Project 1

1 Decision-making based on cockroach aggregation

Reading: Some papers (to be downloaded)[1]

In a petri dish, a population N of cockroaches of the same strain has the possibility to choose between two identical (same size) shadowed areas i ($i = 1, 2$). The model describes the evolution in time of the mean numbers of individuals on each area, $X_i (i = 1, 2)$.

$$\frac{dX_i}{dt} = \text{Flux from } j \text{ to } i - \text{Flux from } i \text{ to } j \qquad i, j = 1, 2 \tag{1}$$

Previous experiments show that individuals explore randomly the petri dish and have the same probability to visit the two sites. Moreover, individuals travel immediately from one area to the other, so at the end of the process no individual is out of the shadowed areas.

The probability to move from shadow area i to shadow area j depends on the probability of leaving the concerned area (ξ_i). At each time step, each individual on area i has a probability to leave its actual area which depends on the number of individuals already on i

$$\xi_i = \frac{\alpha}{\beta + X_i^2} \tag{2}$$

accounting for the fact that as X_i is increasing, the probability to leave i is decreasing. The flux of individuals between the two areas can thus be modeled as [2]

$$\phi_{i \to j} = \xi_i X_i = \frac{\alpha}{\beta + X_i^2} X_i \tag{3}$$

1. (*1/20*) Combine eqs.(1)-(3) and fully express the model equations of the evolution on time of the mean values of individuals on site 1 and 2.

2. (*4/20*) Express analytically the steady states of the system. Compute analytically and /or numerically their stability. Draw the bifurcation diagram of the proportion of individuals on a shadowed area against the total population.

3. (*2/20*) Integrate numerically and try to reach all your steady states for different parameter values and initial conditions.

4. (*2/20*) Draw a phase space portrait with many initial condition for parameter values corresponding to the region after the bifurcation. Is the system dissipative or conservative?

5. (*1/20*) How would your model change if the two shadowed areas were not the same size?

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[1] `http://www2.math.uu.se/~snicolis/teaching003.html`
[2] In the future take the following parameter values : $\alpha = 0.06$, $\beta = 6$.

2 The SIR model in epidemiology

Reading: EDELESTEIN-KESHET, MATHEMATICAL MODELS IN BIOLOGY, CHAPTER 6.6 (TO BE DOWNLOADED)[3]

An infectious disease is introduced in a population. Consider the density of susceptible individuals S, of infected individuals, I and of recovered individuals R. The model equations are then

$$
\begin{aligned}
\frac{dS}{dt} &= \mu - \beta SI - \mu S \\
\frac{dI}{dt} &= \beta SI - \gamma I - \mu I \\
\frac{dR}{dt} &= \gamma I - \mu R
\end{aligned}
\tag{4}
$$

where μ is the natural death rate of the population (taken here equal to the birth rate), β the rate of transmitting the disease by contact between an infected and a susceptible and $1/\gamma$ the infectious period.

1. ($2/20$) The **basic reproductive ratio**, R_0 is defined as the mean number of secondary infection cases caused by an infectious individual in a closed population ($\mu = 0$). Express R_0 in terms of parameters γ and β. Compute the steady state value of S under these conditions (closed population) in terms of R_0.

2. ($1/20$) Show that equations (4) can be reduced to the following two parameter model ($\mu \neq 0$)

$$
\begin{aligned}
\frac{dS}{d\tau} &= 1 - bSI - S \\
\frac{dI}{d\tau} &= bSI - gI - I \\
\frac{dR}{d\tau} &= gI - R
\end{aligned}
\tag{5}
$$

3. ($4/20$) Find the steady state of the model (5) and test its stability in terms of the parameter $R_0^* = b/(g+1)$.

4. ($2/20$) Integrate numerically equations (5) to check your steady state. Make a phase space plot showing the approach to the invariant set.

5. ($1/20$) Comment on the biological meaning of the results.

6. (extra) If you have time and want to practice your skills, try to simulate the system of eqs. (4) or (5).

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[3]`http://www2.math.uu.se/~snicolis/teaching003.html`