

Project I

Reading:

STROGATZ, NONLINEAR DYNAMICS AND CHAOS, CHAPTER 6
EDELSTEIN-KESHET, MATHEMATICAL MODELS IN BIOLOGY, CHAPTER 6

SOME PAPERS YOU CAN DOWNLOAD FROM THE WEBPAGE

1 A linear oscillator

$$\frac{d^2x}{dt^2} = -\omega^2x$$

transformed to two variables through

$$\begin{aligned}\frac{dx}{dt} &= v \\ \frac{dv}{dt} &= -\omega^2x\end{aligned}$$

1. (1/20) For $v_0 = 0$ and $x_0 = 0, 0.1$ and 0.2 as initial conditions and $\omega \neq 0$, integrate numerically the equations, draw the phase space portrait and identify the invariant sets.

2 The Lotka-Volterra oscillator

Consider a population density x of rabbits (the preys) and a population density y of foxes (the predators). Their interactions can be formulated in the following way:

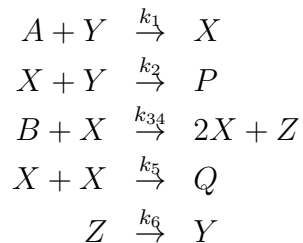
$$\begin{aligned}\frac{dx}{dt} &= ax - kxy \\ \frac{dy}{dt} &= kxy - \gamma y\end{aligned}$$

where a is the birth rate of rabbit's population, k the interaction rate between rabbits and foxes and γ is the death rate of the fox population

1. (3/20) For the parameter values of your choice (don't forget though that parameters are *rate parameters*) and initial conditions equal to $\{x_0 = 0.1, y_0 = 0.1\}$ and $\{x_0 = 1, y_0 = 1\}$, integrate numerically the equations, draw the phase space portrait and identify the invariant sets.
2. (1/20) Discuss the biological meaning of the invariant sets
3. (2/20) Find a chemical equivalent scheme
4. (3/20) Show that the quantity $U = k(x+y) - \gamma \ln x - a \ln y$ is conserved during the dynamics.

3 The Oregonator

The Oregonator model captures the essential features of the **Belousov-Zhabotinsky** reaction. The kinetic scheme can be written as



1. (1/20) Specify from the literature the chemical equivalents of X , Y and Z .
2. (2/20) Write the balance equations for X, Y and Z

3. (3/20) It can be shown that the balance equations can be scaled in the following way

$$\begin{aligned}\frac{1}{\epsilon} \left(\frac{dx}{d\tau} \right) &= y - xy + x(1 - qx) \\ \epsilon \left(\frac{dy}{d\tau} \right) &= -y - xy + z \\ \frac{dz}{d\tau} &= w(x - z)\end{aligned}$$

Integrate numerically the equations and draw phase portrait in projection into x, y plane and in a 3-d perspective for parameter values equal to $\epsilon = 77.27$, $q = 8.375 \times 10^{-6}$ and $w = 0.1610$ and different initial conditions. Identify the invariant set.

4 The Lorenz model of thermal convection

$$\begin{aligned}\frac{dx}{dt} &= \sigma(-x + y) \\ \frac{dy}{dt} &= rx - y - xz \\ \frac{dz}{dt} &= xy - bz\end{aligned}$$

For $\sigma = 10$, $b = \frac{8}{3}$ and $r = \frac{1}{2}, 2, 23.5$ and 28 and initial conditions for x, y and z between $[-1 : 1]$:

- (3/20) Draw phase portrait in projection into x, z plane and in a 3-d perspective
- (1/20) Comment on the qualitative changes as r is gradually increased.