Lecture 1 :

Survey of self-organisation phenomena and complex non-linear phenomena

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Introduction

Dynamical systems : Properties of the solutions generated by a set of evolution equations.

Linear perspective

- System subjected to well- defined external conditions will follow a **unique** course.
- Small change in conditions $\rightarrow$ slight change in the system's response.

Linear world in which the observed effects are linked to the underlying causes by a **law of proportionality**.

$\Rightarrow$

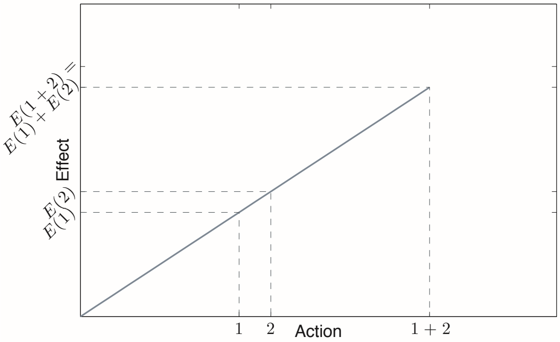
- Very limited view of the real world.
- Most of the phenomena surrounding us **deviate** from proportionality $\Rightarrow$ **Nonlinearity**

Introduction

Consider a combination of actions of two different causes on a system ...

Linear systems

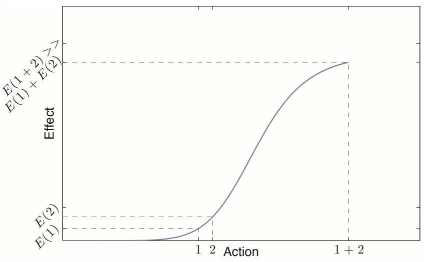
Superposition of the effects of each of these two actions taken individually.



Introduction

Non-linear systems

Small causes may induce large effects.



- Dramatic new effects in the form of unexpected structures and events such as :
  - Generation of spatial patterns
  - Sustained oscillations
  - Abrupt transitions
  - Random looking evolutions known as deterministic chaos.

Introduction

Self- organization, Complexity : Major manifestation of non-linearity.

Objective

Gain insights on the origin of non-linear phenomena of which self- organizaton and complexity appear to be a common denominator.

Introduction

Course description

Practical Information

The course will run from week 43 to week 50 2010, A full timetable can be found from the course webpage,

http://www.math.uu.se/~snicolis/ section, Applied Dynamical Systems

The course organiser is Stamatios Nicolis (snicolis@math.uu.se).

Introduction

Course description

Course description and learning goals

This course aims to provide insight and practice in how dynamical system (i.e. differential equation and difference equation) models can be used to better understand biology, physics and chemistry. The focus will be on formulating models, model analysis, using analytical and numerical solution tools to better understand models and drawing conclusions based on model outcomes. Since the course contents are rather wide a focus will be made on increasing overall confidence in mathematical modeling, rather than deep study of each particular area. In order to pass the course the student should be able to

1. formulate important models treated during the course ;
2. outline the mathematical methods and techniques that are used to analyze these models and understand in what situations these methods can be applied ;
3. understand how to draw a conclusion from a model ;
4. use a computer package to investigate models numerically ;
5. solve standard problems within the areas covered by the course.

Introduction

Course description

Programming Language

The computer tools used in computer labs will mainly be from Matlab software. The first computer lab will introduce some basic symbolic tools included in Matlab to find solutions of typical models and to represent in different manner the properties of these models (phase space, bifurcation diagrams, etc...). We will also see in this first lab how to integrate differential equations numerically.
Matlab is a tool proposed but it is not mandatory to use it. If you know any other language or any other tool, you can use it instead of Matlab.

Examination

Evaluation is based on **three projects** and an **oral examination** on one of the projects.
The relative weights of the three projects are

What is self-organization ?

Self-organization is the spontaneous emergence of spatial, temporal or functional order in systems subjected to constraints.

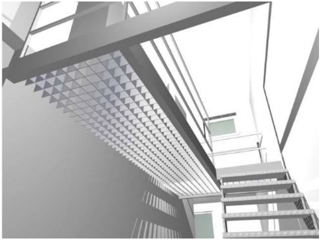
- This new paradigm emerged in the 50's and 60's.
- Opposition between the Newtonian deterministic and static point of view **(being)** and self organising evolutionary **(becoming)** point of view.
- The idea that the dynamics of a system can tend, by itself, to make it more orderly, is very old. The first statement of it was by **René Descartes**, in the fifth part of his **Discours de la méthode**, where he presents it hypothetically, as something God could have arranged to happen, if He hadn't wanted to create everything Himself. Descartes elaborated on the idea at great length in a book called **Le Monde**, which he never published during his life.

