Abstract—Accurate monitoring of the distribution system is performed using state estimation methods. The purpose of these methods is to estimate the most likely state of the grid given various types of redundant measurements. In this paper, we propose a three phase state estimation method that can handle accurately unsynchronized three phase phasor measurements. Unsynchronized phasor measurements, as opposed to synchrophasor measurements, consist in phasor measurements that do not have accurate time stamps. The use of such measurements could be very valuable in unbalanced distribution networks. To handle these measurements, we add unknown synchronizing operators to the state variables. The identification of these additional state variables allows considering any configuration of unsynchronized phasor measurement in a simple and intuitive way. The proposed method is illustrated on a simulated distribution network.

Index Terms—Power distribution, state estimation, phasor measurements, Monte Carlo method, distributed generation

I. INTRODUCTION

Power system state estimation aims to identify the most likely operating state of the network from different type of redundant measurements. These measurements are a combination of real-time field measurements and system operators’ knowledge, for instance load forecasts.

In transmission system, the topic is well covered in the literature [1], [2] and is widely used by networks operators.

In distribution systems, the need for efficient and robust state estimators is growing: these estimators are necessary to benefit of the increasing amount of data gathered by the so-called smart grid. Another driver is the monitoring of voltage limits or lines capacity limits that can be violated by high penetration of distributed generation.

Unlike in transmission systems, distribution state estimation algorithms are characterized by the use of three-phase models to handle unbalanced situations and the lower measurement redundancy [3]-[8]. Some algorithms were specially designed to take advantage of radial or weakly meshed topologies that characterize these networks (e.g. [6]-[8]).

Three phase state estimators were developed to deal with three-phase voltage magnitude measurements, phase power (active and reactive) measurements and current magnitude measurements.

More recently, synchrophasors were added in the state estimation: synchronized three phase voltage and current phasor measurements that can be recorded by Phasor Measurement Units [9], [10].

In this paper, we propose a state estimation method that is able to deal with unsynchronized three phase phasor measurements. This type of measurement can be very valuable in unbalanced situations. To our knowledge, no methods were developed to deal with these measurements.

Actually, some measurement devices are able to compute the phasors from the recorded three phase voltage or current waveform. Therefore these devices provide more information than the phase magnitudes: the angle difference between the phases can also be considered in state estimation. To benefit from this information in state estimation, the devices do not need to be accurately synchronized with a common time reference.

To include these measurements in state estimation, unknown synchronizing operators are added to the state vector. The identification of these additional state variables results in a simple problem formulation.

The paper is organized as follows. In the first section, the state estimation method is presented and we explain how the unsynchronized phasors are handled. The second section gives simulations results and discusses convergence issues. The third section ends the paper with the conclusions.

II. STATE ESTIMATION METHOD

A. Problem formulation

The state of the network is represented by the three phase complex voltage at every node; the voltage is expressed in rectangular form and in abc quantities. Additional states variables are added to consider unsynchronized measurements, as it will be explained later.

From redundant measurements, the state of the system is estimated by solving a weighted least squares problem:

\[
\text{Minimize} \quad J(x) = \frac{1}{2} r^T R^{-1} r \\
\text{Subject to} \quad c(x) = 0
\]

Where \( x \) is the state vector, \( z \) is the measurement vector, \( h(x) \) contains the measurement equations and \( R \) is the covariance matrix of the measurements. \( C(x) \) represents
equality constraints, for instance zero-injection nodes.

This formulation is called the Augmented Matrix Approach [1], [2] that is much more numerically stable than the conventional normal equations formulation.

B. Iterative solution

The solution to this constrained minimization problem must respect the first order optimality conditions of the following Lagragian function:

\[ L = J(x) - \lambda^T c(x) - \mu^T (r - z + h(x)) \]  

i.e:

\[
\begin{align*}
\frac{\partial L}{\partial x} &= 0 \\
\frac{\partial L}{\partial \lambda} &= 0 \\
\frac{\partial L}{\partial r} &= 0 \\
\frac{\partial L}{\partial \mu} &= 0
\end{align*}
\]  

These conditions lead to a system of nonlinear equations that is solved using the Newton-Raphson algorithm, which results into the following iterative solution:

\[
\begin{pmatrix}
R & H & 0 \\
H^T & 0 & C \\
0 & 0 & \lambda
\end{pmatrix}
\begin{pmatrix}
\mu \\
\Delta x \\
\Delta \lambda
\end{pmatrix}
=
\begin{pmatrix}
z - h(x^{k-1}) \\
0 \\
-c(x^{k-1})
\end{pmatrix}
\]  

Where \( H \) and \( C \) are the Jacobian matrices of the measurement equations and the equality constraints, respectively.

