the scanner moves in the cross-range direction during data acquisition, the following subsampling strategies can be considered as initially proposed by [15]:

1) Subsampling in discrete frequencies

Let $k_f = n_f/N_f < 1$ be defined as the subsampling factor in the frequency domain. $n_f$ discrete frequencies can be randomly selected following an interspacing governed by a uniform random distribution, or $d_{n_f} \in [d_{f_{\text{min}}}, d_{f_{\text{max}}}]$. The minimum and maximum interspacings are defined as $d_{f_{\text{min}}}, d_{f_{\text{max}}} \in N_{>0}$, respectively, satisfying $1 \leq d_{f_{\text{min}}} < d_{f_{\text{max}}}$. The data volume produced after subsampling in frequency is directly proportional to $k_f$.

2) Subsampling in the scanning or cross-range dimension

Given the nature of wide-beam measurements, a certain target can be illuminated from several scanning positions which can lead to collecting unnecessary data from a sparsity-based
point of view. This principle described by SAR theory enables the possibility of withdrawing a number of the typically required cross-range positions and still obtain successful results [15], [33]. Scanning positions are randomly selected following the factor \( k_{xy} \), similarly as in frequency subsampling, discarding the full acquisition when a cross-range position is avoided. The maximum inter-spacing between kept cross-range positions is limited by the antenna footprint at a reference range, i.e., the antenna beamwidth.

3) Cross-range and frequency subsampling

A combination of the two previous strategies may lead to significantly lower data volumes proportional to \( k_f \times k_{xy} \). However, heavily subsampling in one dimension may limit the ability to subsample in the other dimension.

### E. DATA PRECONDITIONING

The principle of sparse data recovery lies behind the idea of representing data in a sparse way in some given domain. Often, data in the acquisition domain is not sparse enough or simply not sparse at all and, therefore, an adequate data transformation or preconditioner must be found and incorporated in the CS recovery problem [13].

Recalling (7), data preconditioning can be applied to the optimization problem by transforming both the dictionary \( \tilde{\Theta} = T \Phi \Theta \) and subsampled measurements \( \tilde{y} = T \Phi \tilde{y} \), resulting in:

\[
\tilde{\sigma} = \arg \min_{\sigma} \| \sigma \|_1 \text{ subject to } \| \tilde{y} - \tilde{\Theta} \sigma \|_2^2 \leq \epsilon,
\]

being \( T \Phi \in \mathbb{C}^{M \times M} \) a subsampled version of a given transformation matrix \( T \in \mathbb{C}^{N \times N} \) following the random subsampling sequence as dictated by \( \Phi \).

For the application herein presented, the PCA sparsifying transform will be used. PCA, which has been previously applied successfully to sparsify SAR data [34], is an unsupervised machine-learning method used for extracting features from high-dimensional data. It relies on the availability of a training data set which must be large and representative enough. Given the matrix \( \Theta \) which includes the training data set of the expected returns in its columns, its autocorrelation matrix is calculated as \( R = \Theta^* \Theta (N - 1) \), being \( R \) a square matrix of rank \( M \). Previously, the mean of each atom in \( \Theta \) has been subtracted. The matrix \( \tilde{R} \) is then decomposed as \( \tilde{R} = \Sigma \Sigma^* \), where two new matrices are defined as \( T \in \mathbb{C}^{M \times M} \) or the eigenvectors matrix, \( \Sigma \in \mathbb{C}^{M \times M} \) with the eigenvalues in its diagonal. PCA allows data dimensionality reduction since most of the energy is reduced to few transformed coefficients, therefore discarding the noise subspace, uninteresting during the minimization problem. For the measurements shown in this work, only the first 5% of the PCA coefficients concentrating most of the energy are used.

### F. THE EFFECTS OF SPARSIFYING DATA

The ability of any representation space to make a vector \( x \) compressible or sparser can be estimated by sorting in vector \( x^+ \) all the elements of \( x \) with normalized amplitudes in descending order \( (x^+ \geq x^+_{n+1}) \). An adequate compressibility is reached when \( x^+ \) follows an abrupt decay which can be expressed as the power law \( |x^+_n| \leq C \cdot n^{-1/p} \), being \( p \leq 1 \) and \( C \) any constant [35], [36].

In Fig. 2 the normalized, sorted amplitudes of a full Nyquist-sampled GPR measurement, which includes all three targets and the original measurement noise, are plotted against the original, non-transformed data after being transformed using PCA. The latter shows a slow decay, indicating that sparsity is low and recovery may not be feasible unless the scene shows a certain degree of sparsity by itself. The data transformed using PCA shows higher energy compaction properties, foreseeing potentially better results with respect to non-transformed data in terms of scene reconstruction and regardless of the scene sparsity.

