Mathematical Biology - Lecture 6 – Evolutionary Biology

evolution

Darwin and Lamarck, Darwin and Galton, Darwin and Mendel

gene, genotype, phenotype

survival and reproduction; fitness and efficiency

mutation and selection

genetic algorithms, evolutionary economics

Hardy-Weinberg principle

``Both allele and genotype frequencies remain constant in a population without specific disturbing influences´´

Diploid with two alleles: A and a; genotypes: AA, Aa, aa
homozygous, heterozygous

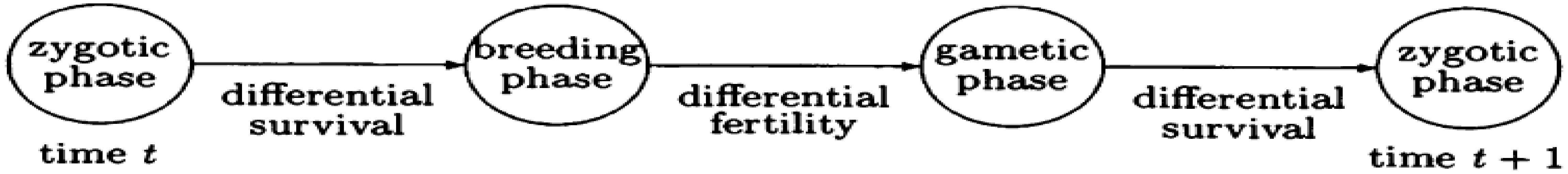
freq(A) = p, freq(a) = q; freq(AA) = p^2, freq(Aa) = $2pq$, freq(aa) = q^2

Punnett square

Hardy's derivation

Positive assortative mating

selection pressure



differential survival and fertility of different genotypes

probability of survival from zygotic to breeding phase $w_x : w_y : w_z$

Fisher-Haldane-Wright equation:

$$p_{n+1} = f(p_n) = \frac{(w_x p_n + w_y q_n) p_n}{w_x p_n^2 + 2 w_y p_n q_n + w_z q_n^2}$$

$$= p_n + g(p_n) = p_n + p_n q_n \frac{\left((w_x - w_y) p_n + (w_y - w_z) q_n\right)}{w_x p_n^2 + 2 w_y p_n q_n + w_z q_n^2}$$

selection pressure

mean fitness of A: $w_p = \frac{w_x p^2 + w_y pq}{p^2 + pq} = w_x p + w_y q$

mean fitness of B: $w_q = \frac{w_z q^2 + w_y pq}{q^2 + pq} = w_z q + w_y p$

Overall mean fitness: $\mathrm{w}' = w_x p^2 + 2 w_y pq + w_z q^2 = w_p p + w_q q$

In terms of the fitness values, the FHW equation becomes

$$p' = \frac{w_p p}{\bar{w}} \quad \text{or} \quad \delta p = \frac{\alpha p}{\bar{w}} = \frac{(w_p - \bar{w}) p}{\bar{w}}$$

selection coefficient s: A advantageous and dominant/recessive

weak selection – replicator equation: $\dot{p} = \alpha p = \left(w_p - \bar{w}\right) p$

game theory

games, strategies, payoffs

prisoner’s dilemma

Nash equilibrium

evolutionarily stable strategy

games with mixed strategies