\( \Delta x \) is found via LU factorization and forward/backward substitution. The iterations are performed until the stabilization of the state variables.

C. Measurement equations

In this subsection, we explain how the functions between measurements and state variables (\( h(x) \) in (2)) are formulated.

1) Synchronized phasor measurements

Synchronized phasor measurements can for instance be provided by Phasor Measurement Units (PMU).

The relationship between a current phasor flow from bus \( k \) to bus \( l \) and the networks node voltages is given below:

\[ I_{kl} = j B_{cl} V_k + Y_{kl} (V_k - V_l) \]  

Or, in real and imaginary parts:

\[\begin{pmatrix} I_{kli} \\ I_{klr} \end{pmatrix} = \begin{pmatrix} G_{kl} & -G_{kl} & -B_{cl} & -B_{kl} & B_{kl} \\ B_{cl} & B_{kl} & -B_{kl} & -G_{kl} & -G_{kl} \end{pmatrix} \begin{pmatrix} V_k \\ V_l \end{pmatrix} \]  

Where all the quantities denote three phase vectors, e.g.:

\[ I_{kl} = \begin{pmatrix} I_{kli} \\ I_{kli} \\ I_{kli} \end{pmatrix} \]  

Therefore, the complex equation (7) contains in fact six real equations. In (8), \( Y_{kl} = G_{kl} + j B_{kl} \) a 3x3 matrix – is the three phase admittance of the line; and \( B_{cl} \) its shunt susceptance.

Since these relations are linear, the Jacobian entries are directly obtained.

Injection current equations are found analogously using the bus admittance matrix (see e.g. [4]).

Voltage phasor measurement equations are related to the associated state variables via a unity matrix.

2) Unsynchronized phasor measurements

Some measurement devices record asynchronously three phase measurements. Thus, they can provide unsynchronized phasor measurements: the magnitude of the three phases and the phase angle of the three phases; but the absolute phase of these phasors remains unknown.

Fig. 1 shows the difference between a synchronized and an unsynchronized phasor measurement.

![Fig. 1. Example of a three phase phasor, its record by a synchronized phasor measurement device (with a measurement error), and its record by an unsynchronized phasor measurement device with a time error of \( \delta \) degrees (here \( \delta = 50^\circ \))](image)

Therefore, we introduce an additional unknown to our problem: a synchronization operator \( e^{j\delta} \) that resynchronizes the device clock to the common time reference.

The common time reference is the one provided by synchronized measurement devices, or, if not available, it is selected arbitrarily amongst one of the unsynchronized voltage measurement device.

As a consequence, the general measurement equation for an unsynchronized measurement is:

\[ h_{\text{unsynch}}(x) = h_{\text{synch}}(x) \cdot e^{j\delta} \]  

With \( \text{unsynch} \) superscript denotes the measurement equation of an unsynchronized device and \( \text{synch} \) superscript the synchronized measurement equation for the same type of measurements (i.e. voltage, current injection or current flow). \( \delta \) represents the unknown angle that allows the resynchronization of the measurement to the time reference.

For instance, for an unsynchronized current phasor flow measurement on line \( kl \), the measurement equation will be:

\[ I_{kl,\text{unsynch}} = (jB_{cl} V_k + Y_{kl} (V_k - V_l)) \cdot e^{j\delta} \]  

In the former equations, there are six real equations as in the synchronized case, but a real unknown (\( \delta \)) is added, so the measurement redundancy is lower than in the synchronized case.
From this development, we finally express the state vector x used in (1)-(2) as:

\[ x = [V_{r1} \ V_{r2} \ \ldots \ V_{rl} \ V_{l1} \ V_{l2} \ \ldots \ V_{ln} \ \delta_{l} \ \ldots \ \delta_{m}]^{T} \]  (12)

where the first 6 n (n is the number of nodes) are the voltage phase state variables expressed in rectangular coordinates (later denoted \( x_{v} \)), and the last m variables are the synchronizing operators associated to the m unsynchronized measurement devices.

The Jacobian of these measurement equations are calculated by the derivation of (10) which leads to:

\[
\frac{\partial h_{\text{unsynch}}(x)}{\partial x_{v}} = \frac{\partial h_{\text{synch}}(x)}{\partial x_{v}} \cdot e^{j\delta}, \quad \frac{\partial h_{\text{synch}}(x)}{\partial \delta} = h_{\text{synch}}(x) \cdot j e^{j\delta} \]  (13)

With \( \frac{\partial h_{\text{synch}}(x)}{\partial x_{v}} \) the Jacobian matrix of synchrophasors measurements.

