Project II: Food recruitment in ant colonies

Reading

Papers that you can download from the website

An ant colony has the choice between two food sources in the experimental setup below.

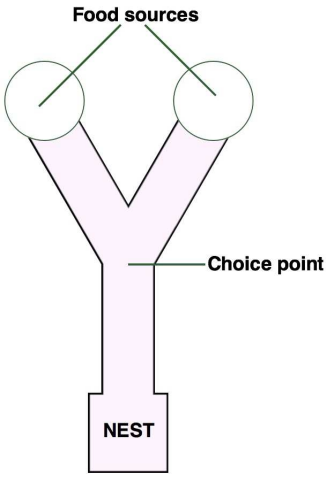

Figure 1: Experimental setup

The following model can capture the phenomenon.

$$\begin{aligned} \frac{dC_1}{dt} &= \Phi q_1 \frac{(k+C_1)^\ell}{(k+C_1)^\ell + (k+C_2)^\ell} - \nu C_1 \\ \frac{dC_2}{dt} &= \Phi q_2 \frac{(k+C_2)^\ell}{(k+C_1)^\ell + (k+C_2)^\ell} - \nu C_2 \end{aligned} \tag{1}$$

From the literature, justify the model, and relate the variables C_1, C_2 and the parameters Φ, q_1, q_2, ℓ and ν to the characteristics of the specific biological system.

In your report, following a short introduction on the subject, the analysis of the model must deal with the following issues[1].

[1] Whenever specific parameter values are needed, take $q_1 = 0.11$,$q_2 = 0.1$, $\nu = 0.01$ and Φ between 0 and 1.

- Show that eqs. (1) can be scaled in the following form

$$\begin{aligned} \frac{dc_1}{dt} &= \phi q_1 \frac{(1+c_1)^\ell}{(1+c_1)^\ell + (1+c_2)^\ell} - c_1 \\ \frac{dc_2}{dt} &= \phi q_2 \frac{(1+c_2)^\ell}{(1+c_1)^\ell + (1+c_2)^\ell} - c_2 \end{aligned} \tag{2}$$

- What are the steady state solutions of the model (2) for $q_1 = q_2$ and for $\ell = 1$, $\ell = 2$. Include details of the calculations.

- What is the stability of these solutions for $\ell = 2$ (find the stability analytically and check by integrating the equations). Construct the bifurcation diagram of c_1 against ϕ, showing the stable branches in full lines and the unstable ones in dashed lines. Determine analytically the critical value ϕ^* where the bifurcation occurs.

- Comment on the biological meaning.

- What are the steady state solutions of the model for $q_1 > q_2$ and $\ell = 2$. Construct the bifurcation diagram of c_1 against ϕ. Find the stability of the different branches by integrating the full equations.

- Summarize the results in a Concluding section. Formulate some speculative remarks about how would you improve your model by i.e., taking into account three or more sources and a C-dependent flux of ants that would saturate with the concentration of pheromone.