### III. THE GPR SYSTEM AND MEASUREMENTS

In order to demonstrate the above-mentioned concepts, we acquired radar data in controlled laboratory conditions at The Netherlands Organization for Applied Scientific Research (TNO) (The Hague, The Netherlands) [37], [38]. To perform GPR measurements of buried mines in a sandbox (see Fig. 3), a scanner carrying a VNA (Rohde&Schwarz FSH6), able to perform 1-D or 2-D scans generating 2-D or 3-D data, respectively, was connected by means of a coaxial cable to a single doubled-ridge Schwarzbeck Mess-Elektronik BBHA 9120A horn antenna. This single antenna was simultaneously used both for transmission and reception of the signals generated by the VNA at a standoff distance of 200 mm from the ground. A total of 301 discrete frequencies were obtained per measurement within the frequency range of 800-2600 MHz, or a step size of 6 MHz. The higher part of the bandwidth ranging from 2200 to 2600 MHz was not used due to limited signal-to-noise ratio (SNR) in that bandwidth as a result of surface roughness and less accurate antenna calibration within that frequency range, limiting the actual
A sandbox hosted at TNO was used to obtain the presented GPR measurements. The scanner carries a portable VNA and horn antenna placed at a standoff distance of 200 mm from the ground.

### TABLE 1: Selected operation parameters of the used GPR.

<table>
<thead>
<tr>
<th>VNA</th>
<th>Antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>R&amp;S FSH6</td>
</tr>
<tr>
<td>Freq. sweep</td>
<td>0.8–2.6 GHz</td>
</tr>
<tr>
<td># freqs., $N_f$</td>
<td>301</td>
</tr>
<tr>
<td>Freq. step, $\Delta f$</td>
<td>6 MHz</td>
</tr>
<tr>
<td>Used freq.</td>
<td>0.8–2.2 GHz</td>
</tr>
<tr>
<td>Aperture</td>
<td>BBHA 9120A</td>
</tr>
<tr>
<td>Length</td>
<td>142×245 mm</td>
</tr>
<tr>
<td>3dB E/H (1 GHz)</td>
<td>45/30°</td>
</tr>
<tr>
<td>3dB E/H (2 GHz)</td>
<td>27/22°</td>
</tr>
<tr>
<td>Antenna-to-ground</td>
<td>200 mm</td>
</tr>
</tbody>
</table>

bandwidth to 800–2200 MHz [39]. The most representative parameters of the GPR system are listed in Table 1.

A 10-m × 10-m area and 3-m deep sandbox hosted at TNO (The Hague, The Netherlands) was used to perform the measurements processed further on in this work. Three types of mines were buried at different depths (between 100 and 300 mm deep) and separated horizontally 750 mm from each other (see Fig. 4 for details). The sandbox contained a homogeneous distribution of thin, dry sand, which ensured a minimal attenuation, a uniform relative permittivity of the ground for all depths and measurement positions and, therefore, uniformity in the data derived from measurements. Under these ideal conditions, the effects of subsampling on target reconstruction and detection can be better quantified without the interference of undesired effects found in real conditions such as possible clutter generated by rocky substrates, isolated rocks buried in the inspected scene or non-uniform water content.

The mines used in this experiment were two AP mines (PMN2 and M14 types) and one typical anti-tank (AT) mine, as described in Table 2. The physical size and metal content (structure and fuse) of those mines are determining factors for estimating the reflectivity of the scene. Most AP mines have a low metal content and are physically small, whereas AT mines are the opposite. It can be derived that, under identical illumination conditions, the AT mine will show a much higher radar cross-section than the AP mines and, thus, produce significantly stronger echoes. Since AP mines must be shallowly buried and because of their low radar cross-section, they can be easily confused in GPR imagery with the first reflection from the air-ground interface or antenna reverberations.

### IV. REMOVAL OF ANTENNA EFFECTS

Due to the variations of impedance between the radar measurement plane, the antenna feed point and the aperture, multiple wave reflections inherently occur within the antenna and between the antenna and the medium. These reflections affect the radar images and may hide targets of interest and produce ghost objects. A radar antenna can be efficiently modeled using the intrinsic, closed-form antenna equation [10], [40], which has never been used before in combination with CS. Such modeling implicitly decomposes the backscattered field in a finite series of homogeneous fields and, for each homogeneous field, accounts for all transmissions and reflections occurring within the antenna. Considering the medium to be in the far-field following the criterion in [41] and the same antenna as simultaneous transmitter and receiver, this radar equation reduces to [10]:

$$ S(\omega) = \frac{S_R(\omega)}{S_T(\omega)} = R_i(\omega) + \frac{T(\omega)G(\omega)}{1 - G(\omega)R_s(\omega)}, $$

where $S(\omega)$ denotes the radar signal expressed here as the ratio between the backscattered field $S_R(\omega)$ and incident field $S_T(\omega)$ at the radar reference measurement plane. $\omega$ is the angular frequency, $R_i(\omega)$ is the global reflection
coefficient of the antenna for fields incident from the radar reference plane onto the source point of the antenna. \( T(\omega) = T_{\text{s}}(\omega)T_{\text{r}}(\omega) \) with \( T_{\text{s}}(\omega) \) being the global transmission coefficient for fields incident from the radar reference plane onto the source point and \( T_{\text{r}}(\omega) \) being the global transmission coefficient for fields incident from the field point onto the radar reference plane, and \( R_{\text{s}}(\omega) \) is the global reflection coefficient for fields incident from the medium onto the field point of the antenna. The Green’s function \( G(\omega) \) is defined as the scattered \( x \)-directed electric field \( E_{\text{s}}(\omega) \) at the field point for a unit-strength \( x \)-directed electric source \( J_{\text{s}} \) situated at the same location at some distance from a planar multilayered medium. The Green’s function is derived using a recursive scheme to compute the global reflection coefficients of the multilayered medium in the spectral domain [42]. The transformation back to the spatial domain is performed by evaluating numerically a semi-infinite integral, for which a fast procedure is applied [43]. The antenna characteristic coefficients \( R_{\text{i}}(\omega) \), \( T(\omega) \) and \( R_{\text{s}}(\omega) \) are obtained through a specific calibration procedure [10], [40]. Once these antenna functions are known, the response of the layered medium only, i.e. \( G(\omega) \), without antenna effects, is calculated from the radar measurements as:

\[
G(\omega) = \frac{S(\omega) - R_{\text{i}}(\omega)}{T(\omega) + S(\omega)R_{\text{s}}(\omega) - R_{\text{i}}(\omega)R_{\text{s}}(\omega)}. \tag{15}
\]

After applying the modeling as in \( S_{\text{R}}(\omega) = S_{\text{R}}(\omega)G(\omega) \), the time zero from the radar reference plane is moved to the antenna phase center. When a local target is present in the multilayered medium, this calibration still provides good results for removing antenna effects [11]. Antenna effects can be filtered out either in far- of near-field conditions [10], [44]. In addition, the method has been validated in real field conditions (e.g., [6], [45], among others) as well as in presence of underground targets (e.g., [37], [39], among others).

In Fig. 5, the effects of the antenna effect removal are visible to the naked eye. The hyperbolic phase history of the AT mine is clearly visible before and after applying the filtering due to its large size and because of being sufficiently far from the strong reflection of the surface and the antenna internal reverberations. In contrast, the smaller and shallower mines are barely detectable without the calibration procedure whereas with it, they both are safely spotted. Moreover, the depths for all targets are corrected as expected.

V. EVALUATION OF EXPERIMENTAL RESULTS

A. OVERVIEW

The subsampling strategies described in Subsection II-D are tested on real GPR measurements in order to evaluate the lower bounds and how robust against noisy measurements the CS reflectivity function recovery approach is. To do so, Monte-Carlo simulations were performed for every combination of subsampling rates in frequency components and scanning positions, and a range of additive white Gaussian noise with a SNR varying from \(-20\) dB to \(+10\) dB. Each combination was executed multiple times using different pseudo-random sampling patterns to decorrelate as much as possible the recovery results from the random selection of samples.

B. THE CA-CFAR DETECTOR

Since the exact ground truth of the scene is not known, the probability of detecting a target under a given subsampling rate, noise contribution, and within a predefined region is assessed by using the CA-CFAR detection approach as...
main metric [46]. Moreover, CA-CFAR detectors have been successfully used in the GPR field for characterizing the performance of the radar [47], [48]. In this work, the tested region contains only one single target. As depicted in Fig. 6, the scene is subdivided into square cells of size $\Delta d \times \Delta xr$ (or $60 \times 80$ mm), that is one depth and one cross-range bin in their respective dimensions. Three types of cells can be identified: the cell or cells under test (CUT), the guard cells and the training cells. It is of common practice to force guard cells to avoid having an undesired contribution or leakage from the CUT into the training cells, which could lead to a raise in the mean noise power and producing artificially high thresholds, possibly jeopardizing the performance of the CA-CFAR detector.