Since, the other measurement equations are not function of the synchronizing operators, the associated derivatives are zero:

\[
\frac{\partial h_{i}(x)}{\partial \delta_{k}} = 0 \]  (14)

if the measurement i is not sampled by the unsynchronized device k.

We remark that an alternative solution would have to write only five measurement equations: three magnitude measurements, and two angle differences between the phases. But the Jacobian entries would have been more complicated. Moreover, a simpler solution with unsynchronized measurements is to reduce the measurements to their magnitude, but the angle difference between the phases is not used; hence the measurement redundancy would be lower than with our method.

3) Power measurements

Power measurements are considered using the equivalent current phasor formulation proposed in [4]: power measurements are converted to equivalent synchronized current phasor measurements using the bus voltage estimate of the previous iteration:

\[
I_{\text{meas.equi.}}^{k} = \left( \frac{P + jQ}{V_{k-1}} \right)^{*} \]  (15)

The weighing of these equivalent equations is also updated at each iteration in function of the assumed error on the power measurements.

Then, depending on the power measurement configuration, the relations with respect to the state variables are the same as for a synchronized current phasor injection or a current phasor flow.

4) Pseudo-measurements

In distribution systems, there are generally not enough measurements to allow the observability of the system. Therefore, pseudo-measurements are introduced. These consist in fact in load modeling obtained from load forecast or historical data.

These measurements are handled in a similar way as the power measurements: they are converted into equivalent current phasor injection at each iteration. With this method, any type of voltage dependent load model can be easily included:

\[
I_{\text{meas.equi.}}^{k} = f(V^{k-1}) \]  (16)

For instance, for a three-phase ungrounded load connected at node i, we use the following constant power load model to represent function f in (16):

\[
I_{i}^{a} = \left( \frac{S_{ab}}{V_{i}^{b} - V_{i}^{a}} + \frac{S_{ac}}{V_{i}^{c} - V_{i}^{a}} \right)^{*}
\]

\[
I_{i}^{b} = \left( \frac{-S_{ab}}{V_{i}^{b} - V_{i}^{a}} + \frac{S_{ac}}{V_{i}^{c} - V_{i}^{b}} \right)^{*}
\]

\[
I_{i}^{c} + I_{i}^{b} + I_{i}^{c} = 0
\]  (17)

Where \( S_{ab} \), \( S_{ab} \) and \( S_{ac} \) are the complex power consumed between phases ab, ac and bc respectively. \( I_{i}^{a} \), \( I_{i}^{b} \) and \( I_{i}^{c} \) the current phasors injection at node i, function of the state variables as explained above. The third equation is expressed as an equality constraint in (2) and is related to the ungrounded nature of the load.

Other type of load connection, e.g. single phase or three phase grounded loads, can be easily represented as well. The power consumed can be also function of the voltage (e.g. with a ZIP load model).

Unmonitored distributed generators can be represented in a similar way.

D. Initialisation of the algorithm

The voltage state variables (\( x_{v} \)) are initialized using a load flow solution. From the load flow solution, the synchronized operators are initialized as well: they are computed such that the shifted measurements fit best (in a least squares sense) to those obtained from the load flow solution.

This initialization procedure is necessary for two reasons. First, the method can fail to converge if the synchronizing operators are initialized too far from the true solution (with an error greater than approx. 60°). Second, when using unsynchronized current phasor measurements, the measurement Jacobian \( (H) \) is singular if a flat start initialization is chosen. This last issue was already met when using current magnitude measurements in conventional state estimators [2].

The choice of using a load flow solution will therefore give a very good starting point provided that the load flow model is close enough to the true network state.

III. SIMULATION EXAMPLE

A. Test system

The single line diagram of the medium voltage (20kV)
distribution test system is shown on Fig. 2. The system is three phase, three wire and is composed of 8.55 km of underground cables and 14.2 km of overhead lines. The furthest node is 11.4 km away from substation (node 1). The total loading of the feeder is 7.1 MVA. Distributed generators are presents at node 11 (0.1MW), 14 (2.5MW) and 21 (2.5MW).