What is self-organization ?

Counter-intuitive ?

An observer in the presence of an organized structure would suspect that there is an external influence behind it.

This intuition is confirmed at first sight by the 2nd law of thermodynamics which, in its most familiar version, stipulates that in an isolated system entropy (traditionally a measure of disorder) is bound to increase.

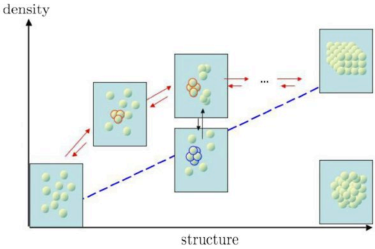


Self-organization and th 2^{nd} law of thermodynamics

Closed system near equilibrium

Crystallization

The randomly moving molecules which are fixed by the crystalline structure pass on the energy of their movement to the liquid phase in which they are embedded. The decrease in the entropy of the crystalline structure is compensated by an increase in the entropy of the liquid. The entropy of the whole system is increasing.



The length scale of such structures is imposed by the intermolecular forces and is thus microscopic. Any form of sustained activity in time (oscillations,...) is ruled out.

Self-organization and the 2$^{\mathrm{nd}}$ law of thermodynamics

Open system far from equilibrium

- Entropy is produced within the system in agreement with the second law and is at the same time being exchanged with the outside world in which the system is embedded. Total system's entropy may now decrease.
- Interplay between internal dissipation and opening to the external environment is at the origin of a host of unexpected behaviours. In particular, macroscopic length scales and temporal activity become now possible.
- Self organisation takes place in **non-linear** systems encountered in many branches of physical science, in presence of nonequilibrium constraints. Because of the existence of **multiple solutions** under certain conditions (contrary to a linear system) there is a choice between different configurations in which the system may settle. **Stability** (response to perturbations) and **bifurcation** (the onset of new regimes) become major issues.

Linearity by examples

Mechanics

- Pendulum in the limit of small deviations from the vertical.
  Acceleration proportional to angle with respect to the vertical.
- Elasticity.
  Elongation of a spring proportional to applied force.

Electromagnetism

- Electric conduction under moderate electric fields.
  Current proportional to voltage.
- Variation of magnetic field as a function of time is proportional to the variation of the electric field as a function of space.
  Variation of electric field as a function of time is proportional to the variation of the magnetic field as a function of space.
  $\Rightarrow$ Combining the two linear relations we obtain a phenomenon of wave propagtion

Physical chemistry, materials science

- Diffusion and heat conduction for moderate concentration and temperature differences.
  Energy flux linear in temperature difference.

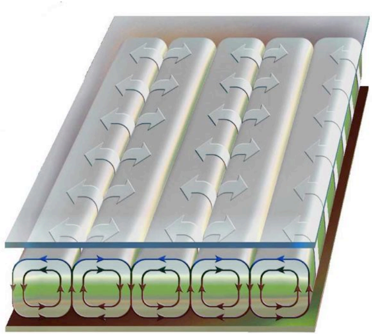
Nonlinearity and self-organization by examples

In Physics : **Rayleigh- Benard cells**

air around a radiator, circulation of earth atmosphere, ...

Horizontal fluid layer heated from below

- Small temperature difference between the plates : the fluid remains at rest.
- Beyond a critical threshold value we witness the onset of thermal convection in which the fluid motion is organised spontaneously in the form of cells (hexagons or rolls) referred as **Rayleigh- Benard** cells, possessing a characteristic length.



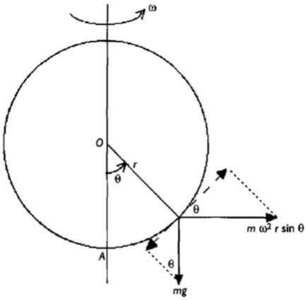
- Structures and events of this kind are a priori extremely improbable. Yet when the appropriate conditions are met they are realised with probability one ! We refer to this as the **"bottom-up"** mechanism.