The following hypothesis model is used by the CA-CFAR detector to determine whether a target is located within the CUT:

$$H_0: \text{only noise is present in the CUT},$$
$$H_1: \text{a target and noise are present in the CUT},$$

which are the null and alternative hypothesis, respectively. The decision is made by the CA-CFAR detector after comparing the test variable $\left| T_C \right|^2$ (the power of the CUT) against the threshold $T_h$:

$$\left| T_C \right|^2 \geq \frac{H_1}{H_0} T_h.$$  \hspace{1cm} (16)

Numerous threshold definitions exist for CA-CFAR [49]–[51]. The one used in this work is defined as $T_h = k \cdot P_n$, where $P_n$ is the average noise power level of the training cells, and $k = n_{RC} \cdot (P_{fa}^{-1/n_{RC}} - 1)$ is a scalar with $P_{fa}$ a selected probability of false alarm and $n_{RC}$ the number of reference or training cells available. The CUT selection is based on the availability of an approximate ground truth obtained from SAR measurements processed using the full data sampled at Nyquist rate.

C. RESULTS: REFLECTIVITY FUNCTION RECOVERY FROM REAL DATA AND SYNTHETIC DICTIONARIES

In Fig. 7, the three buried mines can be clearly seen: the scene reflectivity function $\sigma$ was recovered after using a subsampling factor of 0.5 both in frequency ($k_f$) and cross-range positions ($k_{xr}$) directly applied onto raw VNA measurements. The AT mine is clearly detectable thanks to the spread trace of its energy in cross-range —following the phase history from cross-range positions 0.3 to 0.7 m— whereas the AP mine only becomes detectable after applying the antenna-ground modeling. In any case, the mines positions may be reconstructed correctly using CS provided that adequate subsampling rates be chosen, bypassing the need of performing SAR processing on the non-transformed data. Such promising results were obtained under a favorable set of measurement parameters, that is, no added noise other than measurement noise, antenna-ground modeling, and subsampling rates well above their respective lower limits. After multiple tests, we noticed that the position of the targets may vary slightly in depth and/or cross-range if subsampling rates close to the lower subsampling bounds are selected or strong noise levels are present, although the latter is an improbable hypothesis for measurements under controlled conditions. We assumed that, in both cases, the mismatch between the dictionary and the actual measurements becomes too large, hence increasing target reconstruction uncertainty. This location uncertainty is especially important for the smaller and shallower PMN2 and M14 mines, which show significantly lower backscattered energy than that of AT mines.

D. PERFORMANCE AND ROBUSTNESS ANALYSIS

The lower subsampling bounds for the discussed GPR application can be investigated in terms of the system performance under different reconstruction conditions. Frequency subsampling rates of 0.05, 0.15, and 0.25, and cross-range subsampling rates of 0.1, 0.3, and 0.5 are evaluated in both independent and combined fashions using Monte-Carlo simulations. The CA-CFAR detector yields probabilities of detection ($P_d$) for a given target to be detected after a CS reconstruction and for a probability of false alarm set to $P_{fa} = 10^{-2}$. In order to evaluate the performances in an unbiased way, the number of trials per reconstruction is set to 500, each of them using a different subsampling pattern. Robustness is evaluated by adding white Gaussian noise to the non-transformed data, emulating measurement or sensor noise which artificially worsens the quality of the data, and performing CS reconstructions on those noisy data. The simulated noise level ranges from $-20$ to +10 dB in steps of 2 dB.

The curves depicted in Fig. 8 show the $P_d$ for non-transformed data and PCA-transformed data and several subsampling rates (left column, subsampling in frequency; right column, subsampling in scanning positions) as yielded by the above-mentioned CA-CFAR detector and in agreement with the described Monte-Carlo simulation parameters. The
FIGURE 8: $P_d$ for several subsampling rates for the AT, PMN2 and M14 landmines. Results using the raw and the PCA-transformed data are compared. Values for $k_f$ range from 0.05 to 0.25 (left column), whereas $k_{xr}$ ranges from 0.1 to 0.5 (right column). The SNR ranges from $-20$ to $+10$ dB. For extracting every probability of detection, a $P_{fa} = 10^{-2}$ was chosen and 500 Monte-Carlo trials were run using CA-CFAR. For all figures, the solid and dashed lines show the $P_d$ curves using or not using the ground-antenna modeling, respectively.
TABLE 3: Reconstruction position deviation: percentage of the reconstructions (using PCA and 500 trials) at the correct depth bin, or one or two depth bins away from the ground truth (GT). Several values for \( k_{xrr} \), \( k_f \) and SNR levels are used for each landmine type. The probabilities of detection for each case are also displayed.