In order to illustrate the use of unsynchronized phasor measurements, we consider a measurement set consisting only of this kind of measurement: four measurement devices (M1, M2, M3, M4 on Fig. 2) record phasors with inaccurate time stamps. M1 records voltage and current injection at node 1, M2 the voltage of bus 7 and currents flows on line 7-8 and 7-15, M3 records the current in line 27-28 and M4 the current in line 27-33.

During the state estimation, M1 is used as the reference and three phase shifts are identified to resynchronize the measurements of M2, M3 and M4 with respect to M1.

The standard deviations of the load (DG) models are set to 1/3 of the average load (generation). This is equivalent to a probability of 99% for the load (distributed generator) being between zero and 2 times the average load (generation). The noise is considered to be independent on the three phases of each load model (Sub, Sbc and Sac of (17)) such that an unbalance is possible.

The standard deviations on the real and imaginary parts of the phasor measurements are set to 1% of the phasor magnitude and are uncorrelated.

The measurements used in the state estimation are made as follows. First, a load flow is run using the average load and DG models. The average real measurements are obtained from the load flow solution. Second, normal random noises with the aforesaid standard deviations are added on the real and the pseudo measurements used in the state estimation. Lastly, phase shifts are added to the unsynchronized phasors to represent the inaccurate time stamps (see Table 1).

B. Convergence characteristic

With this measurement set, after 1000 Monte Carlo simulations, we observed that the algorithm converged typically in less than 10 iterations.

To illustrate the convergence of the algorithm and the use of the equivalent current formulation for the pseudo-measurements (16), the values of the cost function and the maximal change of the state variables over the iterations are given on Fig. 3 for one simulation.

The stopping criterion used for the minimization algorithm is the stabilization of the state variables ($\max_i \left| x_i^k - x_i^{k-1} \right| < \varepsilon$). Additional iterations are performed until the update of the equivalent current measurement equations (16) are stabilized as well ($\max_i \left| z_i^k - z_i^{k-1} \right| < \varepsilon$).

Here, both thresholds were set to 1E-4.

In this example, at iterations 5 and 8 the measurement equations (equivalent current equations used for the load models) were updated which explain the sudden increase of the maximal change of state variables. Then, the algorithm converged to a non-zero cost function because of the measurement noise and load modeling errors.

![Fig. 2 Diagram of the test network](image)
C. Accuracy of the state estimate

Once the state of the network has been estimated, other variables describing the network state, such as lines power flows, can be calculated. Moreover, it is possible to compute the confidence level on all the estimated values with the formulas given in the appendix.

The voltage magnitude estimates of phase a with the 95% confidence interval (+/-2σ) are presented on Fig. 4.

![Fig. 4 Estimated value and 95% confidence interval for phase a voltage magnitude](image)

We can see that a flat voltage level is obtained because of the distributed generation. Further, the confidence on the voltage is high because of the two voltage measurements.

The phase shifts of the unsynchronized measurement device with the common time reference have been correctly estimated (see Table 1). An interesting conclusion of this table is that the accuracy on the estimated phase shift is much higher when the device records also the voltage (here M2) than when it does not (here M3 and M4): in the second case, the time shift is only estimated from the load models that are relatively uncertain.

![TABLE I](image)

<table>
<thead>
<tr>
<th>Concerned meas. device</th>
<th>True phase shift</th>
<th>Estimated phase shift</th>
<th>Estimated standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>130°</td>
<td>129.99°</td>
<td>0.46°</td>
</tr>
<tr>
<td>M3</td>
<td>-80°</td>
<td>-81.95°</td>
<td>6.79°</td>
</tr>
<tr>
<td>M4</td>
<td>-10°</td>
<td>-14.74°</td>
<td>6.57°</td>
</tr>
</tbody>
</table>

The plot of the power flows standard deviations of Fig. 5 shows that the uncertainty on the power flow is becoming higher when moving away from the measurement devices. This effect is explained by the load uncertainty.

![Fig. 5 Standard deviation of lines power flows on phase b using the full measurement set, and when not using unsynchronized current measurements in line 27-28 and 27-33](image)

To emphasize the benefit of using current measurements at several locations on the feeder, the power flow standard deviation was also calculated with the current measurements of lines 27-28 and 27-33 removed from the measurement set (Fig. 5). Without these measurements, the increased standard deviation on some lines shows clearly the advantages of adding those unsynchronized current phasor measurements in the state estimation.

D. Effect of uncertain power flow direction

In some networks, because of high distributed generation penetration, the power flows on several lines can have two possible directions (e.g. on line 7-8 in this case) depending on the DGs output and load level.