Self-organization by examples

In Mechanics : **Nonlinear oscillations**

A rigid vertical ring of Radius R in the field of gravity. A mass m is initially placed at an angle θ_0 from the lower end of the vertical diameter and is allowed to move along the ring with no friction

- As long as the ring as a whole is at rest, it will perform a periodic motion around position θ (if $\theta_0 \neq 0$) or will remain fixed forever (if $\theta_0 = 0$)
- if $\omega \neq 0$, as a result of an externally applied torque :
  - if ω is small, the mass m still oscillates around the same equilibrium position as before
  - beyond a critical threshold value, the mass oscillates around either **two new equilibrium** positions corresponding to a non-zero angle θ and placed symmetrically around the vertical diameter.



Self-organization by examples

In Chemistry

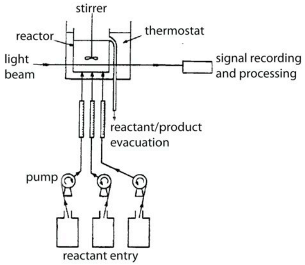
Traditional view :

a unique, homogeneous, time-independent state should eventually emerge from any chemical transformation (naive interpretation of 2nd law of thermodynamics !).

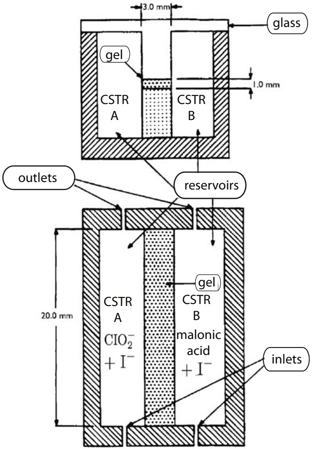
Experimental evidence that this view is incorrect since the 1960's

- Non-trivial transient phenomena in closed reactors.
- Sustained oscillations, chaos and multistationarity in open well-stirred reactors.
- Spatio-temporal patterns (wavefronts, steady state (Turing)) in open unstirred gel-type reactors.

Experimental setup for an open well-stirred chemical system



Experimental setup for an open unstirred chemical system

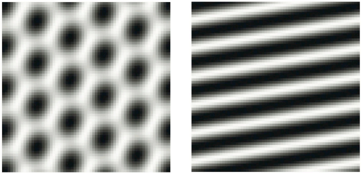


Self-organization by examples

In Chemistry :Belousov Zhabotinsky (i)

Oxydation of an organic acid by an oxydizing agent in the presence of redox couple, e.g., malonic acid by Potassium bromate in presence of Cerium^{3+}, *Cerium*$^{4+}$

Two-dimensional reaction-diffusion patterns feat. hexagons and bands



- Chemical feedback in the form of autocatalysis.

- Non-trivial patterns in the form of excess of certain substance in preferred region of space.
- Characteristic dimension of the pattern is **intrinsic** (not determined primarily by the system's size and the boundary conditions).
- Sharp threshold separating this state from the one where chemicals are homogeneously distributed (**symmetry breaking**).

Self-organization by examples

In Chemistry : Belousov Zhabotinsky (ii)

Sustained oscillations

- Chemical feedback in the form of autocatalysis

Wave propagation in a 2D layer of BZ reagent

- Oxidation fronts emanating from certain points and propagating in the layer.

Self-organization by examples

In Biology : Synchronization - coherent behavior in time

Fireflies

Slime mould

- Cells/Individuals taken separately are moving/oscillating randomly/int their own frequence.
- Remarkably, at the colony level, cells/individuals collectively organize to move/oscillate coherently/synchronously.

Self-organization by examples

In Biology : Self-assembly

Fish schools

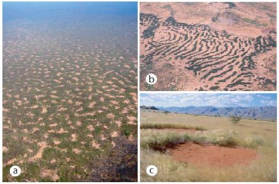
Starlings flock

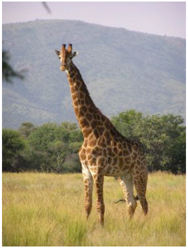
- Individuals aggregate and adopt a coherent motion, presumably through a mechanism of imitation.

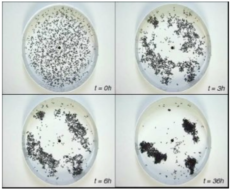
Self-organization by examples

In Biology : Pattern formation

Vegetation	Animals furs/skins	Ants cemeteries







- Rather than being randomly distributed in space, plants/cells/individuals tend to concentrate in preferred region of space.

Self-organization by examples

In human systems :

Mexican waves

Stock exchange

- Human systems do not escape from the self-organization paradigm : Mexican waves, competing financial agents in stock exchange but also synchronized clapping in concert halls, pedestrian trafic, ...