<table>
<thead>
<tr>
<th>( k_{xrr} )</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>( k_f )</th>
<th>0.05</th>
<th>0.15</th>
<th>0.25</th>
</tr>
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<tbody>
<tr>
<td>SNR (dB)</td>
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<td></td>
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<td>SNR (dB)</td>
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<tr>
<td>( P_d(%) )</td>
<td></td>
<td></td>
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<td></td>
<td>( P_d(%) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% at GT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>% at GT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% ±1 bin</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>% ±1 bin</td>
<td></td>
<td></td>
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<tr>
<td>% ±2 bins</td>
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<td></td>
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<td></td>
<td>% ±2 bins</td>
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</table>

Table 3.A Reconstruction position deviation for the AT landmine.

<table>
<thead>
<tr>
<th>( k_{xrr} )</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>( k_f )</th>
<th>0.05</th>
<th>0.15</th>
<th>0.25</th>
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<tbody>
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<td></td>
<td></td>
<td>SNR (dB)</td>
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<tr>
<td>( P_d(%) )</td>
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<td></td>
<td>( P_d(%) )</td>
<td></td>
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</tr>
<tr>
<td>% at GT</td>
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<td></td>
<td></td>
<td></td>
<td>% at GT</td>
<td></td>
<td></td>
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<tr>
<td>% ±1 bin</td>
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<td>% ±1 bin</td>
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<td></td>
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<tr>
<td>% ±2 bins</td>
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<td></td>
<td>% ±2 bins</td>
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</table>

Table 3.B Reconstruction position deviation for the PMN2 landmine.

<table>
<thead>
<tr>
<th>( k_{xrr} )</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>( k_f )</th>
<th>0.05</th>
<th>0.15</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>SNR (dB)</td>
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<tr>
<td>( P_d(%) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( P_d(%) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% at GT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>% at GT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% ±1 bin</td>
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<td></td>
<td></td>
<td></td>
<td>% ±1 bin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% ±2 bins</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>% ±2 bins</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.C Reconstruction position deviation for the M14 landmine.

thicker blue curves correspond to the \( P_d \) obtained using the full Nyquist-rate data set and the optimum matched-filter-based SAR processing [52] to provide a comparison of the reconstruction performances using CS against those of the theoretical optimum processing.

In Fig. 8(b) and 8(d), two very similar performances are obtained for the AT landmine where \( P_d \) reaches well above 0.9 for SNR \( \geq 0 \) dB and \( k_{xrr} \geq 0.1 \). The PCA transform produces negligible \( P_d \) increases when the filtering is used, whereas the increase is more noticeable if not used. In fact, during the dimensionality reduction step, the discarded noise subspace contains the undesired effects caused by the internal reverberations of the antenna and the air-ground interface, therefore partially filtering out those effects and increasing the \( P_d \). In both cases, for \( k_{xrr} \geq 0.3 \) and SNR \( \leq -10 \) dB the \( P_d \) curves appear to outperform the performances using the matched filter SAR processing. This is due to the very bright return from the AT landmine and simplification of the reconstructed scene produced by the BPDR algorithm and the fact that scene sparsity is forced: the cells used for noise learning and thresholding purposes happened to be less affected by the added Gaussian noise. In Fig. 8(a) and 8(c), it can be seen that for \( k_f = 0.05 \) and \( k_f = 0.10 \) PCA outperforms the non-transformed data results, reaching \( P_d \geq 0.9 \) for SNR = -4 dB and SNR = -6 dB, respectively. However, the use of PCA does not significantly improve performances for \( k_f > 0.1 \).

As for the plastic landmines, PMN2 and M14, the effect of using PCA is less evident although it follows the same reasoning: it helps increasing the \( P_d \) when frequency subsampling is applied (especially for lower \( k_f \) figures) but it is negligible when subsampling happens in the cross-range dimension (valid for all \( k_{xrr} \) figures). Nevertheless, it is worth mentioning that the antenna-ground modeling employed for ground reflection removal enables unmasking and detection of these small landmines, which would remain undetectable otherwise (\( P_d \leq 0.2 \) for all cases studied).