However, unsynchronized current measurements do not provide the power flow direction unless the measurement device records also the voltage. The lack of directional measurement can therefore give convergence problems or the algorithm could converge to a local minimum having a reverse flow direction to the actual one.

The problems associated to the lack of directionality were similarly met when using current magnitude measurements [2].

As an example, in our test system, we removed the voltage measurement at node 7 of M2, making the flow directions on lines 7-8 and 7-15 unknown. With such measurement set, we observed that the algorithm could fail to converge or estimate a reverse flow direction. Directional measurement at this location is thus mandatory because of distributed generation at nodes 11, 14 and 21.

However, in our test case, directional measurements on lines 27-28 and 27-33 are not mandatory because the following subnetworks (lateral 27-31 and lateral 27-38) are passives.

We stress that these convergence problems are not met when using paired unsynchronized voltage and current phasors, synchronized phasor measurements or power measurements.

IV. CONCLUSIONS

In this paper, we propose a new three phase state estimation method that can consider unsynchronized phasor measurements in an intuitive way. This approach takes
advantage of the angle difference between the phases, resulting in more redundancy than simply reducing these measurements to magnitude measurements. The use of such measurements in the state estimation should increase the estimation accuracy in unbalanced conditions.

Simulations of a radial network with distributed generation were used to illustrate the method. It was shown that the algorithm estimates accurately the time stamps difference between the measurement devices. The impact of the load models uncertainties on the state estimate was illustrated and the increase of state uncertainty when removing some measurement devices was illustrated as well. There is therefore a concern for optimal measurement device placement that would reduce the total uncertainties on the state estimate.

Finally, another conclusion of the simulations is that, because of convergence issues, the network operator should avoid placing unsynchronized current phasor measurements on lines where the flow direction is unknown.

V. APPENDIX

A. Covariance of the state estimate

The covariance matrix of the state variables is calculated as follows.

At the optimal solution, by linearizing the measurement equations and the constraints (2) we have:

\[
\begin{bmatrix}
R & H & 0 \\
H^T & 0 & C^T \\
0 & C & 0
\end{bmatrix}
\begin{bmatrix}
R^{-1}F \\
\hat{x} \\
\hat{\lambda}
\end{bmatrix}
= \begin{bmatrix} z \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}
\]

Or, equivalently:

\[
\begin{bmatrix}
R^{-1}F \\
\hat{x} \\
\hat{\lambda}
\end{bmatrix}
= \begin{bmatrix}
B_1 & B_2 & B_3 \\
B_1^T & B_4 & B_5 \\
B_3^T & B_5^T & B_6
\end{bmatrix}
\begin{bmatrix} z \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}
\]

With B, the inverse of the so-called Hachtel matrix. From this, we deduce that:

\[
\hat{x} = B_5 z
\]

Therefore, it can be shown that the covariance of the state variable is:

\[
\text{cov}(\hat{x}) = B_2 R B_2^T
\]

From the covariance of \(x\), we can further find the covariance of any linear transformation of \(x\). If \(F\) is a matrix representing a linear combination of the state variables, then:

\[
\text{cov}(F\hat{x}) = F \text{cov}(\hat{x}) F^T
\]

If the function is nonlinear, we can use (22) by linearizing the function: \(f(x_0 + \Delta x) = f(x_0) + F\Delta x\)

The function \(f\) can for example represent the power flow in a line computed from the estimated state. Therefore, from the state variables and its covariance matrix, we are able to compute all the other variables describing the network (e.g. power flow in branches) and their covariance matrix.

VI. REFERENCES


VII. BIOGRAPHIES

Pierre Janssen  (S’2010) was born in Brussels, Belgium on September 28, 1987. He received the M. Sc. Degree in Electrical Engineering from the Université Libre de Bruxelles, Brussels, in 2010. Mr. Janssen is currently preparing a Ph. D. with research emphasis on smart distribution networks monitoring, protection and control.

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Jean-Claude Maun received the M.Sc degree in mechanical and electrical engineering in 1976 and the Ph.D. degree in Applied Sciences in 1981, both from the Université Libre de Bruxelles (ULB), Brussels, Belgium. He joined the Electrical Engineering Department of this university in 1976 and he is now professor and Dean of the ULB Engineering School. He has been leading research projects in the field of the design of digital protections for Siemens as a consultant. An emeritus member of the Société des Electriciens et Electroniciens français and a recipient of many academic awards, Prof. Maun is an recognized expert in electrical networks’ safety and protection systems as well as, more generally, in power transport and distribution networks and in decentralized electricity generation.