Project III

1 Climate variability: **Main project**

Reading Papers that you can download from the website

Consider the following model supposed to describe the coupling between the mean ocean temperature deviation (θ) and the sea ice extent deviation (η) from some reference state

$$\begin{aligned} \frac{d\eta}{d\tau} &= -\phi_2\eta + \phi_1\theta \\ \frac{d\theta}{d\tau} &= -\psi_1\eta + \psi_2\theta - \psi_3\eta^2\theta \end{aligned} \tag{1}$$

In your report, following a short introduction on the subject, comment briefly the different terms in the equations. The subsequent analysis of the model must deal with the following issues[1].

- Show that eqs. (1) can be scaled in the following form

$$\begin{aligned} \frac{dx}{dt} &= -x + y \\ \frac{dy}{dt} &= -\alpha x + \beta y - x^2 y \end{aligned} \tag{2}$$

- Find the steady state solutions of the model (2) and compute their stability.
- Integrate numerically the equations beyond the instability point(s), if any.
- Comment on the relation between your results and climate variability.

[1] Whenever specific parameter values are needed, take $\alpha = 6.4$ and vary β

2 Extra exercise : partial differential equations

Reading Edelestein-Keshet, Mathematical Models in Biology, chapter 10

A substance X diffuses on a membrane of width ℓ embedded in a medium of fixed concentration C_0 and undergoes simultaneously a first order irreversible reaction

$$X \xrightarrow{k} P$$

1. Write down the reaction-diffusion equation for the instantaneous local concentration C and the appropriate boundary conditions.
2. Determine its stationary solution.
3. Do you expect this solution to be stable and if so, why.