In general, for all the results presented, the performance curves appear to be less affected when subsampling in the cross-range dimension is applied. As an example, we compare separately the subsampling coefficients \( k_f = k_{xrr} = 0.2 \), for which the final data volumes prior to the CS reconstruction are the same. In all cases, the obtained \( P_d \) show worse results for \( k_f = 0.2 \) than \( k_{xrr} = 0.2 \), causing a 0.15 \( P_d \) difference for the cases Fig. 8(a) and 8(b) at SNR = -10 dB or Fig. 8(g) and 8(h) for SNR = 0 dB; and reaching up to a 0.2 \( P_d \) difference for SNR = 6 dB, for Fig. 8(k) and 8(l). On the one hand, this difference is present under the less favorable conditions (low SNR) for the AT landmine experiments (very bright target) and under all favorable conditions for the other two landmines experiments. On the other hand, tests using \( k_{xrr} = 0.05 \) showed that the \( P_d \) drastically dropped to unexploitable results since in most cases there were not enough cross-range measurements to
adequately reconstruct the hyperbolic phase history of the target or directly there were no available measurements at all (several adjacent cross-range measurements may be skipped, creating a shadowing effect in the target region). From these results it is derived that the performance is more sensitive to subsampling in frequency than in cross-range although the worsening appears more gradually in the former, dropping faster to $P_d$ values close to 0 for the latter.

In Fig. 9, the different $P_d$ for a combined cross-range-frequency subsampling strategy are displayed. Fig. 9(a) ($k_{xf} = 0.1$) presents overall low $P_d$ values for any $k_f$-SNR combination, since the total amount of data (proportional to $k_f \times k_{xf}$) appears to be insufficient even if high SNR’s are considered. In this case, $P_d = 0.83$ is obtained for the combination $k_f = 0.25$ and SNR = 10 dB with non-transformed data, and $P_d = 0.89$ with PCA-transformed data. As expected, as $k_{xf}$ increases, $P_d$ reaches 1 for lower $k_f$ values, even under fairly noisy experiments. The benefit of using PCA —thanks to its robustness against noise— is shown in Fig. 9(c) and 9(f); the two figures show a similar behavior for the Monte-Carlo experiments using $k_f = 0.25$. The $P_d$ results for PCA are $0.02 - 0.03$ higher than those of non-transformed data. As $k_f$ decreases down to $k_f = 0.1$, it can be seen that the $P_d$ (Fig. 9(c)) remains above 0.95 for PCA and for $SNR = -2$ dB, whereas for non-transformed data (Fig. 9(f)) this only happens for $k_f \geq 0.2$. If more favorable conditions are assumed ($SNR \geq 10$), $P_d > 0.9$ or $P_d = 1$ can be obtained just by keeping 1.5% of the original data with a combined subsampling strategy of $[k_{xf} = 0.3, k_f = 0.05]$ or 2.5% with $[k_{xf} = 0.5, k_f = 0.05]$ (Fig. 9(e) and 9(f), respectively).

The same experiments for the combined subsampling strategy are performed for the PMN2 and M14 landmines, whose results are depicted in Fig. 10. The diagrams obtained from experiments not using the antenna-ground modeling are not displayed since no landmine detections were possible without ground reflection and reverberation removal, regardless of the noise or subsampling strategy combination. Moreover, only the PCA-transformed results are displayed given the almost identical results produced using the non-transformed data. For the two mines tested, the probabilities of detection are clearly lower than those of the AT landmine, which is expected due to their much lower backscattered energy. For the most severe cross-range subsampling ($k_{xf} = 0.1$), $P_d$

<table>
<thead>
<tr>
<th>k_{xf}</th>
<th>Non-transformed data</th>
<th>PCA-transformed data</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>0.3</td>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
<tr>
<td>0.5</td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

FIGURE 9: $P_d$ vs. SNR for several subsampling rates $k_{xf} = 0.1 - 0.5$ and $k_f = 0.05 - 0.25$ for the AT landmine. Left column, original non-transformed data; right column, PCA-transformed data. A probability of false alarm of $10^{-2}$ was chosen and 500 Monte-Carlo trials were run. A CA-CFAR detector was used to extract the $P_d$ values.
achieves a maximum figure of 0.576 for the PMN2 mine and 0.463 for the M14, both for \( k_f = 0.25 \). If \( k_{xr} = 0.5 \) is used, \( P_d \) raises to 0.858 and 0.56, respectively, also for \( k_f = 0.25 \).

### E. POSITIONING DEVIATION AFTER RECONSTRUCTION

In the subtables of Table 3 the effects in the positioning of positive targets with respect to the ground truth for different SNR levels when using random subsampling are displayed as the percentage of times targets are reconstructed within the correct depth bin or one or two depth bins away from the ground truth. As a complementary metric to \( P_d \), these tables give a global idea of how trustworthy target positioning is after CS reconstruction, in this case on PCA data sets. From Subtable 3.A it can be extracted that reconstruction location deviations stay minimal when cross-range subsampling is applied and under unfavorable noise conditions of \( \text{SNR} = 0 \) dB. In the worst subsampling case \( (k_{xr} = 0.1) \), the reconstructions in the ground truth depth bin reach 85.4% whereas 12.5% happen one bin away and the rest, three or more bins away. Keeping the same noise conditions, 96.9% of the reconstructions are in the ground truth for \( k_{xr} = 0.3 \) and 100% for \( k_{xr} = 0.5 \). Correct positioning of the reconstructions shows to be less affected by frequency subsampling and added noise, since \( k_f = 0.15 \) yields comparable results to \( k_{xr} = 0.5 \) in terms of \( P_d \) and positioning, both reaching 100% for \( \text{SNR} \geq 0 \) dB.

