Project 2 : Predator-prey model

Reading: Some papers (to be downloaded)[1], Britton, Essential Mathematical Biology, §2

Imagine that in an ecosystem, we are interested by the competition between two species. One is a prey (x) and the other is a predator (y). In their general form the evolution equations of x and y can be written as follows

$$\begin{aligned}\frac{dx}{dt} &= \text{birth rate} - \text{death rate due to the presence of the predator} \\ \frac{dy}{dt} &= \text{reproduction rate due to the presence of the prey} - \text{death rate} \qquad (1)\end{aligned}$$

In absence of predators, the population growth and death is modelled as a logistic growth (i.e., *à la* Verhulst) involving two parameters, r (growth rate[2]) and K, the carrying capacity. As for the predators, their reproduction rate (and thus the death rate of the preys) depends on the density of preys caught per time. This can be writen as

$$\eta(x) = \frac{\mu x}{1+x} \qquad (2)$$

where μ can be viewed as a rate of "searching for preys". The reproduction rate of the predators and the death rate of preys due to the predators is then $y\eta(x)$. Finally it is assumed that the death rate of the predators is linear and equal to νy.

1. (*2/20*) Combine eqs. (1)-(2) and the different rates and fully express the model equations of the evolution on time of the density of predators and preys.
2. (*4/20*) Find analytically the steady-states of the system and express the Jacobian matrix and eigenvalues for all solutions.
3. (*5/20*) Express analytically the condition for instability of the non-trivial solution.
4. (*5/20*) Integrate numerically the equations and represent for different parameter values and initial conditions the time evolution of x and y and the corresponding phase portraits.
5. (*2/20*) Looking at your phase space portraits, what is the main differences between this model and the classic Lotka-Volterra predator-prey model (see lecture 3 and papers) ?
6. (*2/20*) Simulate your equations to take into account noise. Take inspiration with the file **simulation.m** given in the first lab.

[1] `http://www2.math.uu.se/~snicolis/teaching003.html`
[2] in the following take $r = 1$