Subtables 3.B and 3.C show the same comparison for the PMN2 and M14 mines. In those cases, target positioning is significantly more affected as frequency subsampling and added noise become more severe, especially for the M14 which presents overall lower \( P_d \) figures than PMN2. Despite being close to the surface, the very low returns of these objects make them more vulnerable to mismatches with the training dictionary \( \Theta \), hence producing depth inaccuracies after target reconstruction and detection. Given the smaller size of these landmines, fewer cross-range positions are needed to describe their hyperbolic phase history than for the much larger AT landmine. This explains that PMN2 and M14 present similar positioning deviation to those of the AT landmine when cross-range subsampling is applied and under relatively benign noise conditions (\( \text{SNR} \geq 0 \)).

**FIGURE 10:** \( P_d \) vs. SNR for several subsampling rates \( k_{xr} = 0.1 - 0.5 \) and \( k_f = 0.05 - 0.25 \) for PCA-transformed data. Left column, results for the PMN2 landmine; right column, results for the M14 landmine. A probability of false alarm of \( 1 \times 10^{-2} \) was chosen and 500 Monte-Carlo trials were run. A CA-CFAR detector was used to extract the \( P_d \) values.
TABLE 4: Processing times in seconds for one full measurement (2.5 m × 0.9 m covered in 54 patches) for individual and combined subsampling rates.

<table>
<thead>
<tr>
<th>k_{xf}</th>
<th>Original data</th>
<th>PCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_{xf}</td>
<td>0.05 0.15 0.25 1</td>
<td>0.05 0.15 0.25 1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.96 0.99 1.03 2.03</td>
<td>1.04 1.17 1.49 19</td>
</tr>
<tr>
<td>0.3</td>
<td>0.97 1.05 1.07 2.84</td>
<td>1.16 2.22 4.83 193</td>
</tr>
<tr>
<td>0.5</td>
<td>1.06 1.09 1.14 3.62</td>
<td>1.32 4.38 17.4 946</td>
</tr>
<tr>
<td>1</td>
<td>2.19 2.21 2.82 –</td>
<td>4.75 35.2 161 –</td>
</tr>
</tbody>
</table>

F. PROCESSING BURDEN INCREASE

The additional processing steps required to sparsify the measured data can hinder the ability of a system to produce near-real-time results, especially if the reconstruction problem is extended to a third dimension. A possible alternative for GPR measurements that would reduce the computational burden is subdividing the total scene into smaller scene patches to be reconstructed. Dictionaries describing those ranges would be smaller and easier to handle. Table 4 displays the total processing times for reconstructing a full 2.5m × 0.9m scene (along-track and depth, respectively) using Matlab R2015b on a 24-core Intel® Xeon® E5649 2.53 GHz. A total of 54 scene patches of dimensions 0.30m × 0.15m covered the full measurement and every patch was reconstructed using several subsampling rates, whether individually or in a combined fashion. The involvement of PCA in the data sparsification stage makes the computational time grow exponentially with respect to the data subsampling rate, whereas using non-transformed data yields a linear processing time increase. The processing times using non-transformed data remain under 4 seconds for all tested subsampling strategies, which can be considered as real-time processing. Processing times for combined range-cross-range subsampling strategies and PCA are considerably higher although they may as well be considered real time or near-real time. However, larger values of only range or cross-range subsampling rates, without combination, may reach dozens or hundreds of seconds to complete one full reconstruction. The results presented in this article were obtained using a computer cluster, running parallel tasks on 25 cores for ca. 8,000 core hours.

VI. CONCLUSIONS AND FUTURE WORK

A thorough evaluation of the application of CS on GPR measurements has been described in this work. The reverberations within the antenna as well as the antenna-to-ground interactions are removed from the collected data after incorporating the described intrinsic antenna modeling. This modeling creates a much clearer imagery and enables detecting even small, non-metal targets very close to the prominent reflections produced by the air-ground interface.

When performing a CS reconstruction, the observed data can be preconditioned to achieve better performance by using transformation bases such as PCA. The fact that PCA is data-dependent makes it a potentially better tool for compacting the energy while discarding the contribution from the noise subspace, although the processing requirements may become computationally prohibitive for larger scenes or moderate subsampling rates. PCA not only leads to dimensionality reduction but also to at least equal P_d figures than using the randomly sampled non-transformed data. This is valid even if it is assumed that the measurement domain is already sparse or near-sparse.

This study reveals that if PCA and the antenna-ground modeling are combined, the AT landmine can be detected in at least 90% of the cases using only 5% of the conventional frequency samples for an SNR of -4 dB or higher, which is a 4-dB SNR gain with respect to the non-transformed data result. For frequency subsampling rates of 15% or higher, PCA does not provide any performance increase but, on the contrary, it unnecessarily increases the computational burden exponentially as k_f grows.

Under controlled laboratory conditions (SNR = 10 dB), frequency and cross-range subsampling rates can go as low as 5% and 10%, respectively, and yet achieve 100% of correct detections of the AT landmine. These results advocate that CS can be safely applied to GPR and imply drastic data volume reduction for all GPR types and lower measurement times for stop-and-go systems. The smaller, plastic landmine PMN2 obtains similar results using a subsampling factor of 20% for both in frequency and cross-range, whereas the M14 landmine achieves P_d ≥ 0.9 for k_{xf} = 0.4 and P_d ≈ 0.7 for k_f = 0.25, far from the results using the full measurements and a matched-filter-base SAR processing algorithm.

When randomly subsampling both in frequency and along-track positions, even lower data volumes can be obtained although the performance is affected. For the AT mine, PCA has proven to provide better performance rates than non-transformed data, especially under noisy environments, thanks to the noise subspace rejection when dimensionality reduction is performed. Detection probabilities above 90% are obtained using just 1.5% of the original Nyquist data volume (that is, 30% of the along-track positions and 5% of the frequencies). However, PCA preconditioning does not improve performances for the other two tested mines due to their much lower reflected energy.

Future work aims at applying the recent advances in CS showing that it is possible to use a priori knowledge of the measured scene to improve scene reconstruction and target recovery. This is known as side information [53]–[59] and uses signals which present a certain degree of correlation and repetition with the actual measured signals. For the application presented in this work, the recovery algorithm will be adequately modified into an ℓ_2 - ℓ_1 minimization problem to include side information, which can be described as a GPR measurement of the ground in a target-free area.

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EDISON CRISTOFANI obtained his Bachelor and Master of Engineering in telecommunications and electronics from the Autonomous University of Barcelona (ETSE-UB, 2006) and the Technical University of Catalonia (ETSETB-UPC, 2009), respectively. Until 2010, he was with the TSC Department (ETSETB-UPC) as a research fellow in a national security project. Since 2010, he is a radar and signal processing engineer at the CISS Department, Royal Military Academy of Belgium. Since 2014, he is also a PhD candidate. His main interests are radar and SAR principles as well as compressed sensing. He is also appointed as national representative in two NATO task groups (ET-SET-093 and RTG-SET-236), both on compressed sensing.

MATHIAS BECQUAERT received the engineering degree (with hon.) from the Faculty of Polytechnics, Royal Military Academy, Brussels, Belgium, in 2007. He is currently pursuing the Ph.D. degree at the Department of Electronics and Informatics, Vrije Universiteit Brussel, Brussels, Belgium. After a tour of duty of three years as signal officer in the Army, he became a Teaching Assistant with the Department of Communications, Information, Sensors, and Systems, Royal Military Academy. His current research interests include compressed sensing, nondestructive testing, mm-wave imaging, and trough-the-wall radar sensing.
NIKOS DELIGIANNIS received the Diploma degree in electrical and computer engineering from the University of Patras, Greece, in 2006, and the Ph.D. degree (Hons.) in applied sciences from Vrije Universiteit Brussel, Belgium, in 2012. He is currently an Assistant Professor with the Electronics and Informatics Department, Vrije Universiteit Brussel. From 2012 to 2013, he was a Post-Doctoral Researcher with the Department of Electronics and Informatics, Vrije Universiteit Brussel. From 2013 to 2015, he was a Senior Researcher with the Department of Electronic and Electrical Engineering, University College London, U.K. and also a Technical Consultant on Big Visual Data Technologies with the British Academy of Film and Television Arts, U.K. His current research interests include big data processing and analysis, machine learning, Internet-of-Things networks, and distributed signal processing. He has authored over 80 journal and conference publications, book chapters, and two patent applications (one owned by iMinds, Belgium and the other by BAFTA, U.K.). He was a recipient of the 2011 ACM/IEEE International Conference on Distributed Smart Cameras Best Paper Award and the 2013 Scientific Prize FWO-IBM Belgium.